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POTENTIAL FLOW IN TUBE BANKS

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ABSTRACT

This paper presents an analytical solution for potential cross-flow in doubly-periodic in-line and staggered tube banks. The solution, in the form of a power series for the complex potential, is consistent with the well-known solution for a single cylinder. Results are tabulated for in-line square, rotated square, and equilateral triangle geometries for pitch-to-diameter ratio $1.25 \leq s/d \leq 2$. Pressure contours and flow nets are presented for selected cases, together with local pressure coefficient and wall velocity data.

Écoulement potentiel dans les faisceaux tubulaires

Dans ce papier on présente un solution analytique pour un écoulement potentiel aux travers des faisceaux de tubes arrangés double-périodiquement en ligne et en quinconce. La solution, sous forme de série de puissance pour le potentiel complexe, est cohérente avec la solution très populaire pour un seul cylindre. Des résultats sont présentés pour des faisceaux à disposition carrée droite, carrée tournée et à triangles équilatéreaux pour de rapports distance entre rangées/diamètre des tubes $1.25 \leq s/d \leq 2$. Les distributions spatiales des nets de pression et des flux d'écoulement sont données pour certain cas, et sont accompagnées des coefficients locales de pression et les vitesses aux parois correspondantes.

INTRODUCTION

In-line and staggered tube banks are commonly-employed in heat exchangers, Beale [1]. Knowledge of potential flow furnishes insight into some salient features of real flow within the passages of tube banks, where local variations in pressure may be large in comparison to row losses due to viscous effects. Analytical solutions are also of use when calculating the rate of heat transfer, e.g. in liquid metals, Hsu [2], and other working fluids, and in the analysis of flow induced vibrations, Païdoussis [3]. In addition they provide convenient benchmarks for assessing the accuracy of numerical schemes.

In this context, the early method of Knight and McMullen [4] was meritorious, however the authors chose to represent the flow with a series of even and odd complex harmonic functions. The latter could not possibly satisfy the appropriate Dirichlet boundary conditions; e.g. for an in-line tube bank, lines mid-way between rows or columns must be iso-potentials or constant stream-lines, as appropriate. Suh et. al. [5] proposed a solution for the complex potential, $W = \phi + i\psi$, by the introduction of doublets at the location of the centre of each tube. Dalton and Helfinstine [6] and Yamamoto [7] explain the need to introduce secondary, tertiary, and higher-order image-doublets, when such a scheme is employed, in order that the normal velocity be zero, $u_n = 0$, at the cylinder wall. Otherwise the circular shape of the cylinders is lost, and the reader may verify that for tube banks with $1.25 \leq s/d \leq 2$, the distortion is significant. The solution given in [5] also violates the Dirichlet boundary condition in ψ, above.

Chen [8] and Païdoussis et al. [9] obtained Fourier series of W, for bundles of staggered cylinders containing a maximum of 19 tubes (5 rows with 3-4-5-4-3 tubes) and 7 tubes (3 rows containing 2-3-2 tubes) respectively. The experimental results of Žukauskas et al. [10] show that for relatively small numbers of rows, the flow is not fully-developed, thus data from the first and last three rows or so, of the analytical solution, must be discarded as must those from the sides of the bundle: Because multiple summations are performed in [6-9] an alternative more economical solution is sought, for a single unit of the bank, corresponding to a rectangular (in-line) or rhombic (staggered) primitive region; one which exploits the doubly-periodic nature of the bank geometry, and results in a reduction in required computational effort.

RESULTS AND DISCUSSION

The required expression has the form,

$$W = \sum_{n=0}^{\infty} B_{2n+1}\left[\left(\frac{z}{r_0}\right)^{2n+1} + \left(\frac{r_0}{z}\right)^{2n+1}\right] \tag{1}$$

where $z = x + iy$, and $r_0 = d/2$ is the cylinder radius.

The B_{2n+1} coefficients are evaluated numerically, as described in the Appendix. Table 1 is a tabulation of B_{2n+1} for in-line square, rotated square, and equilateral triangle configurations in the range $1.25 \leq s/d \leq 2$. References [11,12] contain tabulations over a wider range, as well as further details about the derivation. Numerical tests showed that Eq. (1) was accurate over a wide range of s/d, though in the limit $s/d \rightarrow 0$, series truncation errors may be significant. In the limit $s/d \rightarrow \infty$, B_1 is the dominant term, and the correct solution for a single cylinder, Milne-Thomson [16], is obtained.

Table 1. B_{2n+1} coefficients for in-line square, rotated square, and equilateral triangular tube banks with $\Delta\bar{\phi} = 1$.

	IN-LINE SQUARE			ROTATED SQUARE			EQUILATERAL TRIANGLE		
n	$s/d = 1.25$	1.50	2.00	1.25	1.50	2.00	1.25	1.50	2.00
1	-0.539462E+00	-0.495840E+00	-0.418097E+00	-0.762914E+00	-0.701223E+00	-0.591278E+00	-0.585575E+00	-0.548782E+00	-0.470648E+00
3	0.437758E-01	0.192961E-01	0.514655E-02	-0.619083E-01	-0.272888E-01	-0.727832E-02	-0.401025E-08	-0.149609E-08	-0.378739E-09
5	-0.261192E-02	-0.263948E-03	-0.702128E-05	0.369381E-02	0.373278E-03	0.992959E-05	-0.140845E-01	-0.441370E-02	-0.673687E-03
7	0.156349E-02	0.322275E-03	0.271509E-04	0.221111E-02	0.455766E-03	0.383972E-04	0.469115E-03	0.164725E-04	0.796347E-07
9	-0.172776E-03	-0.801672E-05	-0.665822E-07	-0.244342E-03	-0.113374E-04	-0.941614E-07	0.569148E-10	0.596372E-11	0.250357E-12
11	0.423873E-04	0.370929E-05	0.981671E-07	-0.599447E-04	-0.524573E-05	-0.138829E-06	0.614662E-04	0.620890E-05	0.168587E-06
13	-0.984698E-05	-0.199089E-06	-0.507929E-09	0.139257E-04	0.281554E-06	0.718320E-09	-0.139395E-04	-0.162829E-06	-0.139991E-09
15	0.147569E-05	0.474841E-07	0.391010E-09	0.208695E-05	0.671527E-07	0.552972E-09	-0.544624E-12	-0.142286E-13	-0.988929E-16
17	-0.517246E-06	-0.424797E-08	-0.323752E-11	-0.731497E-06	-0.600754E-08	-0.457854E-11	-0.326440E-06	-0.853419E-08	-0.410912E-10
19	0.618705E-07	0.604550E-09	0.152044E-11	-0.874981E-07	-0.854963E-09	-0.215022E-11	0.219184E-06	0.833923E-09	0.127180E-12
21	-0.272652E-07	-0.861956E-10	-0.189552E-13	0.385589E-07	0.121899E-09	0.268067E-13	0.643906E-14	0.298319E-16	0.337402E-19
23	0.309936E-08	0.797511E-11	0.594992E-14	0.438316E-08	0.112785E-10	0.841446E-14	0.286518E-08	0.118919E-10	0.100327E-13
25	-0.145051E-08	-0.171478E-11	-0.104621E-15	-0.205133E-08	-0.242506E-11	-0.147957E-15	-0.263770E-08	-0.310053E-11	-0.833099E-16
27	0.173409E-09	0.109909E-12	0.232569E-16	-0.245238E-09	-0.155435E-12	-0.328902E-16	-0.100570E-15	-0.616218E-19	-0.103175E-22
29	-0.775141E-10	-0.341008E-13	-0.556644E-18	0.109622E-09	0.482258E-13	0.786974E-18	-0.388841E-10	-0.171320E-13	-0.244950E-17
31	0.102558E-10	0.161366E-14	0.871333E-19	0.145038E-10	0.228207E-14	0.128675E-18	0.291287E-10	0.962090E-14	0.447577E-19
33	-0.413948E-11	-0.681624E-15	0.000000E+00	-0.585411E-11	-0.963963E-15	-0.408735E-20	0.174686E-17	0.138208E-21	0.000000E+00
35	0.620428E-12	0.256195E-16	0.000000E+00	-0.877418E-12	-0.362315E-16	-0.508147E-21	0.635668E-12	0.266444E-16	0.000000E+00
37	-0.220725E-12	-0.136861E-16	0.000000E+00	0.312153E-12	0.193558E-16	0.000000E+00	-0.326477E-12	-0.269414E-16	0.000000E+00
39	0.377854E-13	0.437500E-18	0.000000E+00	0.534366E-13	0.625673E-18	0.000000E+00	-0.308052E-19	0.000000E+00	0.000000E+00

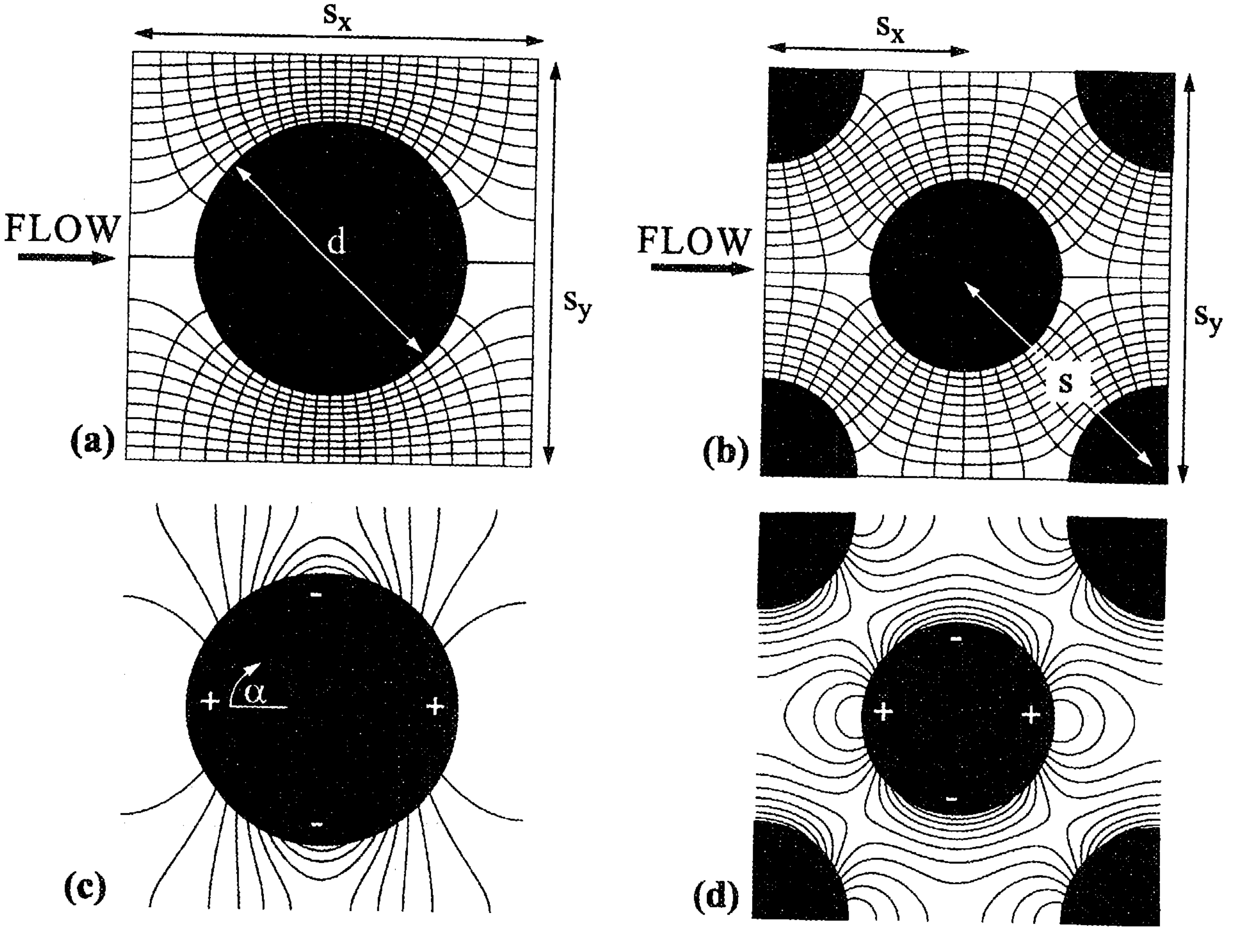

Figure 1. (a) Contours of velocity potential, ϕ, and stream function, ψ, for in-line square and (b) rotated square banks, with s/d=1.5, (c) and (d) show pressure contours for the same configurations.

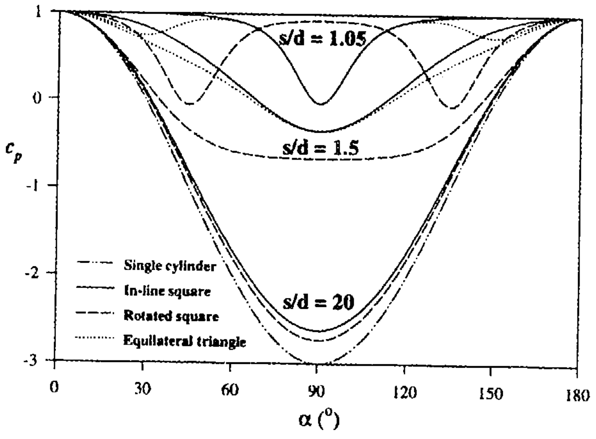

Figure 2. Pressure coefficient, c_p, vs. the angle, α, from the front of the cylinder.

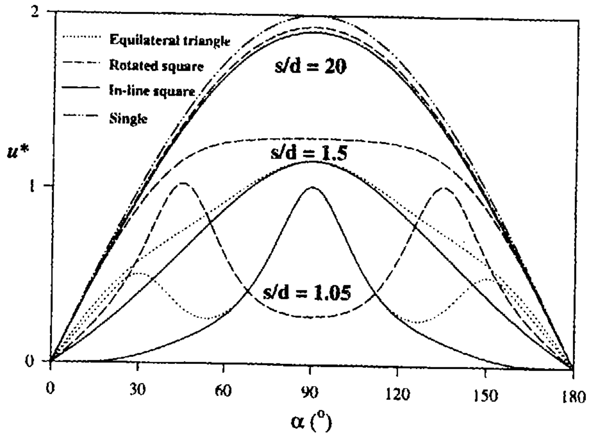

Figure 3. Normalized tangential velocity, $u^* = u_t/\overline{u}_m$, around the cylinder wall. $\overline{u}_m$ is the bulk velocity in the minimum cross-section.

Figure 1 shows contours of velocity potential, ϕ, stream function, ψ, and pressure, p, for in-line and rotated square banks, with $s/d = 1.5$. The latter are calculated from the complex velocity $U = -dW/dz$, using Bernoulli's law. For the in-line geometry, contours of ϕ at $x = \pm s_x/2$ and of ψ at $y = \pm s_y/2$ are straight lines; for both in-line and staggered geometries, the bifucating stream-lines at $y = 0$ form well-defined circles, i.e., both inner and outer boundary conditions are satisfied. Pressure maxima occur at $\alpha = 0°$ and 180°, minimum at ±90°, with a saddle-point in the free-stream between cylinders, Fig. 1(c,d). These 'checkerboard' pressure distributions may possibly contribute to the shear-layer instabilities and wake-switching effects known to occur in viscous flow in tube banks, Beale and Spalding [17].

Figure 2 shows values of pressure coefficient, $c_p = 1-(p(0)-p(\alpha))/\frac{1}{2}\rho\bar{u}_m^2$, while in Fig. 3 normalized velocity, $u^* = u_t/\bar{u}_m$, around the cylinder wall is displayed. For $s/d = 20$, the distributions approach those for a single cylinder; with $c_p = 1$, $u^* = 0$ at $\alpha = 0°$ and 180°, and $c_p \rightarrow -3$, $u^* \rightarrow 2$ at $\alpha = 90°$. The in-line and equilateral triangular bank distributions are essentially identical. For moderate $s/d = 1.5$, there is less variation in c_p and u^*, the maximum u^* of 1.1 to 1.3 corresponding to c_p values of -0.34 to -0.67. For highly compact banks, there are substantial differences in the three distributions: For in-line banks u^* is a maximum of slightly over unity at $\alpha = 90°$. In rotated square banks, two distinct u^* extrema occur across the diagonals at 45° and 135°. Equilateral triangular banks exhibit an intermediate behaviour, with one global extremum at $\alpha = 90°$, and two local extrema at $\alpha = 45°$ and 135°. The two staggered banks exhibit similar distributions in the range 60°-120° (around the minimum cross-section): note the global maximum is the same value for all geometries.

CONCLUSIONS

A theoretical solution for potential flow in tube banks was derived in terms of a power series for complex potential, W. Results were tabulated for the three main types of bank configuration, in-line square, rotated square, and equilateral triangle. For very large pitch-to-diameter ratio, s/d, the solution approaches that for a single cylinder, regardless of bank type. For s/d in the range 1.25-2, typical of many engineering applications; velocity and pressure distributions differ substantially from those obtained from the solution for a single cylinder with fluctuations in c_p being substantially less pronounced than for a single cylinder, and somewhat dependent on geometry. Velocity and pressure distributions were found to be more affected by s_y/d than s_x/d. For very compact banks, velocity and pressure distributions were found to be strongly dependent upon bank type: For staggered banks, the minimum cross-sectional area lies across the diagonal at $\alpha = 45°$ and 135°, consequently the velocity is a maximum and the pressure is a minimum in this region with the flow and pressure distributions exhibiting diagonal symmetry as $s/d \rightarrow 0$. The results are important indicators for real-flow, where local pressure variations around the tube wall are large in comparison to the row pressure drop.

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Prof. J.P. Gostelow provided some stimulating discussion on the subject matter described in this paper.

APPENDIX: MATHEMATICAL DETAILS

Let it be assumed that the cylinders lie in the complex plane at $\Omega_{m,n} = 2m\omega_1 + 2n\omega_2$ where $\omega_1 = s_x/2$, $\omega_2 = is_y/2$ (in-line) or $\omega_1 = s_x/2 + is_y/2$, $\omega_2 = s_x/2 - is_y/2$, (staggered), s_x and s_y are stream-wise and crosswise pitches, and W is composed of linear and doubly-periodic terms. Assume,

$$W = -2\Delta\bar{\phi}\frac{x}{s_x} + \sum_{s=0}^{\infty} A_{2s+1} W_{2s+1} \tag{2}$$

$\phi_{2s+1} = \mathrm{Re}(W_{2s+1})$ are zero on appropriate symmetry lines, and A_{2s+1} are selected to satisfy $u_n = 0$ at r_0. For problems involving one cylinder, the use of circular harmonics is well-known Batchelor [13]. W_{2s+1} are chosen similarly, as derivatives of sinks at $\Omega_{m,n}$, non-dimensionalized according to,

$$W_{2s+1} = \frac{s_x^{\;2s+1}}{(2s)!}(-1)^{s+1}\left(\frac{1}{z^s} + \sum_{m,n}{}' \frac{1}{(z-\Omega_{m,n})^s}\right) \tag{3}$$

The prime-sign indicates the value $m = n = 0$ is excluded. This may be expanded as a Laurent series,

$$W_{2s+1} = \left(\frac{z}{s_x}\right)^{-2s-1} + \sum_{n=0}^{\infty} \alpha_{2n+1\,2s+1}\left(\frac{z}{s_x}\right)^{2n+1} \tag{4}$$

The double summation in Eq. (2) has been reduced to a single one, resulting in significant saving in the computational time required to obtain a solution. Thus,

$$\phi_{2s+1} = \rho^{-2s-1} + \sum_{n=0}^{\infty} \alpha_{2s+1\,2n+1}\rho^{2n+1}\cos(2n+1)\theta \tag{5}$$

where $z/s_x = \rho e^{i\theta}$ With $\partial\phi/\partial\rho = 0$ at $\rho = \mu$, where $\mu = r_0/s_x$. It can readily be shown that W is of the form Eq. (1) with,

$$B_{2n+1} = \frac{A_{2n+1}}{\mu} \tag{6}$$

The A_{2n+1} coefficients are evaluated as follows,

$$A_{2n+1} = \sum_{r=0}^{\infty} A_{r\,2n+1} \tag{7}$$

$$A_{01} = -2\Delta\bar{\phi}\mu^2, \quad A_{0\,2n+1} = 0, \quad n \geq 2 \tag{8}$$

$$A_{r\,2n+1} = \mu^{4n+2}\sum_{s=0}^{\infty} \alpha_{2s+1\,2n+1} A_{r-1\,2s+1}, \quad r > 1 \tag{9}$$

with,

$$\alpha_{2s+1\,2n+1} = \frac{(2n+3)(2n+2)}{2s(2s-1)}\alpha_{2s-1\,2n+3} \tag{10}$$

$$\alpha_{1\,2k-1} = -\frac{c_k}{{s_x}^2} = -\frac{1}{{s_x}^2}\frac{3}{(2k+1)(2k-1)(k-3)}\sum_{n=2}^{k-2}(2n-1)(2k-2n-1)c_n c_{k-n}, \quad k \geq 4 \qquad (11)$$

where $c_k = \sum_{m,n}{}' \, 1/\Omega_{m,n}{}^{2k}$ are the coefficients of the Laurent series for Weirstrass' ζ- function, and c_1, c_2, and c_3 are computed according to Ling and Tsai [14,15].

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