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Kinematics of Filament Winding on Slender Mandrels

INTRODUCTION

Filament winding is now widely used to make reinforced plastic (RP) structures featuring continuous fibers. The special case of helical filament winding involves a lathe-type winder in which the filament, fed from an axially-travelling carriage, is wound on a simply rotating mandrel.

Helical filament winding in its simplest form is used to make long circular-cylindrical pipes in which specific fiber angles $+\theta_\mu$ and $-\theta_u$ are desired over the longest possible length. The process requires a "start-up" stage as well as a "reversal" stage at each end of the mandrel.

The analysis of the kinematics of the winding process, i.e., the determination of the filament angle as a function of mandrel and carriage velocities as well as the geometry of the set-up, is trivial in the case of circular-cylindrical mandrels for regions away from the ends where start-up and reversal occur. It does not appear, however, that an analysis of the kinematics of start-up and reversal has been reported.

The objective of our study was to conduct a theoretical analysis of the kinematics of start-up and reversal in such a manner that results in the form of mathematical expressions or generalized charts would be helpful in the development of computer-aided design or manufacturing programs (CAD/CAM), as well as in the rapid "rough and ready" estimation of process parameters. An experimental verification of the results in selected cases was also carried out.

This study also supports our ongoing more general study of the kinematics of helical filament winding on more complex slender mandrels (elliptic cross sections and conical mandrels) [1,2].

ANALYSIS OF HELICAL FILAMENT-WINDING

The circular-cylindrical mandrel has a radius R_M and a corresponding circumference $\lambda_M = 2\pi R_M$ which is used as the basic reference dimension for reduced quantities defined later.

A schematic representation of a lathe-type winder is shown in Figure 1.

The filament is guided at a point D (delivery point) which moves with the carriage along the delivery line (axis). The position of the delivery point D relative to a suitable reference axial position is denoted x_D.

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The filament contacts the mandrel at a point C (contact point) which moves along the contact line (axis). The position of the contact point C relative to the reference axial position is x_C.

For the rationalization of the analysis, it is convenient to define several reduced axial positions for delivery or contact points (subscripts D and C, respectively) as follows:

$$X = x/\lambda_M \tag{1}$$

$$X' = X/\tan\theta_u' \tag{2}$$

$$X'' = X'/T_g' \tag{3}$$

The quantities θ_u' and T_g' will be defined later.

The fixed distance between the delivery line and the contact line is δ. A dimensionless delivery distance parameter D is defined as $D = \delta/\lambda_M$ and another parameter G is defined as $G = D/T_g'$.

The length of filament between C and D is referred to as the filament free span. The angle between the free span line C-D and the contact line corresponds to the filament placement angle on the mandrel (assuming no slippage). It is referred to as the axial filament angle and deonoted θ. Its complement $\theta' = \pi/2 - \theta$, which is referred to as the cross filament angle, is more convenient for use in calculations.

During what is referred to as steady operation, the velocity of rotation of the mandrel N, as well as the velocity of axial translation of the carriage v, are fixed.

Far enough from the ends of a long mandrel, in steady operation, a permanent regime can be achieved, which corresponds to a uniform filament angle θ_u' (or $-\theta_u'$) such that

$$\tan\theta_u' = v/(2\pi R_M N) \tag{4}$$

If the carriage velocity varies during start-up or reversal, the corresponding peiod of time is referred to as the gradual start-up or reversal stage; it is followed by a steady stage when the carriage velocity is constant and eventually by the permanent regime defined earlier. The combination of gradual and steady stages in this case is referred to as the transitory regime over which the filament angle is not uniform.

Time t is the fundamental variable in the analysis. The time origin ($t = 0$) corresponds to the onset of delivery point motion for start-up or to the delivery point "dead" position for reversal.

The duration T_1 of a mandrel revolution is the basic reference time for reduced quantities; it is related to the mandrel rotation velocity as $T_1 = 1/N$.

A reduced time t' is defined as $t' = t/T_1$.

The reduced gradual stage duration T_g' is defined as T_g' is defined as $T_g' = T_g/T_1$.

A reduced time t'' is defined as $t'' = t/T_g$ or t'/T_g'.

Three distinct cases are considered for start-up and reversal: the "instantaneous" case in which it is assumed that the steady stage is reached instantly; the "linear gradual" case in which the delivery velocity increases linearly to its steady value over a gradual stage duration T_g; the "sinusoidal gradual" stage in which the delivery velocity increases, according to the first quarter of a sinusoidal function, to the steady value over the duration T_g. This last variation corresponds to that encountered in chain/yoke mechanisms found in many conventional winders.

For the instantaneous case, the reduced delivery position X_D' can be expressed in the range $0 < t' < \infty$ as:

$$X_D' = t' \tag{5}$$

and

$$X_D' = -t' \tag{6}$$

for start-up and reversal, respectively.

For the gradual cases, the reduced delivery position X_D'' can be expressed as follows:

For start-up, in the range $0 < t'' < 1$,

$$X_D'' = 0.5\, t'' \tag{7}$$

and

$$X_D'' = (2/\pi)(1 - \cos(\pi/2)t'') \tag{8}$$

for linear and sinusoidal cases, respectively.

For reversal, in the range $-1 < t'' < 1$,

$$X_D'' = -0.5\, t'' \tag{9}$$

and

$$X_D'' = (2/\pi)(1 - \cos(\pi/2)t'') \tag{10}$$

for linear and sinusoidal cases, respectively.

Other relationships apply for the steady range ($1 < '' < \infty$) [3].

The geometric principle of analysis of the kinematics is illustrated in Figure 2 for distant delivery. The corresponding governing equation is

$$\tan\theta' = (X_D - x_C)/\delta = dx_C/dy_C \tag{11}$$

where $dy_C = \lambda_M N\, dt$.

EXPERIMENTAL WORK

A computer-controlled laboratory helical winder has been designed and built for experimentation related to our long-term research program [1]. This machine has been used to produce experimental data in the case of instantaneous start-up and reversal.

A sheet of paper is wrapped as a sleeve over the mandrel, and a polyester thread, slightly restrained at the delivery point, is wound over the mandrel. The thread path is marked on the paper sleeve which is subsequently unwrapped for analysis. No significant slippage of the thread occurs on the paper under most experimental conditions.

RESULTS

Start-up was analyzed for an arbitrary initial filament angle θ_o' but since, in practice, the filament is most likely to be initialy set at 90° to the mandrel axis, results are discussed here for the special case where $\theta_o' = 0$.

Instantaneous Start-Up and Reversal

For instantaneous start-up, the integration of the governing Equation (7), coupled with Equation (1), leads to the following closed-form expression for the filament angle variation:

$$Z = 1 - \exp(-t'/D) \tag{12}$$

where

$$Z = \tan\theta'/\tan\theta_u' \tag{13}$$

Theoretical results for the evolution of the filament angle θ' from zero towards the uniform angle θ_u' (permanent regime) are given in Figure 3 for a reduced delivery distance $D = 1$ (the distance δ is equal to 3.14 times the mandrel diameter). For the case of $\theta_u' = 45°$, an angle $\theta' \cong 32°$ is reached after the first rotation ($t' = 2$).

Corresponding experimental results plotted on the same scales are shown in Figure 4. They are in good accord with the theoretical predictions.

For instantaneous reversal, integration of Equation (7) coupled with Equation (2) gives:

$$Z = -1 + 2\exp(-t'/D) \tag{14}$$

Theoretical results for the evolution of the filament angle θ' from $+\theta_u'$ towards the opposite uniform angle $-\theta_u'$ are given in Figure 5 for $D = 1$. For the case of $\theta_u' = 45$, the angle θ' becomes zero (filament normal to the mandrel and "dead" position of the contact point) at a reduced time $t' \cong 0.7$, i.e., after about 70 percent of a rotation.

Corresponding experimental results plotted on the same scales in Figure 6 are also in good accord with the theoretical predictions.

Gradual Start-Up and Reversal

For linear gradual start-up, integration of Equation (7) coupled with Equation (3) gives:

$$Z = t'' - G[1 - \exp(-t''/G)] \tag{15}$$

for $0 < t'' < 1$. A different expression applies for the range $1 < t'' < \infty$ [3].

For sinusoidal gradual start-up, integration of Equation (7) coupled with Equation (4) gives:

$$\begin{aligned} Z = [&\sin(\pi/2)t'' - (\pi/2)\, G\cos(\pi/2)t'' \\ &+ (\pi/2)\, G\exp(-t''/G)]/B^2 \end{aligned} \tag{16}$$

where $B = [1 + (\pi/2)G^2]$ for $0 < t'' < 1$.

Theoretical results for the evolution of the filament angle θ' from zero towards the uniform angle θ_u' (permanent regime) are given in Figures 7 and 8 for linear and sinusoidal cases, respectively. They correspond to a choice of $G = 1$, i.e., a reduced delivery distance $D = 1$ and a reduced gradual stage duration $T_g' = 1$, for example. For the case of $\theta_u' = 45°$, the curves show that angles $\theta' \cong 20°$ or 24° are reached at the end of the gradual stage ($t'' = 1$).

For linear gradual reversal, integration of Equation (7) coupled with Equation (5) gives:

$$Z = -t'' + G\{1 - \exp[-(t'' + 1)G]\} \tag{17}$$

for $-1 < t'' < 1$.

For sinusoidal gradual reversal, integration of Equation (7) coupled with Equation (6) gives:

$$Z = \{(\pi/2)\, G\cos(\pi/2)t'' - \sin(\pi/2)t'' + (\pi/2)\, G^2\exp[-(t'' + 1)/G]\}/B^2 \tag{18}$$

where $B = [1 + (\pi/2)G^2]$ for $-1 < t'' < 1$.

Theoretical results for the evolution of the filament angle θ' from $+\theta'_u$ towards $-\theta'_u$ are given in Figures 9 and 10 for linear and sinusoidal cases, respectively, when $G = 1$. For the case of $\theta'_u = 45$ the angle θ' becomes zero at reduced times $t'' \cong 0.85$ or $t'' \cong 0.78$, i.e., at about 85 or 78 percent of the acceleration period.

Delivery Overshoot, Contact Undershoot

Winding under distant delivery conditions requires a travel of the delivery point which can be significantly longer than the travel of the contact point, i.e., the length of the wound product. Our analysis of the reversal permits the quantitative determination of this delivery overshoot (or contact undershoot), and our experiments for the case of instant reversal agreed with the theoretical predictions [3].

Filament Slack

It was experimentally observed that under certain reversal conditions (instant reversal for $\theta'_u = 60°$, for example) the portion of filament in the free span temporarily lost its tension. This phenomenon was analyzed theoretically and, for the above conditions, filament slack was predicted over the time range $0 < t' < 0.4$, in good accord with the experimental observations [3].

CONCLUSIONS

The kinematics of helical filament winding on circular-cylindrical mandrels including start-up and reversal can be modelled mathematically, leading to closed-form analytical results suitable for incorporation into computer-aided design and manufacturing programs. Comprehensive numerical results can be presented in just a few charts, allowing design or process engineers to make quick estimates.

The analysis of the kinematics of helical-type filament winding on mandrels of more complex shapes, which is underway in our group, should help designers and processors in the development of slender hollow composite structural elements.

ACKNOWLEDGEMENTS

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BIOGRAPHY

Jean-Michel Charrier

Dr. Charrier has been on the staff of the Department of Chemical Engineering at McGill University, Montreal, since 1969. He obtained his engineering degree from ENS Arts et Metiers in France and his M. Eng. and Ph.D. from the Institute of Polymer Science in Akron. His teaching and research interests in the properties and processing of polymeric systems encompass plastics, rubbers and composites. He spent a sabbatical leave in polymer-related industries in Europe and recently a three-year partial leave at the Industrial Materials Research Institute of the National Research Council of Canada in Montreal. He was the 1982–83 President of the Quebec Section of the Society of Plastics Engineers.

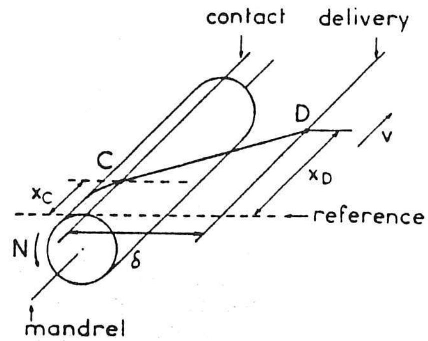

FIGURE 1. Helical filament winding geometry.

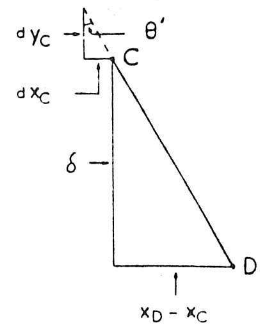

FIGURE 2. Geometric principle of kinematic analysis.

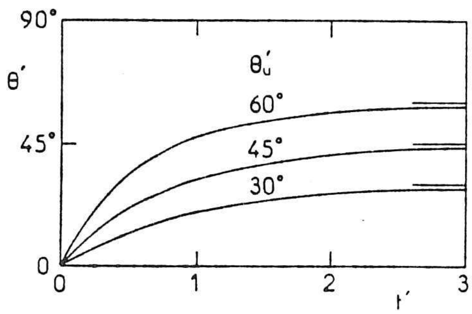

FIGURE 3. Instantaneous start-up; theoretical results.

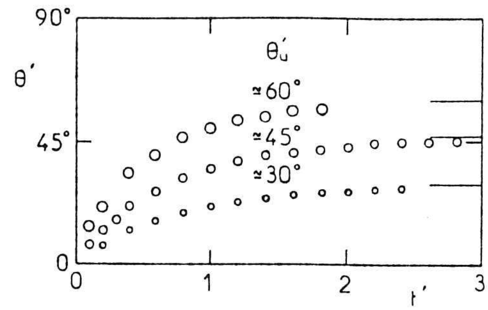

FIGURE 4. Instantaneous start-up; experimental results.

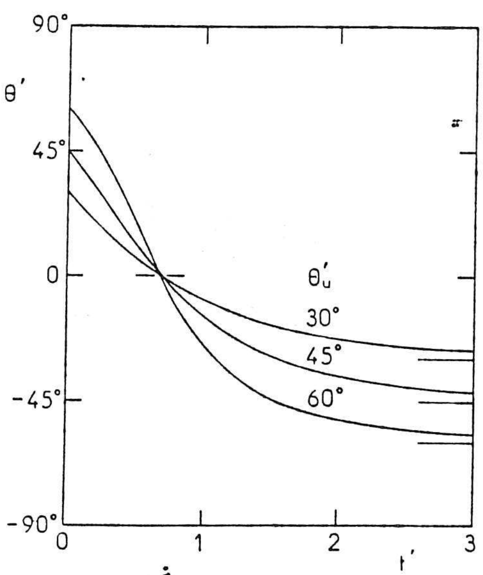

FIGURE 5. Instantaneous reversal; theoretical results.

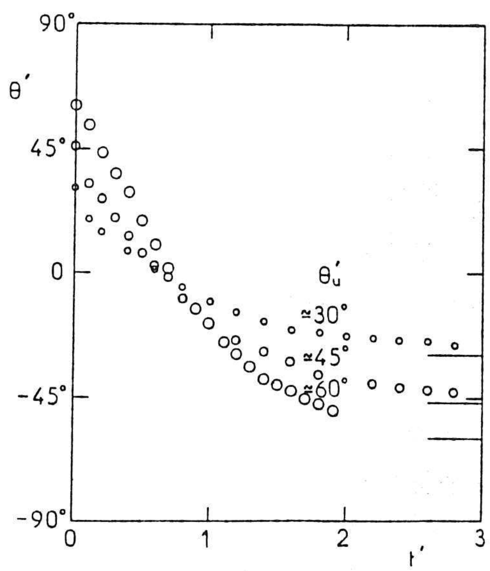

FIGURE 6. Instantaneous reversal; experimental results.

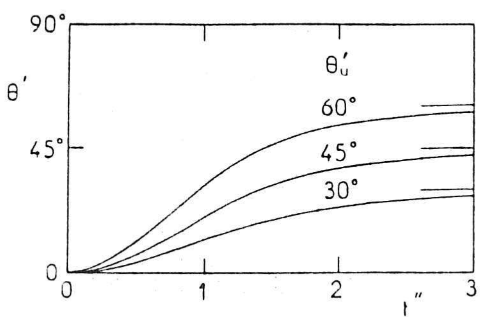

FIGURE 7. Linear gradual start-up.

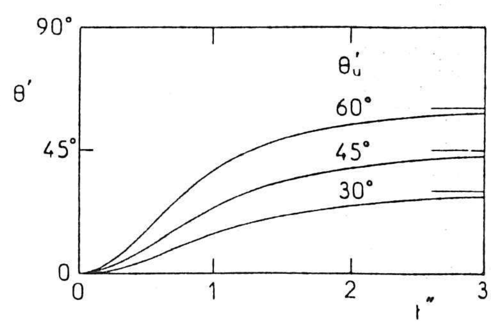

FIGURE 8. Sinusoidal gradual start-up.

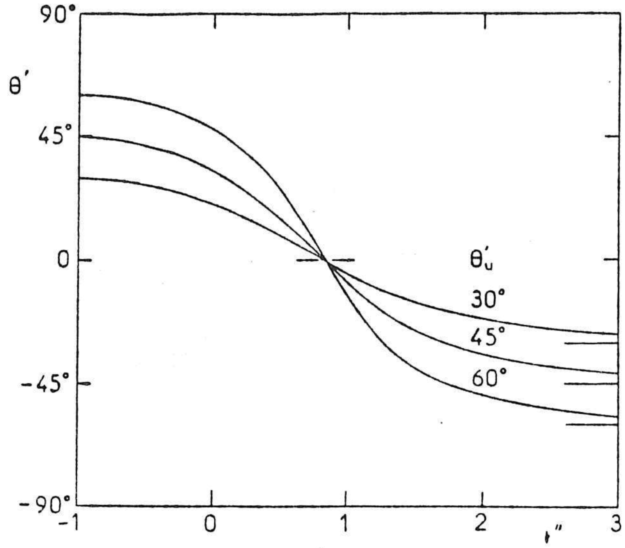

FIGURE 9. Linear gradual reversal.

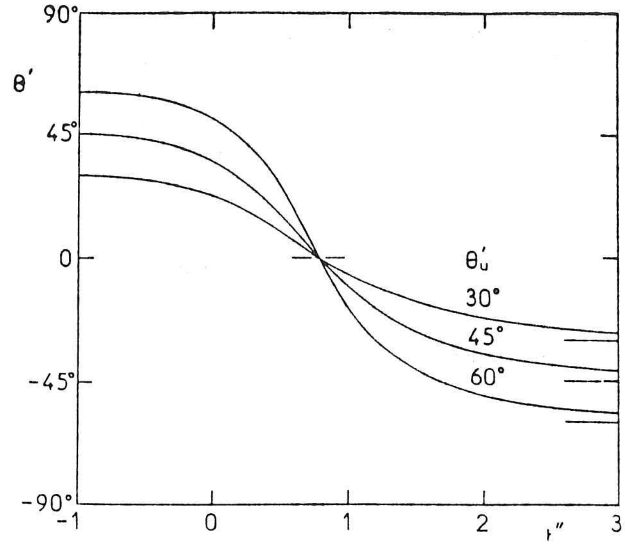

FIGURE 10. Sinusoidal gradual reversal.