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THE INITIAL CREEP OF COLUMNAR-GRAINED ICE

PART II. ANALYSIS[1]

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ABSTRACT

Observations on the initial creep behavior of columnar-grained ice are analyzed by assuming that the creep strain at a given time has a power-law dependence on the applied constant compressive stress. The exponent for the stress was time-dependent during transient creep. For first load it started at a low value, increased to a maximum of about 2.23 approximately 75 minutes after the application of the load, and decreased thereafter. For reload it started at a high value and decreased continuously to a constant value of 1.46 by 100 minutes after the application of the load. Creep rates at a given time, calculated from the observed power-law dependence of the creep strain on stress, also had a power-law dependence on stress for time greater than about 25 minutes after the application of the load. The observations are shown to be in agreement with observations by Krausz (1963) on the deflection rate of ice beams and by Steinemann (1954) and Glen (1958) on the stress-dependence of the minimum creep rate during secondary creep. The observations indicate that the creep rate during secondary creep varies approximately as $t^{-0.5}$.

Part I (Gold 1965) presented information on the difference in creep behavior, due to constant compressive load, between columnar-grained ice previously undeformed and the same ice after it has been subjected to deformation. It was shown that structural changes occurred in the ice during first load in association with an unusual behavior of the creep rate during transient creep. Evidence of the formation of small-angle boundaries after the creep strain exceeded about 0.1% was obtained by thermal etching. The formation of internal cracks during the transient creep stage was visually evident.

In the present paper the observations are analyzed by assuming a power-law dependence of creep on stress. From the empirical relationships obtained, the relationship between creep rate and stress and its change with time is determined. This relationship is used to interpret the creep behavior of ice beams reported by Krausz (1963). The results of the present observations are compared with observations by Steinemann (1954) and Glen (1958) on the stress-dependence of the minimum creep rate of ice during secondary creep.

STRESS-DEPENDENCE OF THE CREEP

In Fig. 1(a) log creep strain for first load is plotted against log stress for times t = 1 minute, 100 minutes, and 350 minutes. The corresponding observations for reload are given in Fig. 1(b) for times t = 1 minute, 100 minutes, and 300 minutes. From these figures it is apparent that the relationship between the creep strain and the stress at a given time is given approximately by an equation of the form

$$(e)_t = A(t)\sigma^{n(t)}, \tag{1}$$

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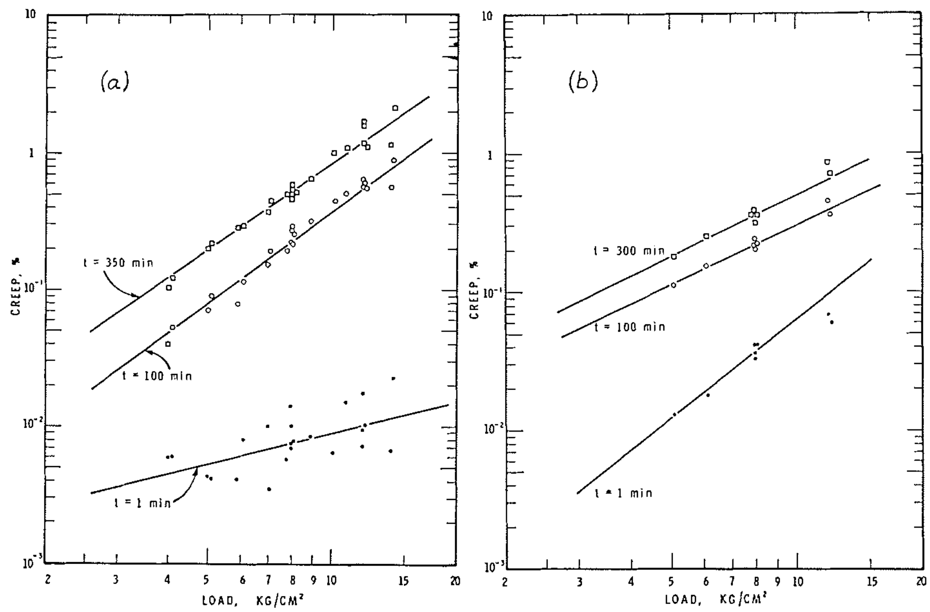

FIG. 1(*a*). Dependence of creep on constant compressive stress for first loading (time = 1, 100, and 350 minutes; temp. = -9.5 ± 0.5 °C). (*b*) Dependence of creep on constant compressive stress for reloading (time = 1, 100, and 300 minutes; temp. = -9.5 ± 0.5 °C).

where $(e)_t$ is the creep strain at time t, σ is the constant compressive stress, and $A(t)$ and $n(t)$ are constants for given time t. For first load there is considerable scatter for time $t = 1$ minute, due in part to the erratic initial creep behavior of ice, as was mentioned in Part I. The relative scatter was reduced appreciably by $t = 10$ minutes. For reload, creep observed for high stresses ($\sim$14kg/cm²) appears to deviate from a power-law dependence on stress, being less than expected for t less than 100 minutes and greater than expected for t greater than 100 minutes as may be seen in Fig. 1(*b*).

The least-squares fit for given times was calculated for first load using stress as the independent variable, and values for n were obtained. Although only eight reload tests were carried out, it was decided to analyze these along with the first-load tests to show the marked difference in deformation behavior between the two loading situations. The line through reload observations was located by eye. Calculated values for n for first load and reload are plotted against time in Fig. 2; subscript 1 refers to first load and 2 to reload.

The marked difference in the characteristics of deformation for first load and reload is clearly evident. For example, the initial value of n for the first load is quite small, increases to a maximum at t equal to approximately 75 minutes, then gradually decreases. Unfortunately, the load times were not long enough to establish the existence of constant n_1. For reload, the value for n is relatively large initially and decreases continuously to a constant value of 1.46 for t greater than 100 minutes.

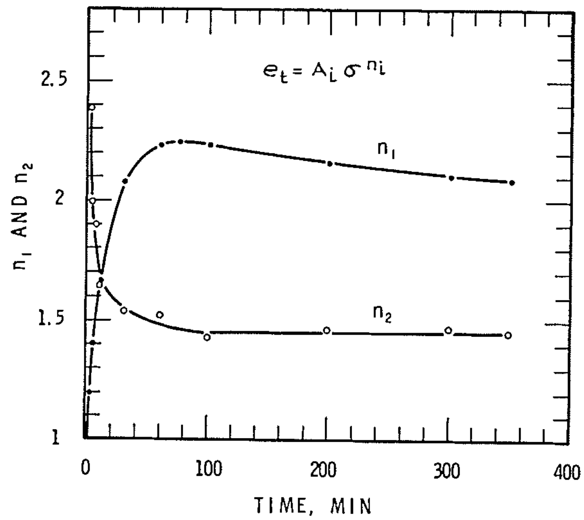

FIG. 2. Time-dependence of n_1 (first load) and n_2 (reload).

STRESS-DEPENDENCE OF CREEP RATE

The dependence of creep rate on stress at given times can be determined from the results of the foregoing analysis. Before proceeding with the calculations, however, it will be useful to modify equation (1). For given time

$$A = e_0/\sigma_0{}^n,$$

where e_0 is the creep strain at time t due to stress σ_0. Substituting this expression into equation (1) gives

$$e(\sigma, t) = e_0(\sigma/\sigma_0)^n, \tag{2}$$

an equation that is more satisfactory dimensionally, particularly under conditions of varying n. Differentiating this expression with respect to time gives

$$\frac{\partial e(\sigma, t)}{\partial t} = \left(\frac{\sigma}{\sigma_0}\right)^n \left[\frac{\partial e_0}{\partial t} + e_0 \frac{\partial n}{\partial t} \ln \frac{\sigma}{\sigma_0}\right]. \tag{3}$$

In Fig. 3, log e, obtained from the least-squares fit to the first-load observations, is plotted against log t for various values of σ. It may be seen that because the value of n_1 is time-dependent, the shape of the log e vs. log t curve is not independent of stress, as it would be if n were constant. It is of interest that extrapolation of the experimental results to $\sigma = 1.3$ kg/cm^2 gives e equal to a constant for about the first 10 minutes of loading. Extrapolation of the observations to loads greater than 25 kg/cm^2 gives $e \propto t^{0.55}$ for times greater than 100 minutes after application of the load.

It was possible, for given time periods and stress, σ, to determine simple functions giving the time-dependence of n and e. The derivatives of n and e_0 with respect to time were determined from these functions and $\partial e/\partial t$ calculated

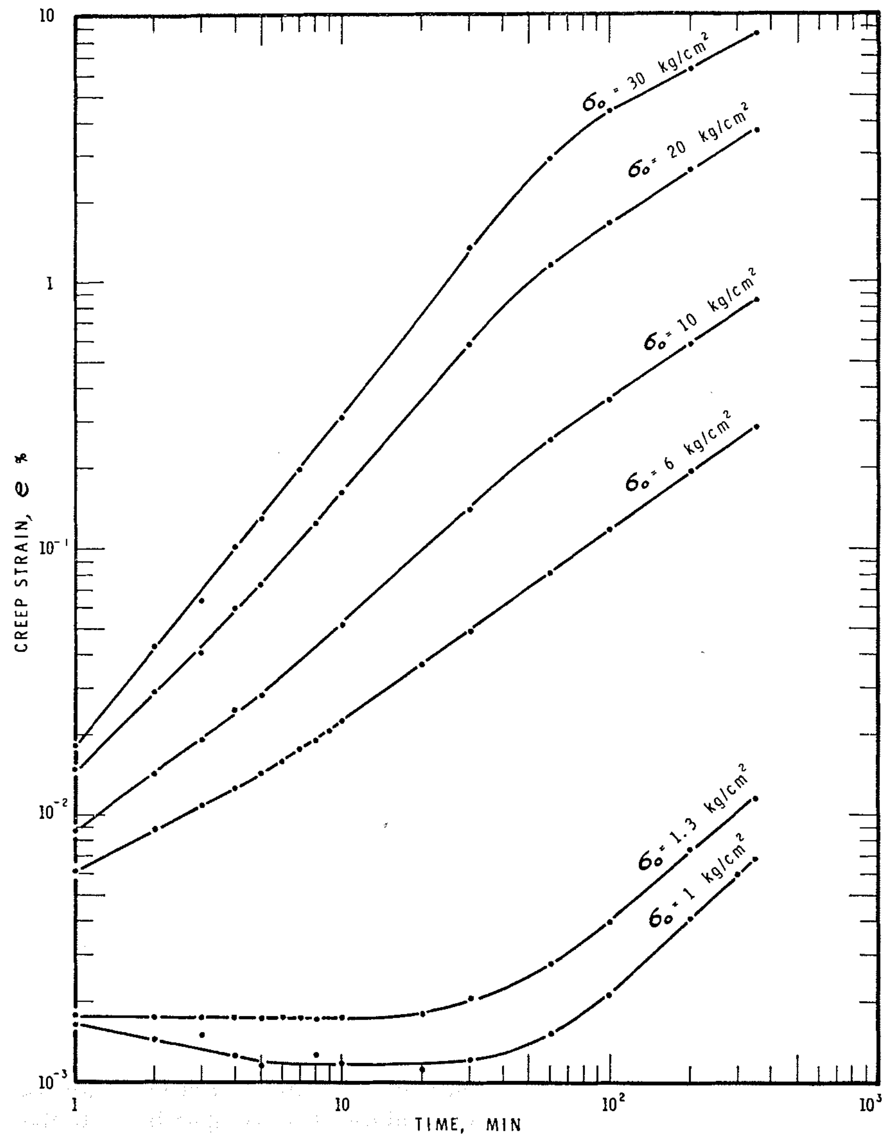

FIG. 3. Creep curves for given stresses obtained from least-squares fit to first-load observations (temp. = -9.5 ± 0.5 °C).

for given times and σ = 4, 6, 8, 10, 12, and 14 kg/cm², by the use of equation (3). Calculated values are given in Table I. The values for σ = 4 and 14 kg/cm² are plotted against time in Fig. 4. It may be seen that for first load the creep rate has a high value initially, decreases to a minimum within the first 10 minutes of loading, then rises to a maximum for t between 10 to 20 minutes. There is evidence of a plateau in the creep rate between 20 to 40 minutes after the application of the load, following a fairly rapid decrease from the maximum.

TABLE I

Calculated creep rate for given time and load (in kg/cm²) for first load (temp. −9.5 ± 0.5 °C)

Time (min)	Creep rate (%/min × 10⁴) for a load of:					
	4	6	8	10	12	14
1	18.0	32.8	48.2	63.7	79.1	95.1
3	9.9	21.7	36.3	53.0	71.2	92.4
5	7.6	18.0	32.0	49.1	68.9	91.9
7	6.2	15.7	29.1	45.7	65.8	90.2
10	5.9	15.3	28.9	47.1	69.1	95.3
15	5.2	14.4	28.5	47.9	72.3	102.0
20	4.8	13.3	26.8	45.7	70.0	100.0
25	4.7	12.6	24.8	42.0	64.0	91.3
30	4.4	11.9	23.8	40.8	62.9	90.5
40	4.0	11.0	22.5	39.0	60.8	88.4
50	3.8	10.4	21.0	36.3	56.0	81.9
60	3.8	9.9	19.6	33.2	51.1	73.5
70	3.7	9.5	18.4	30.8	46.8	66.7
80	3.7	9.2	17.4	28.7	43.2	61.0
100	3.6	8.6	16.0	25.7	37.8	52.2
150	3.3	7.7	13.8	21.4	30.8	41.9
200	3.1	7.1	12.5	19.3	27.6	37.6
250	3.0	6.7	11.7	18.0	25.6	34.7
300	2.9	6.4	11.2	17.0	24.2	32.5
350	2.8	6.1	10.7	16.1	22.3	30.7

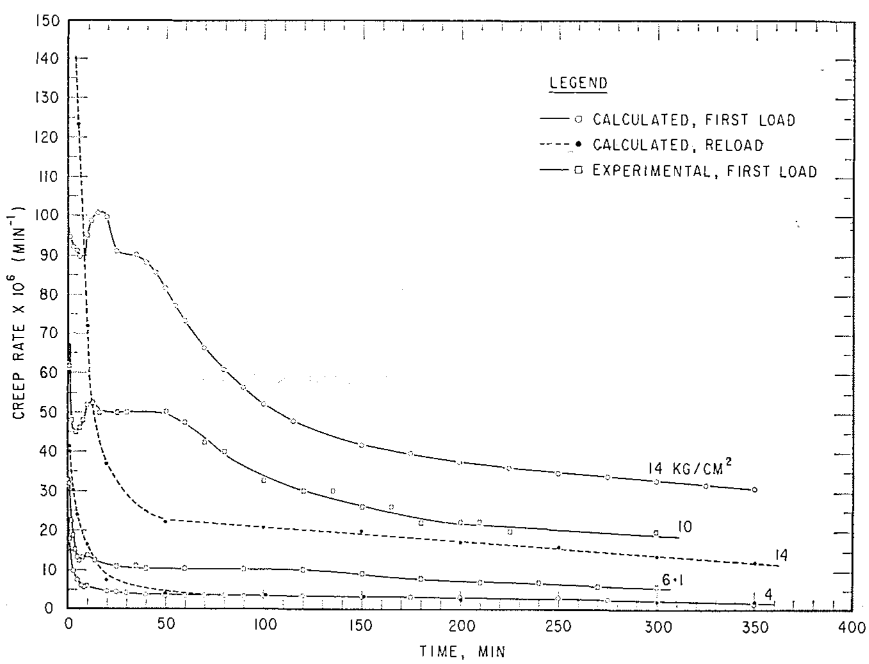

FIG. 4. Time-dependence of creep rate for first load and reload (temp. = −9.5 ± 0.5 °C).

Thereafter the creep rate decreases smoothly and continuously. Creep rates calculated directly from observations show the same features; two examples are presented in Fig. 4.

Creep rates were determined also from the reload observations. The calculated values for stress equal to 4 and 14 kg/cm^2 are shown in Fig. 4. In contrast with first load, the creep rate has a high initial value and decreases continuously with time, most of the decrease occurring within the first 50 minutes.

The logarithm of the calculated creep rate at given times is plotted against the logarithm of the stress for first load in Fig. 5(*a*) and reload in Fig. 5(*b*). For t greater than about 25 minutes the dependence of creep rate on stress for both first load and reload is closely approximated by

$$\partial e(\sigma, t)/\partial t = B(t)\sigma^{m(t)}, \tag{4}$$

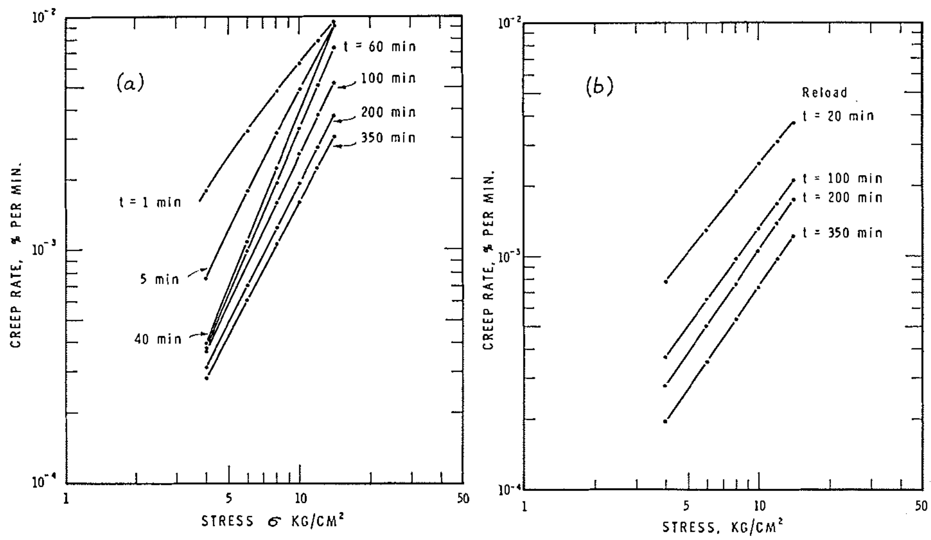

FIG. 5(*a*). Stress-dependence of creep rate at given times for first load (temp. = −9.5 ± 0.5 °C). (*b*) Stress-dependence of creep rate at given times (temp. = −9.5 ± 0.5 °C).

where B and m are constants for given t. For time less than 25 minutes the plot of logarithm of the creep rate against logarithm of the stress was not linear, as would be anticipated from equation (3) in view of the dependence of n on time.

Values of m and B were determined for t greater than 20 minutes and plotted against time in Figs. 6 and 7, respectively. From equations (2) and (3) it may be seen that m should equal n if n is constant, as it was for reload for t greater than about 150 minutes. For first load, n_1 was still changing at the time the load was removed; correspondingly, m_1 also was still decreasing.

Figure 7 shows that for $100 < t < 350$ minutes, B_2 has a power dependence on time. The exponent of this dependence was found to be equal to −0.53, in

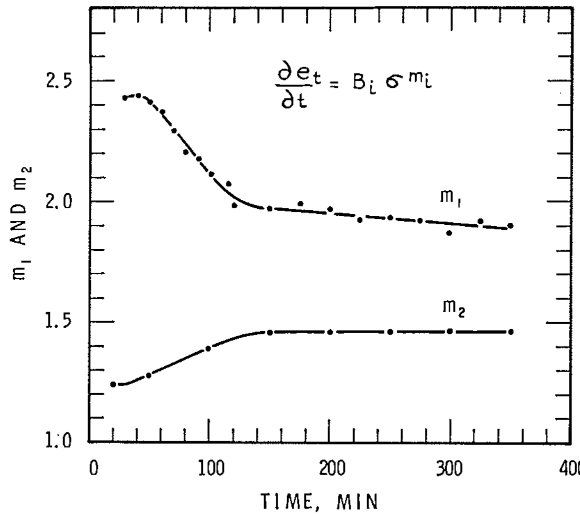

FIG. 6. Time-dependence of m_1 (first load) and m_2 (reload).

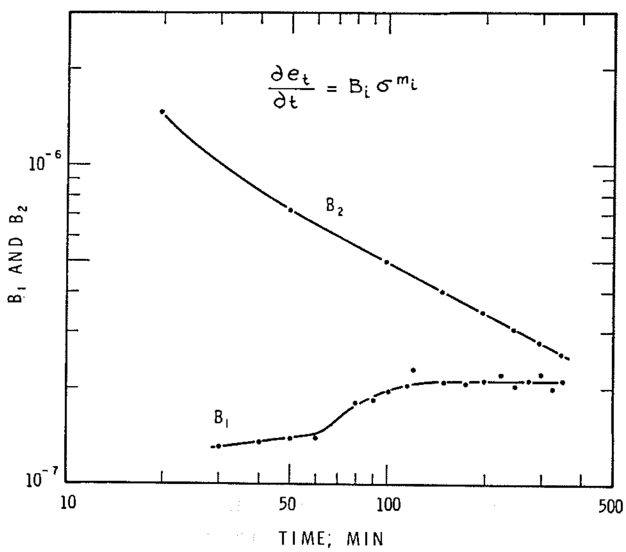

FIG. 7. Time-dependence of B_1 (first load) and B_2 (reload).

good agreement with the exponent for the time-dependence of the creep for t equal to 300 minutes given in Table I, Part I. For $150 < t < 350$ minutes, therefore, the analysis gives for the creep rate for reload

(5) $$\partial e(\sigma, t)/\partial t = 5.6 \times 10^{-4} t^{-0.53} \sigma^{1.46}\% \text{ per minute.}$$

An empirical equation can be obtained as well for first load for $150 < t < 350$ minutes, namely,

$$\partial e(\sigma, t)/\partial t = 2.13 \times 10^{-5} \sigma^{2.04-4.5\times 10^{-4} t};$$

but this equation is not in a form that can be readily related to creep theory as is equation (5). It is of interest that if the time between first load and reload (about 20 hours) is substituted in the above equation, the exponent obtained for the stress is about equal to that observed for the reload tests.

COMPARISON WITH PREVIOUS WORK

Steinemann (1954) and Glen (1958) have presented information on the stress-dependence of the minimum creep rate of granular ice in the secondary creep stage. The present investigations were not continued until that creep rate was attained. Furthermore, the stress-dependence was established for given time, whereas the condition of minimum creep rate used by Glen and Steinemann would be more closely associated with given creep strain. The dependence of the creep rate on stress and creep strain can be calculated from equation (1). Assuming that n is constant and $A = Dt^p$ (D and p constants),

$$e(\sigma, t) = Dt^p\sigma^n. \tag{6}$$

Differentiating with respect to time,

$$\partial e(\sigma, t)/\partial t = pDt^{p-1}\sigma^n. \tag{7}$$

Solving for t in equation (6) and substituting in equation (7) gives

$$\partial e(\sigma, e)/\partial t = pD^{1/p}e^{p-1/p}\sigma^{n/p}. \tag{8}$$

Assuming that with time the values for p and n for first load approach those for reload, i.e. $p = 0.47$, $n = 1.46$,

$$\partial e(\sigma, e)/\partial t = pD^{1/p}e^{-1.13}\sigma^{3.11}.$$

The exponent for the stress, 3.11, is in good agreement with that obtained by Glen (1958) (3.17 ± 0.1 for a temperature of −1.5 °C). Steinemann (1954) did not obtain a linear dependence of log creep rate on log stress, but found that the exponent increased from about 1.80 at $\sigma = 1$ kg/cm^2 to about 4.16 at $\sigma = 14$ kg/cm^2, being equal to about 3.0 for $4 < \sigma < 8$ kg/cm^2. If the creep rate of ice is, in fact, approximately inversely proportional to the creep strain, as indicated by equation (8), a possible explanation for the curvature in the dependence of log creep rate on log stress obtained by Steinemann is that the minimum creep rates occur for different amounts of creep strain for the stresses used in his experiments. Steinemann's observations did indicate a tendency for the onset of tertiary creep to occur at a smaller creep strain for large loads (15 kg/cm^2) than for small loads (4 kg/cm^2). A similar effect may exist for the minimum creep rate. Glen's observations were for loads less than 10 kg/cm^2.

Glen (1958) found that for granular ice near the melting point the observed creep strain could be approximated reasonably well by the Andrade law:

$$e = \beta t^{\frac{1}{3}} + kt.$$

Application of the Andrade law to the present observations yielded poor fit for first load and only fair fit for reload. No attempt was made to fit the observations with polynomials containing powers of t other than $\frac{1}{3}$ and 1.

As the dependence of creep strain on stress was usefully approximated by a power-law function, it may be of interest to point out certain features of the exponent, n. The value of n was found to be primarily dependent on time and independent of stress or the amount of deformation at a given time. Taking into consideration the evidence of formation of small-angle boundaries and internal cracks, and the consequences of recovery, it would appear that n is in some way related to the formation and rate of operation of the microscopic elements responsible for creep. The characteristics of these elements to which n is related must, however, be dependent primarily on time and only to a minor degree, if at all, on applied stress. It may be of interest that for first load and t between 1 and 20 minutes n was proportional to $\ln t$. For first load and t between 150 and 350 minutes, and reload and t between 0.5 and 100 minutes, n appears to be proportional to $-\ln t$. It is unfortunate that the first-load observations were not continued long enough to establish whether, with continued deformation, the value for n_1 approaches the constant value observed for n_2.

Readey and Kingery (1964) observed for single crystals of ice deformed in tension (under conditions of constant strain rate) a power-law dependence of the creep rate on stress. They found that the exponent n decreased with strain from about 2.5 to about 1.5. The range in creep strain over which they observed this change was much greater than that in the present experiments. Higashi, Koinuma, and Mae (1964) made observations on single crystals of ice similar to those of Readey and Kingery, giving particular attention to a "yield drop" phenomenon that occurs for creep strain about 1%. They observed a power-law relationship between the applied constant strain rate and the maximum stress that occurred, the exponent for the stress being 1.53. The strain rates applied were such that the maximum stress did not exceed about 4.5 kg/cm^2. These observations suggest that the influence of stress on the creep rate of ice may be basically the same for both single crystals and polycrystals.

APPLICATION OF RESULTS TO BEAM EXPERIMENTS

Krausz (1963) conducted first-load and reload experiments on ice beams made from columnar-grained ice, and observed deformation behavior similar to that recorded in the present work. The ice beams were subjected to a constant bending moment of 11.4 kg-cm/cm (25 in.-lb/in.). The long axis of the columnar grains was perpendicular to the face to which the load was applied, so that the principal stresses had the same orientation with respect to the grain boundaries as in the compression tests.

The load distribution associated with the applied bending moment, if we assume that the beam deforms elastically, is shown in Fig. 8 for a beam 2.54 cm thick. As the ice creeps, it would be expected that the load distribution would change. It was assumed that after a sufficient length of time following the application of the load the creep rate perpendicular to the plane upon which the bending moment acts would be linearly proportional to the perpendicular distance from the neutral plane. Krausz's observations indicated that the

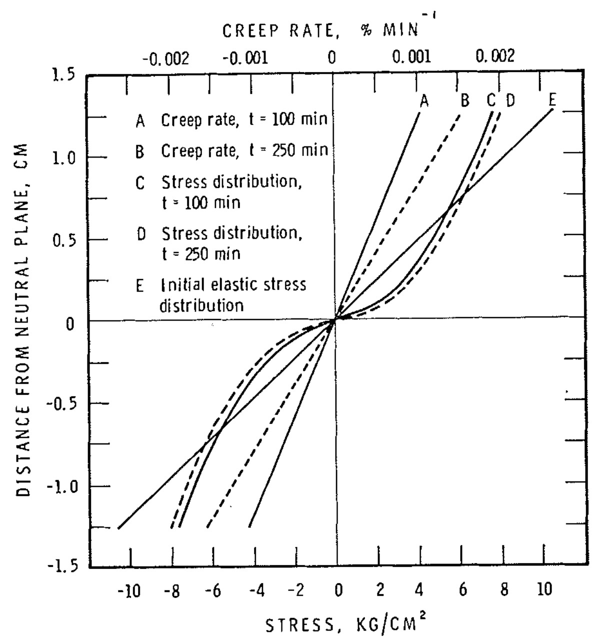

FIG. 8. Creep rate and stress associated with constant bending moment of 11.4 kg-cm/cm (temp. = -9.5 ± 0.5 °C).

neutral plane remained at the center of the beam throughout deformation. By the use of the above assumption, creep rates were determined from observed deflection rates of a beam 2.54 cm thick at times t equal to 60, 100, and 250 minutes. From these creep rates, stresses were determined by using the dependence of creep rate on stress for first load obtained from the simple compression experiments.

Assumed creep rates and resulting load distribution through the beams for t equal to 100 and 250 minutes are given in Fig. 8. The calculation indicates that the stress near the surface of the beam has relaxed considerably and that near the center it has increased from the initial elastic distribution; but there is very little difference between the distributions obtained for either time. The stress calculated for $t = 60$ minutes was a little less than that for $t = 100$ minutes within 0.63 cm of the surface, and a little greater over the remaining center section of the beam. The bending moment calculated graphically from the mean of the stress distributions for t equal to 100 minutes and 250 minutes was found to be 10.5 kg-cm/cm^2 (23 in.-lb/in.), within 10% of that applied. For $t = 60$ minutes the bending moment was somewhat less.

Maximum deflection rate in the above beam test occurred at t between 60 and 100 minutes after the application of the constant bending moment. Analyses of the deflection rates for a second beam 3.05 cm thick showed similar agreement between calculated and applied bending moments for time equal to or greater than that associated with the maximum deflection rate. For times less than that at which the maximum deflection rate occurred, the bending moments calculated according to the foregoing assumption were smaller than that applied. These observations indicate that during first load of columnar-grained ice beams not only does the unusual creep behavior affect the deformation, but there is also an influence associated with the transition from the initial elastic to the final plastic condition. It would be expected that when the load is first applied the stress distribution would tend to be that associated with elastic deformation. According to the results of the compression tests, therefore, the creep rate near the surface would be considerably larger than the linear assumption would predict from the observed deflection rates.

The observed deflection rates indicate that the middle section of the beam, where the stresses initially are less than for the fully plastic condition, largely determines its initial deformation behavior. As the stress near the surface relaxes, the center section of the beam must carry more of the load. Because of the characteristics of the dependence of creep rate on stress for first load, the increase in stress over the middle section results in an increased creep rate and associated deflection rate. On the basis of this interpretation, Krausz's observations indicate that transition from the elastic to the plastic behavior requires from 1 to 5 hours for ice loaded under the conditions of the experiments. The transition for the reload condition appears to take place in less time and without a maximum in the deflection rate, although an inflection might be present (see Krausz 1963, Fig. 3).

Krausz's observations were made on beams between 2.28 and 3.05 cm thick. The deflection rate was very sensitive to beam thickness. Two beams could not be considered geometrically equivalent unless they were machined to a given thickness to a tolerance smaller than $\pm 0.5\%$. Beams 2.29 cm thick failed within half an hour of the application of the load.

The reason for the very sensitive dependence of deflection rate on thickness for beams about 2.54 cm thick subjected to a bending moment of 11.4 kg-cm/cm becomes clear when the power-law dependence of creep rate on stress is taken into consideration. The stress at the surface of a beam for a given bending moment varies inversely as the square of the beam thickness. For beams 2.29 to 3.05 cm thick subjected to a bending moment of 11.4 kg-cm/cm, the range in maximum stress at the surface is about 6 kg/cm^2 to 11 kg/cm^2. The corresponding creep rates for first load and t = 100 minutes are 8.7×10^{-4} and 3.7×10^{-3}%/min, a 25% reduction in beam thickness causing an increase of over 300% in the creep rate at the surface. Failure of the beam 2.29 cm thick within half an hour of the application of the load is to be expected from the results of investigations now in progress on the stress-dependence of the time to formation of internal cracks in ice.

CONCLUSIONS

Analysis of creep observations for columnar-grained ice subject to compressive stress between 4 and 14 kg/cm^2 perpendicular to the long axis of the grains showed that the stress-dependence of the creep at a given time for first load and reload can be usefully approximated by a power-law function between t equal to 1 and 360 minutes. The value for the exponent of the stress varies continuously with time during transient creep. For reload it tends to a constant value of 1.46 for t greater than 100 minutes. The stress-dependence of the creep rate at a given time can be usefully approximated by a power-law function only for t greater than about 25 minutes. The flow law obtained for first load agrees with observations on the deflection rate of beams for times greater than that associated with the maximum deflection rate, if it is assumed that the creep rate of the beam varies linearly with perpendicular distance from the neutral plane. Extrapolation of the present results to give creep rates for a given amount of creep yields an exponent for the stress in the flow law in good agreement with the values obtained by Steinemann (1954) and Glen (1958). Observations during reload indicate that the creep rate during secondary creep is approximately proportional to $t^{-0.5}$.

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