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A NUMERICAL SIMULATION OF WHEEL WEAR

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Abstract

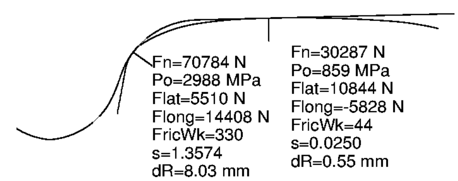
The conditions at the wheel/rail contact have a dramatic impact on safety, ride quality and cost of maintenance. Appreciating its importance, the US Federal Railroad Administration (FRA) initiated a project to improve both the understanding of and application of an improved wheel/rail interaction on Amtrak's Northeast Corridor. One item delivered as part of this programme is an improved wheel profile. This profile is designed to reduce wheel flange wear for the Amtrak vehicles without introducing wheelset hunting.

The comparative wear performance of the new wheel has been evaluated against existing wheels by simulating the wheel-rail interaction over 800 kilometres of running. The simulation is based on the principle of pummelling and employs a quasi-static curving model. The model is validated through comparison with NUCARS predictions. The improved wheel design is found to exhibit a 25% reduction in flange wear compared with the current wheel profile.

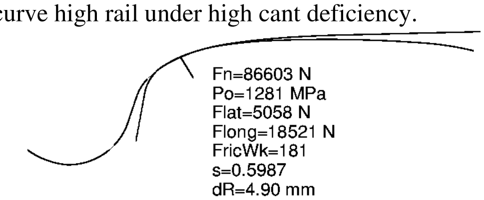
INTRODUCTION

Amtrak's Northeast Corridor (NEC) is a mixed traffic system that operates 240-km/h Acela passenger trains between Washington and Boston on tracks shared with much slower and heavier freight trains. The traffic mix runs the gamut between slow moving heavy axle load trains operating at considerably under the balanced elevations (maximum 150 mm) and high speed trains running at up to 175 mm of cant deficiency over an alignment that includes curvatures between 2 and 8 degrees for almost 4% of its length. This operation places unique demands on the wheel/rail system and provides a particularly challenging maintenance environment in which to operate a high-speed service.

The Acela trainsets currently employ the "Amtrak Standard" wheel profile, which is a 1:40 coned wheel based on an early AAR profile. This shape has proven relatively stable from a dynamics perspective but suffers from high rates of wheel flange wear. Following extensive analysis of the existing worn wheel shapes and various aspects of the wheel/rail interaction, the authors engineered an improved wheel profile, called the AMTK-NRCC, to improve curving to minimise wear, but maintain the adequate stability of the Amtrak-Standard wheel. The improvements in curving performance when run against the average, mild curve high-rail profile with high (150 mm) cant deficiency are summarised in Figure 1.



A) Amtrak Standard wheel on the Amtrak mild curve high rail under high cant deficiency.



B) AMTK-NRCC wheel on the Amtrak mild curve high rail under high cant deficiency.

	Amtrak-Standard	AMTK-NRCC
Contact type	Non-conformal 2-pt	Conformal 1-pt
Max normal contact stress (Po)	100%	≈50%
Total frictional energy, high rail	100%	48%
Steering Moment	100%	215%
L/V (low Rail)	100%	92%
Effective Conicity	1:40	Unchanged

Figure 1: Comparison between the Amtrak-Standard AMTK-NRCC wheel shapes.

Before field-testing could be approved, a much more detailed analysis was required to not only qualify the performance improvements but also to assure no problems are encountered when it is run against the broad spectrum of conditions that exist on the NEC. The authors have applied PUMMEL™, their wheel-rail analysis process, to this task.

The authors define pummelling as the process of controlling the distribution-of and severity-of wheel/rail contacts. This control is exercised by managing the wheel and rail profiles, with due consideration to the type of bogies and the contribution of friction management strategies. Pummelling is used to synergistically manage both the amplitude and frequency of contact at different points on the running surfaces, minimising the instantaneous wear and ensuring a favourable distribution over the surface. For a wheel, the primary benefit of this strategy is that the wheel profile geometry remains stable for a longer period of time. This stable profile is slower to wear and slower to hollow, thereby retaining its designed favourable shape as long as possible – a shape that was presumably prescribed in the first place to minimise wear and/or fatigue, or to provide the stability required.

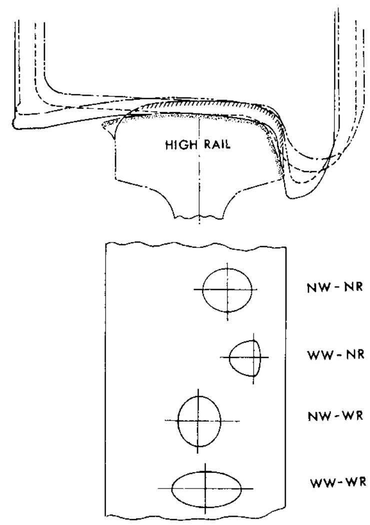
A number of software tools have been developed by engineers at the Centre for Surface Transportation Technology (CSTT) of the National Research Council of Canada for evaluating the wheel/rail interaction. PUMMEL™ is simply a variant that facilitates this analysis for a large number of cases. In a typical pummelling analysis, a new rail profile design might be subjected to loading by 1000 to 3000 axle passes (e.g. Figure 2), using wheel profiles measured from appropriate vehicles. The vehicles that are modelled to run against that rail will each have properties representative of that vehicle type, and the types of vehicles are characteristic of the fleet that the rail is known to encounter. In the case of a newly designed wheel, it will be placed under an appropriate car model and run over a representative sample of measured rail shapes along a track profile representative of the actual railway. In this paper we discuss a wear simulation that was undertaken to compare the performance of the newly designed AMTK-NRCC wheel with the existing Amtrak Standard wheel[1].

THE CSTT PUMMELLING MODEL

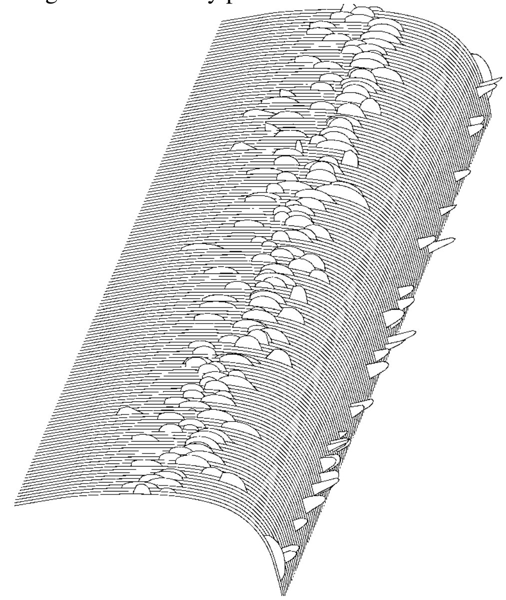
The CSTT pummelling model includes a quasistatic curving simulation (QCS, [1]) that positions the two linked wheelsets of a bogie onto the rail to balance the gravitational, creep and suspension forces. Inputs to the model include the rail profiles (which have been measured at 1.2 - 3 metre intervals over 1600 km of track), friction coefficients, detailed measurements of super-elevation, curvature and track gauge, braking/acceleration torque on each wheelset, and the shear and bending stiffness of the vehicle suspension.

[1] Note that track and wheel tread inspections show that rolling contact fatigue is of little practical importance under the current Amtrak maintenance regime. Validation of stability, besides a simple effective conicity approach, awaits the development of a reliable vehicle model.

This model was used to "run" the new and existing wheel profiles over the length of the Northeast corridor and predict the wheel/rail performance at each track location. Performance is evaluated through a variety of indices including steering moment, conicity, contact stress and wear. This paper focuses on wheel performance with respect to wear only.



A) The shape and position of the contact patch depends primarily on the rail/wheel profiles and curving demands at any particular time.



B) Twenty-two freight car bogies with wheels at various stages of wear contact the rail at a range of locations with differing severity.

Figure 2: The location and amplitudes of contact are critically dependent on the specific wheel and rail shapes.

The QCS is capable of extremely detailed analysis that includes rail rotation associated with lateral loads, a numerical elastic contact model (following the outline of Kik and Piotrowski [2]) and a non-linear traction creepage characteristic [3]. In a typical analysis, these items are all disabled due to the dramatic increase in computation time otherwise incurred. But as Knothe and Le The show [4], the use of a numerical wheel/rail contact model *can* yield substantial differences in the wear pattern compared with the elliptical model. The authors plan to examine this issue in future studies. In the current study, a Hertzian calculation is employed, with an on-line, adaptive curve-fitting algorithm that determines the profile curvature in the vicinity of the point of first contact. The creep force calculations employ Kalker's USETAB [5], with the addition of a μ/0.6 multiplier on the creep coefficients, as per British Rail [6].

Ideally, a fully dynamic analysis would be employed in the pummelling analysis - and indeed that is the intention for the future. A dynamic analysis *will* provide a more variation in the relative position of the wheel and rail profiles and may therefore have a significant impact on the wheel/rail interaction calculations. But another perceived benefit, the inclusion of dynamic load, is not believed to be of significant advantage since the maximum normal contact stress for elliptical contacts (P_o) is proportional to the $1/3^{rd}$ power of the normal load. Even a doubling of the contact load (either dynamic or static) results in only a $2^{1/3}$ => 26% increase in stress. To verify this presumption the QCS was used to assess the relative importance of measured track geometry and measured rail profiles on the pummelling output.

A bogie with unworn wheel profiles was run through the body of a 650 metre long, 4-degree curve. Rail profiles were measured at 212 locations through the curve. Complete track geometry data is available at 0.3m intervals. The typical train speed is 80 kph for Amtrak vehicles over this curve. The four cases of Table 1 were examined. Case 1 involves only a single super-position with the mean track geometry and an average of the high and low rails. The remaining cases involve many simulations - 212 in this example. The output from the QCS pummelling analysis is shown in Figure 3. When measured track gauge, curvature and super-elevation were applied, it was found that the QCS predicted only small changes in the instantaneous vertical and lateral wheel-rail load. Consequently the normal contact stress changed little along the curve (Figure 4, Top). The effect of using measured rail profiles is more dramatic (Figure 4, Bottom) – the contact distribution changes significantly. Since computation time for the last three cases is effectively the same, the most accurate technique - case 4 – is the standard approach in the CSTT pummelling analysis.

Current dynamics packages *are* reasonably adept at conducting simulations over long sections of track, but frequent changes to the rail profiles (e.g. every metre) are very cumbersome to implement. The authors believe that the use of a statistically representative sample of measured rail and wheel profiles, in combination with measured track geometry parameters, provides more realistic assessments of the wheel rail interaction than current dynamic analysis systems are capable of doing, *even given the limitations of a quasi-static analysis*.

	Track geometry	
Rail Profiles	Mean values	As-measured
Numerical Average	Case 1	Case 2
As-measured	Case 3	Case 4

Table 1: Test matrix to evaluate the importance of rail profiles on pummelling.

COMPARISON BETWEEN NUCARS™ AND PUMMEL™

A comparison was made between the quasi-static analysis of the CSTT pummelling model, called PUMMEL™, and a full dynamic simulation as implemented by NUCARS™. The same measured track geometry (including leading and trailing spirals on either end of a four degree curve) were input into both software packages, and using the unworn Amtrak-Standard wheel on the average high and low rail profiles, the wheel/rail interaction was evaluated at many points. The results are shown in Figure 5. The angle of attack and lateral position results are very similar for both models, with the NUCARS™ analysis showing a much more dynamic system, as expected. The quasi-static model nicely captures the position and amplitude of the many contact points, see Figure 5C&D. The inability of the quasi-static model to realistically model spiral negotiation or consider roughness in super-elevations, lateral and vertical profile is primarily responsible for the small difference in tread-wear and normal stress distributions. The slight difference in the wheel flange contact position is due to difficulties in interpolating the NUCARS™ output data.

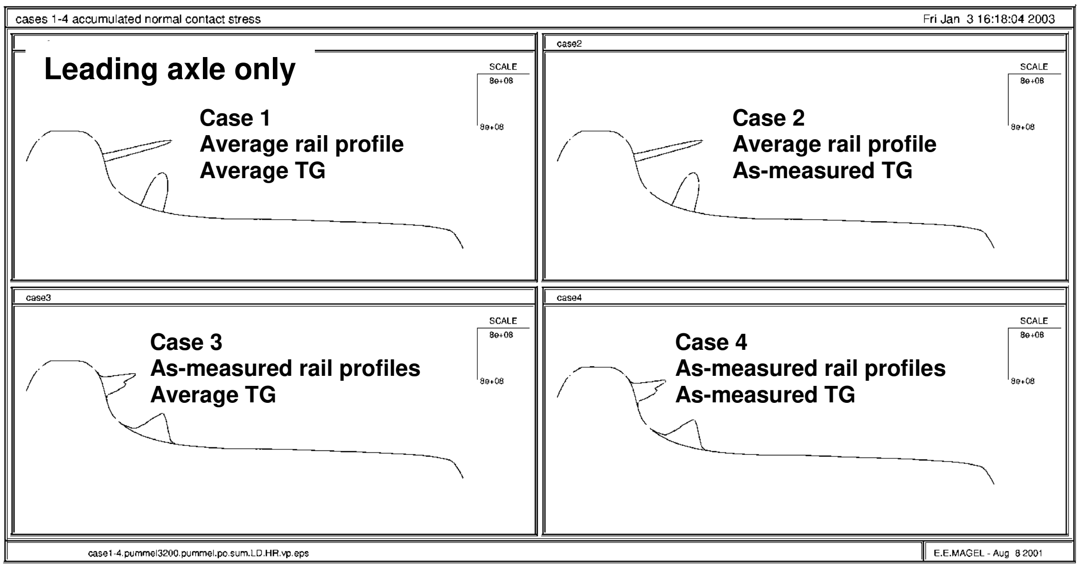


Figure 3: Pummelling results for cases 1-4 of Table 1.

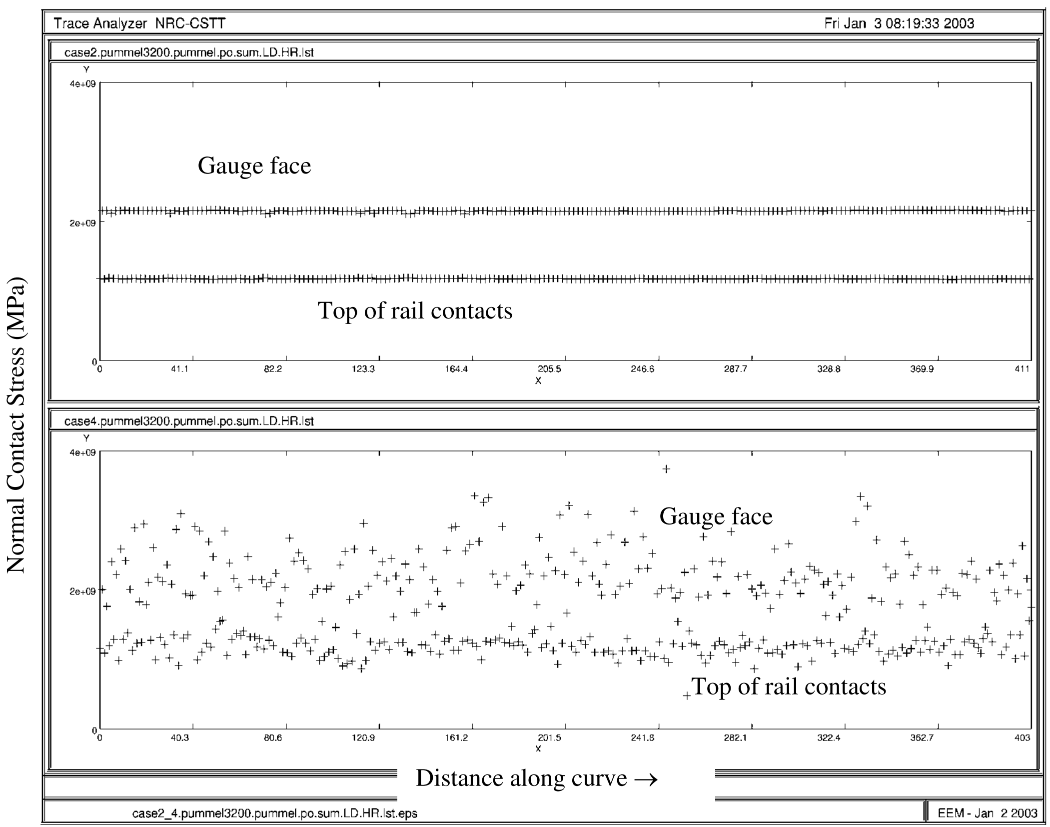
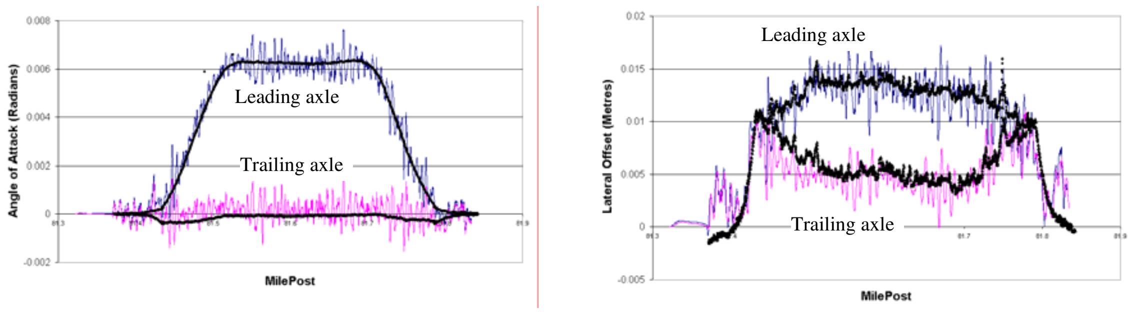
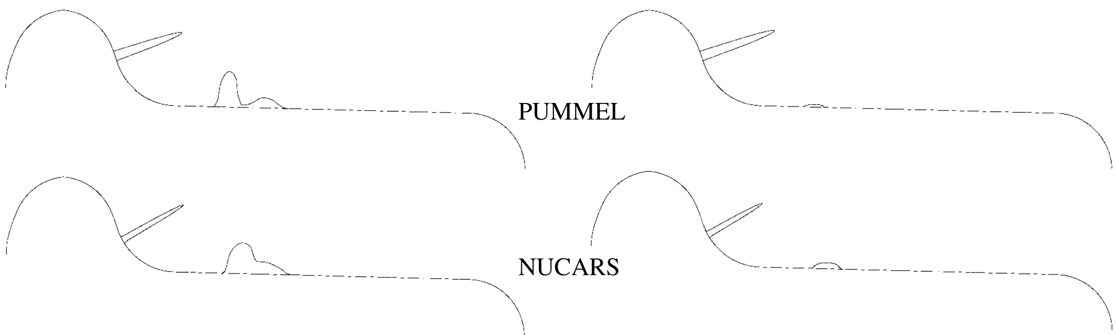


Figure 4: Normal contact stress (P_o) for cases 2 (Top) and 4 (Bottom). Case 1 uses the same rail profile throughout the curve but with as-measured track geometry, while case 4 includes measured rail profiles at about 200 intervals through the curve. Small track perturbations have little impact in a quasi-static analysis, whereas variations in the rail profile have a significant impact in any type of wheel/rail analysis.



A) Wheelset angle of attack (QCS is the dark lines) B) wheelset lateral position (QCS is the dark lines)



C) Distribution of normal contact stress (leading axle, high wheel) D) Distribution of frictional work (leading axle, high wheel)

Figure 5: Comparison between the predictions of CSTT's quasi-static PUMMEL™ model and NUCARS™

SIMULATING WHEEL WEAR ON THE AMTRAK NORTHEAST CORRIDOR (NEC)

The available track information for the Northeast Corridor is of particularly good quality, which greatly assists in achieving a good wear simulation. Through its partnership with the Federal Railroad Administration, Amtrak and Ensco Inc., CSTT was able to access a number of data streams for use in its wear simulation:

- Rail profiles measured at 1-2.5 metre intervals[2] were available for almost all of the north- and south-bound runs totalling 1600km of track.
- Track geometry data is updated every 2-4 weeks and stored at one foot (254mm) intervals for the mainline tracks in the Northeast Corridor. Currently the pummelling model uses the measured super-elevation, track gauge and curvature in its simulations.
- Vehicle characteristics
  - An Amfleet coach ("conventional" Amtrak rolling stock dating to the late 1970's) is used in this simulation. This vehicle regularly runs up to 175 km/hr. Amtrak provided the values of axle load, centre of gravity, wheelbase and axle back-to-back.
  - Although the pummelling model is able to simulate flexible suspension bogies, this work was undertaken using a rigid truck of Amfleet dimensions, since the Amfleet bogie *is* very stiff and the solution times are much quicker when compared with a flexible truck model.
- The braking torques and vehicle speeds were taken from the Amtrak T-16 inspection car, which is effectively an Amfleet coach. The strain-gauged wheels on this car provide measurements of the longitudinal forces at the contact patch that are converted into a net torque on the wheelset by summing the left and right signals (to remove the equal-and-opposite creep components) and multiplying the remaining signal by the average wheel radius. Since tread brakes were disabled on the T16 (to avoid thermal damage to the strain gauges), the disc-braked torque was scaled up according to the 60:40 (disc:tread) designed brake ratio used under normal running to more closely represent typical loads on the Amfleet rolling stock. As the T-16 vehicle is towed by a revenue service

[2] High-speed (approx. 40 mph) continuous rail profile measurements were made by Advanced Rail Management Corp. using their hy-rail mounted, laser-based rail profile measuring system.

train, its braking duty and speed profile should be representative of typical NEC service.

- The one parameter not yet properly accounted for is the friction characteristic. Friction values measured several years earlier with the Portec high-speed (30-40 km/hr) rail tribometer were used to justify the levels for this run (μ=0.5), since those levels were considered to still be applicable today on most of the track segments. Ideally, a more recent high-speed run, or even regular measurements with a portable push-tribometer would be used. On the other hand, there remains considerable debate on the validity of those values in high-speed rail/wheel contacts. It is also well known that the traction coefficient decreases with increase in speed e.g. [7]. Improved characterisation of friction at the rail/wheel contact patch remains one of the "final frontiers" in modelling of the wheel/rail interaction.

A typical screen view of the data management system used to warehouse, view and export the many data streams employed in the model is shown in Figure 6.

Worn wheel data was taken from a revenue Metroliner car that was part of an earlier study. Wheel profile measurements were made with a Miniprof™ instrument after approximately 0, 1600, 2900, 5100 and 15600 km of running since the last retruing. All eight wheels of the case study car were measured. Although the individual wheel measurements are used in the pummelling runs, for the purpose of determining wheel wear rates, the wheels from each time period were aligned and an average worn wheel profile generated. The average profiles were then used to determine the progression of wear with kilometres of running through the NEC (Figure 7).

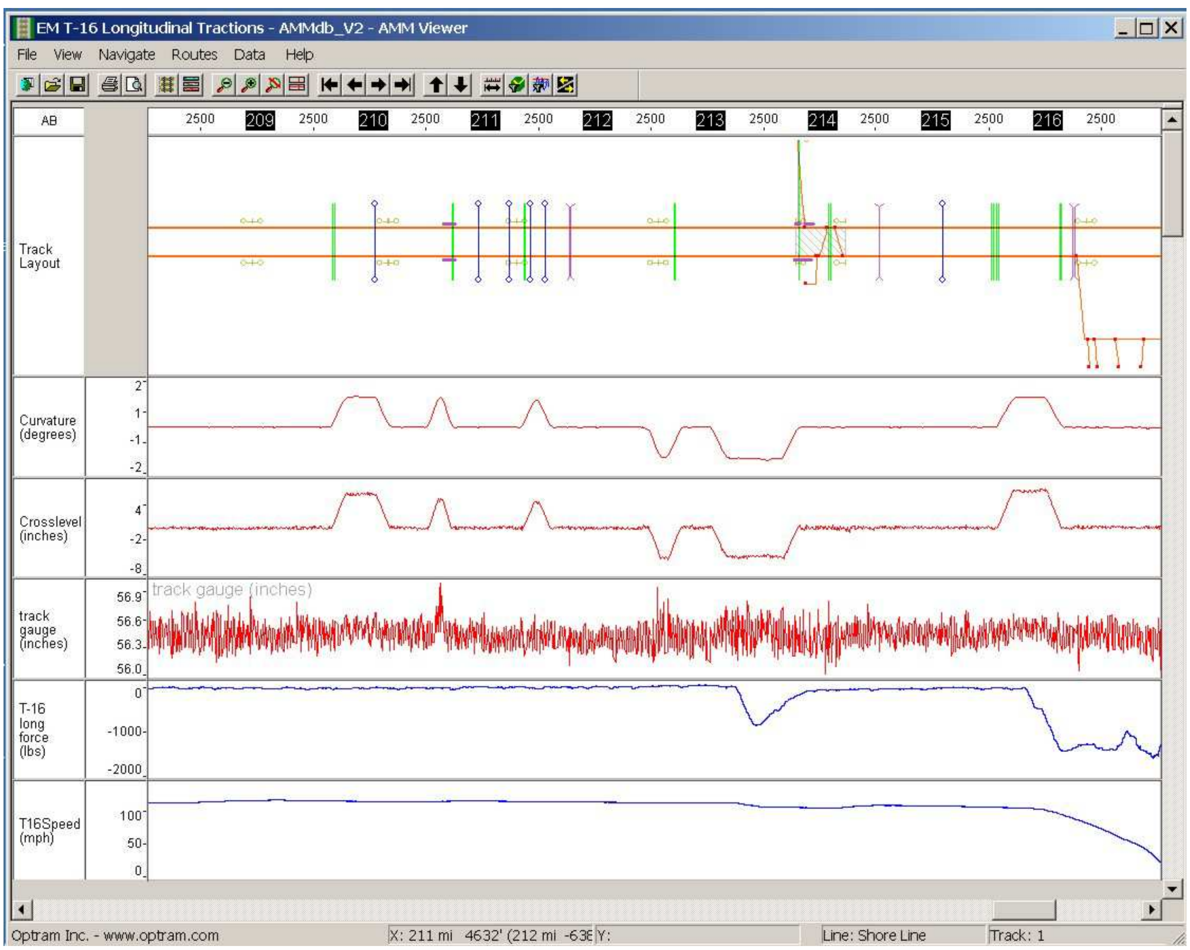


Figure 6: An example of the track and operating conditions for the "test" vehicle. This screen-capture, using Amtrak's AMM viewer shows ten miles (16km) of typical track used in the simulation. Also shown are the net wheelset longitudinal force and speed of the test car.

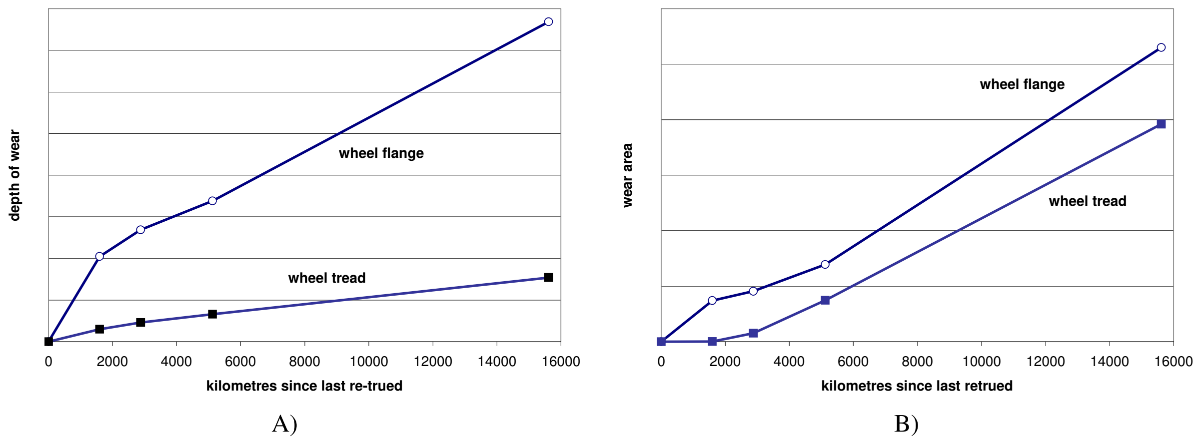


Figure 7: Progression of wear on the Amfleet vehicle with kilometres of running, in terms of A) depth and B) area.

Before conducting a full system run, several smaller cases were examined over a 51km stretch of track. Because of the extremely large data set, calculations are made only every 8 metres or so, resulting in a total of about 5800 analyses for each case.

1. The Amtrak Standard wheel profile *with* braking torques applied. This is the base-case scenario that represents the current configuration on the NEC.
2. The Amtrak Standard wheel *without* wheelset torques was run to investigate the practical influence of braking torques on wheel/rail wear for a coach vehicle.
3. A measured set of wheels that were initially the Amtrak Standard profile, but after 1600 km of running. These wheels were measured with a MiniProf™ in an earlier FRA project.
4. The AMTK-NRCC wheel, with wheelset torques applied. This shape is more conformal to the rail in curves and is therefore expected to provide a noticeable reduction in wear as compared with the base case (#1 above).

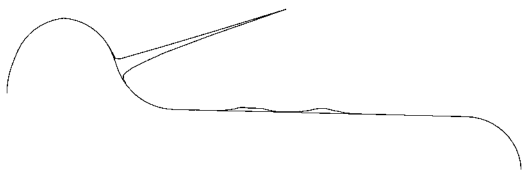
The results of these four cases are summarised in the vector plots of frictional work distribution shown in Figure 8. The removal of braking torques was found to have surprisingly little influence – compare Figures 8A and 8B. In fact, the difference in frictional work at the wheel tread was only about 2%, while at the gauge-face it was negligible. Subsequent consideration of the braking conditions showed that the frictional demands were not very large on these coach cars. Creepages in the order of 0.2% are sufficient to generate the measured longitudinal forces at the wheel/rail contact (about 1000-lbs/4550N total) for a friction coefficient of 0.4 and wheel load of 61,000N. Although the braking torque data *was* used through the remainder of the analyses, the improvement to the wear simulation for a coach car with non-powered axles will not justify the exhaustive effort required to both obtain and synchronise the data for future efforts. It should be noted that the sustained traction borne by powered wheelsets *will* likely play a significant role in wear and should not be neglected for that case. The AMTK-NRCC wheel profile in Figure 8D experienced noticeably less flange contact than both the worn and unworn Amtrak Standard wheels.

The comparison between Figure 8B and C shows that the wheel shape after the first 1600 km of wear has relatively little impact on the distribution of wear in the next cycle.

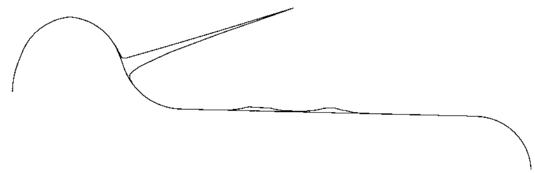
A last test run was made to determine the impact of sample frequency. An interval of 30m was used instead of the "normal" 8m. The output plots were nearly identical, with the flange spike being 3.4% shorter for the larger step size. A 30-metre interval was used in the remainder of this study.

A "complete", 800 km wear simulation

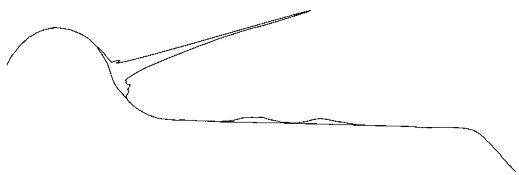
The CSTT pummelling model was next applied to evaluate the performance of several wheels as they ran the entire length of the Amtrak Northeast corridor from Washington to Boston. Presuming similarity, and due to time restraints, the return journey was not simulated in this exercise. Runs were performed at approximately 30m intervals over the 800-km from Washington to Boston, a total of over 16000 distinct bogie/track simulations and about 34000 contacts per wheel. As noted in the preceding section, the change in wear distribution over an 800 km distance is minor, and so each run was made with the same wheel shape for the entire 800-km. Five sets of wheels were investigated: the unworn AMTK-NRCC wheel, the unworn Amtrak Standard wheel, and the same set of four measured worn wheels after 1600, 5120, 15600 kilometres of running.



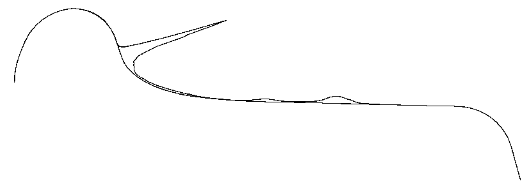
A) Amtrak Standard wheel without braking torques applied



B) Amtrak Standard wheel profile with braking torques applied.



C) A lightly worn (1600-km of running) Amtrak Standard wheel, with braking torques applied.



D) The AMTK-NRCC wheel profile, with braking torques applied.

Figure 8: Calculated frictional work distribution for four different wheel profile/braking-torque combinations after running over 51km of Northeast Corridor track.

The distributions of frictional work for these runs are shown in Figure 9 for the right wheel of the leading wheelset. As the wheel profiles wear-in, the distribution of wear over the flange widens, reducing the severity of the flange peak.

The AMTK-NRCC profile, which is partly based on the actual worn geometry of Amtrak Acela wheels, shows convincingly the lowest level of flange "wear". This wheel profile presents a more conformal contact geometry on average with the gauge corner of the rails than the Amtrak Standard and exhibits a broader contact distribution. The total frictional work dissipated at the flange is 25% lower, and the peak amplitude is less than half of that on the unworn Amtrak Standard wheel. Statistically, the CSTT profile encountered 1/3rd less 2-point contacts on the NEC. The benefits of the new wheel shape are expected to increase further when matching rail profiles are designed and implemented at a later stage of the program.

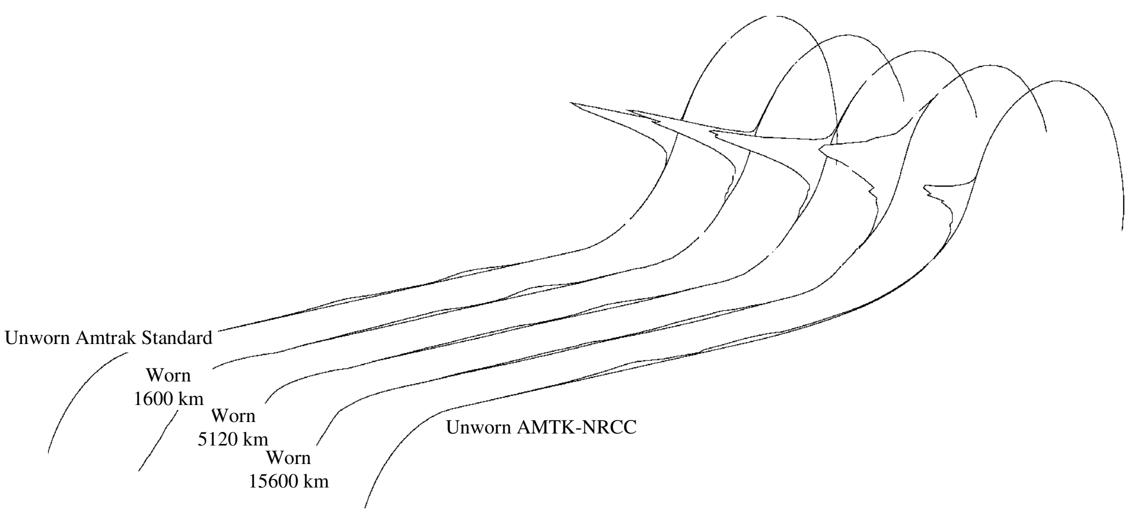
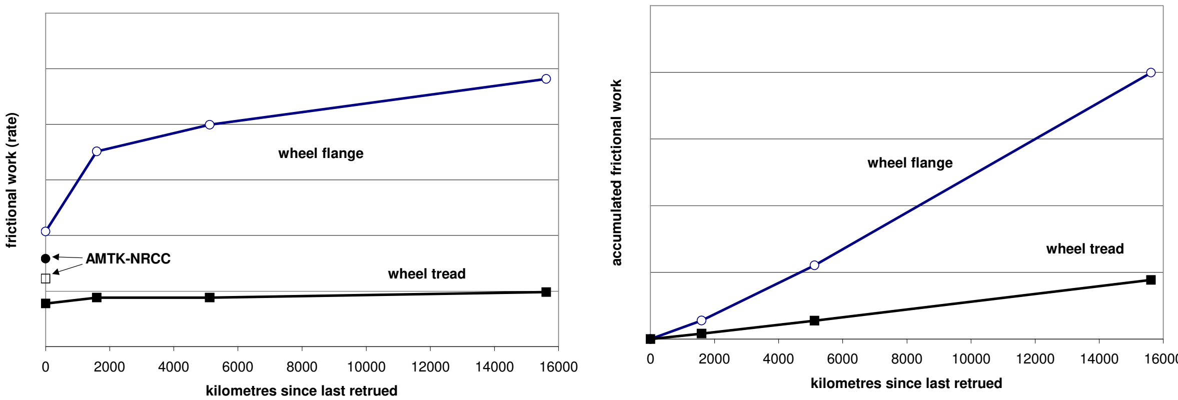


Figure 9: Frictional work dissipated at the wheel tread for the full-length (800-km) pummelling runs. Plots are shown for the leading axle, right wheel only.



A) Rate of frictional work dissipated at the wheel tread and wheel flange.

B) Total frictional work dissipated at the wheel flange and tread.

Figure 10: Predictions of the frictional work dissipated at the flange and tread of the wheel at various stages of wear, using measured wheel profiles on vehicles that run the length of Amtrak's Northeast Corridor.

Correlating frictional work with wear

The conventional understanding is that the rate of wheel/rail wear is proportional to the frictional energy (tangential force x sliding distance) dissipated in the contact zone [8,9]. The constant of proportionality varies over several orders of magnitude and depends highly upon the particular properties of the wear system, including the strengths of the surfaces, the rates at which particles are extruded from the interface, the normal stress levels and slip velocities. The wear map of Lim et al [10] shows that the wear mechanism varies with sliding speed and contact pressure.

Shear induced failure (delamination) has been suggested by several authors e.g. [9] and [11] as the dominating mechanism of wear at the wheel/rail contact. In other studies, wear is classed as Type I to Type III [9] or even type IV [12], with a larger number referring to mechanisms of increasing wear rates. The rail-gauge-face / wheel-flange-wear mechanism is more closely allied with Type III wear while the top of rail is typically subject to the milder Type I, and sometimes Type II, wear rates.

We expect the wear at any point on the wheel surface to be proportional to the frictional work dissipated there. The results of Table 2 thus suggest that the wear coefficient at the tread is about seven times (i.e. 36.5/5) that at the flange. But since the small contact zones at the flange are particularly sensitive to curve-fitting errors (and would therefore benefit greatest from a non-Hertzian model) these intense concentrations of frictional work at the flange should be considered qualitative only. A much more robust technique is to compare the areas of metal lost with the total frictional work dissipated over the flange and tread. This shows that the wear coefficient at the tread is about 4.5/1.35≈3 for the field worn Amfleet wheels under fairly dry friction conditions.

	Wear		Frictional work	
	Area	Depth	Rate	Peak
Flange	1.35x	5x	4.5x	36.5x
Tread	1x	1x	1x	1x

Table 2: Comparison of the levels of wear and frictional work for the flange and tread regions on the wheel, after 15600 km of running

PUMMEL™ as a wheel profile design tool

The Amtrak Standard wheel exhibits a strong 2-point contact on the current worn high-rail shapes and therefore has very little ability to steer in curves. As it wears-in, the rate of frictional work dissipation is predicted to progressively increase (Figure 10A), at least for the first 15000-km examined in this study. Broadening of the contact band offsets this increase in frictional work, such that flange thinning and tread wear remain relatively constant after the first 1600-km (Figure 7). Field observations show that eventually, after about 65000-km of running, the flange root of the Amtrak Standard profile wears in, after which some small number of high-rail contacts can be carried by better steering, lower wear, single point contacts. The new CSTT designed wheel shape, in comparison, carries about 1/3 of the high-rail contacts with single point contact right from the beginning. The AMTK-NRCC wheel therefore is predicted to exhibit better steering characteristics and lower flange wear than any of the unworn or lightly worn Amtrak Standard wheels.

But while wheel wear *is* an important consideration, improvement in wear performance must be achieved without compromising other important characteristics. The CSTT pummelling model calculates several parameters for each wheelset superposition, including contact stresses, effective conicity, rolling radius difference, tractions and steering moments. This paper focussed on wheel wear, since that is the primary focus of this design program. A freight railroad may be most concerned with plastic flow and rolling contact fatigue, while another high-speed rail program may focus on ride quality. PUMMEL™ can be used to undertake the same sort of detailed analysis that others perform against one or two nominal shapes [e.g. 13], but to carry it out at many thousands of locations along the track. PUMMEL™ evaluates the wheel shape against the full distribution of measured rail shapes and track geometry, which in the authors' opinion can provide a comprehensive picture of the expected wheel performance characteristics.

CONCLUSIONS

A quasi-static curving analysis of the wheel rail interaction has been developed and its output compared against NUCARS™. The axle alignment, normal contact forces and distributions of wear all show good agreement. CSTT's PUMMEL™ model was used to simulate the wheel-rail interaction over roughly 16000 measured rail pairs during an 800-km run of the Amtrak Northeast corridor. This paper discusses specifically the wear performance of several wheel shapes. A wheel profile designed by CSTT is predicted to reduce wheel flange wear by at least 25% when compared to the existing Amtrak Standard wheel profile.

All models are only as good as the data on which they depend. The simulation described in this paper uses very detailed measurements of track geometry and profiles but unfortunately the friction conditions are currently known in only very rough terms. Since the friction characteristic at the wheel/rail interface dramatically affects many facets of a vehicle/track simulation including the contact position, tractions and damage at the contact, one focus of future work will be to better define the friction characteristic.

Acknowledgements

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