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SECURITY CLASSIFICATION OPEN

SUBJECT A STUDY OF THE STREAMLINE PATTERN IN A TWO-DIMENSIONAL ELBOW WITH A CASCADED OUTLET AS DETERMINED BY RELAXATION METHODS.

PREPARED BY John D. Bannister

ISSUED TO

SUMMARY

The streamline pattern for a two-dimensional 45 degree elbow with a cascaded outlet is developed by relaxation methods. It is found that the suction side splitters which are at appreciable incidence to the flow tend to produce irregularities in the elbow discharge.

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A STUDY OF THE STREAMLINE PATTERN IN A TWO-DIMENSIONAL ELBOW WITH A CASCADED OUTLET AS DETERMINED BY RELAXATION METHODS.

1 INTRODUCTION

It is proposed to drive the wing-embedded fans for the fan-in-wing VTOL propulsion scheme by tip turbines. This concept necessitates distributing the working fluid with uniform mass flow and velocity around the periphery of the fan and directing it at the proper angle to impart its energy to the tip-turbine blades.

To accomplish this effect a volute with nozzle openings in one radial wall is to be developed. Splitters are to be positioned in the outlet to impart a 25 degree helix angle to the working fluid flow vector. Flow acceleration is to be achieved by convergence of the nozzle passage normal to the splitters. A sketch of this proposed volute and nozzle scheme is shown in Figure 1.

This study was undertaken to determine the incidence effects of the splitters on the flow through the volute and stator section.

2 ANALYSIS

To maintain continuity and obtain uniform distribution of mass flow and velocity to the nozzle inlets, a uniform reduction in the cross-sectional area of the volute is necessary. It is assumed that this area reduction will be achieved by initial convergence of the "cylindrical" walls to width of the nozzle inlets, and subsequent convergence of the radial walls to zero cross-sectional area. If the volute is unrolled, the cylindrical surface bisecting the nozzle passages becomes a plane. And if the resulting configuration is sectioned through this plane, a two-dimensional flow field is exposed which is very similar to the simple elbow shown in Figure 2.

Being a two-dimensional elbow, this model cannot reveal the three-dimensional flow effects of the converging "cylindrical" wall portion of the volute. However, the two-dimensional picture of this flow may be approximated by the diffusing part of the elbow (i.e., including the first three splitters).

To accentuate the incidence effects on the splitters a 45 degree, in lieu of a 25 degree, elbow was studied. Since in the prototype the nozzle acceleration is to take place in a plane normal to the plane being studied, the model elbow was laid out to produce no net acceleration. The nozzle passage width to splitter thickness ratio was maintained the same for the model as for the prototype.

The streamline pattern was obtained by two-dimensional flow theory and relaxation methods as set out in the Appendix.

3 RESULTS

The streamline pattern for the 45 degree elbow with four equally spaced splitters is shown in Figure 3.

It is seen that flow near the pressure side of the elbow is turned by the walls, upstream of the splitters, so that the splitters are at zero incidence. Thus the flow alignment performance of the splitters is negligible in this section.

The flow near the suction side of the elbow tends to approach the splitters at a positive incidence. Figure 3 shows that where the splitters are at incidence there is an irregular distribution of flow through the nozzles. One would expect that this effect is exaggerated by the 45 degree turn of the model as against the 25 degree prototype turn. Also, to mitigate this effect the splitter leading edge could be cropped to allow the flow to accomplish better alignment with the splitters.

In any case, the splitters do not seem to perform to advantage in turning the flow.

4 CONCLUSION

This study indicates that it may be possible to reduce the prototype splitter profile pressure loss, without loss of flow turning effectiveness, by cropping the leading edges.

5 REFERENCE

1 Hildebrand, F.B. Methods of Applied Mathematics.

APPENDIX

FLOW THEORY AND RELAXATION METHOD OF SOLUTION

1 Two-dimensional Potential Flow

In a two-dimensional flow field where the velocity $\overline{V}$ has x and y components u and v, the conditions for continuity and irrotationality are respectively:

$$\operatorname{div} \overline{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

and

$$\operatorname{curl} \overline{V} = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 \qquad (2)$$

Potential flow may be described by two functions of position which are defined as follows:

Potential function ϕ ; grad $\phi = \overline{V}$,

or $u = \frac{\partial \phi}{\partial x}$ and $v = \frac{\partial \phi}{\partial y}$

Stream function ψ ; $u = \frac{\partial \psi}{\partial y}$

and $v = -\frac{\partial \psi}{\partial x}$

Substituting the definition of potential function into equation (1) and the definition of stream function into equation (2) one obtains

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \qquad (3)$$

and

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0 \qquad (4)$$

From equations (3) and (4) a flow net may be developed whereby lines of ψ = a constant describe flow streamlines and lines of ϕ = a constant describe velocity equipotentials. For the purposes of this study only the stream function was considered.

2 Relaxation Method of Solution

(a) Finite Difference Equations.

Superimposed on the flow field is a grid of such spacing that the stream function may be assumed linear between nodes. Consider a portion of this grid as shown in Figure 4 (a).

Then
$$\frac{\partial \psi}{\partial x} = \frac{\psi_1 - \psi_0}{a} = \frac{\psi_0 - \psi_2}{a}$$

and
$$\frac{\partial\left(\frac{\partial \psi}{\partial x}\right)}{\partial x} = \frac{\frac{\psi_1 - \psi_0}{a} - \frac{\psi_0 - \psi_2}{a}}{a}$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{\psi_1 + \psi_2 - 2\psi_0}{a^2}$$

similarly
$$\frac{\partial^2 \psi}{\partial y^2} = \frac{\psi_3 + \psi_4 - 2\psi_0}{a^2}$$

Then if the stream function at node (o) satisfies Laplace's equation,
$$\psi_1 + \psi_2 + \psi_3 + \psi_4 - 4\psi_0 = 0 = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}$$

Otherwise, the degree of approximation to a solution of Laplace's equation is represented by the residual,
$$R_0 = \psi_1 + \psi_2 + \psi_3 + \psi_4 - 4\psi_0 \qquad (5)$$

(b) Internal Nodes.

The effect of a change in the trial stream function on the residuals at an internal node and its adjacent neighbours may be obtained by considering equation (5) at the said nodes. A graphical representation of this effect is the relaxation pattern shown in Figure 4 (b), where the encircled numbers represent residual changes for a change in stream function of plus one at node (o).

(c) Boundary Nodes.

Consider a node near a boundary as shown in Figure 5 (a). The boundary has reduced the arms 02 and 04 to the proportions $m = \frac{\overline{02}}{a}$ and $n = \frac{\overline{04}}{a}$ respectively. The finite difference equations become,

$$\frac{\partial \psi}{\partial x} = \frac{\psi_1 - \psi_0}{a} = \frac{\psi_0 - \psi_2}{m a}$$

and

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{\psi_1 - \psi_0 - \frac{\psi_0}{m} + \frac{\psi_2}{m}}{a^2}$$

similarly,

$$\frac{\partial^2 \psi}{\partial y^2} = \frac{\psi_3 - \psi_0 - \frac{\psi_0}{n} + \frac{\psi_4}{n}}{a^2}$$

Then

$$R_0 = \psi_1 + \frac{\psi_2}{m} + \psi_3 + \frac{\psi_4}{n} - \left(2 + \frac{1}{m} + \frac{1}{n}\right)\psi_0 \qquad (6)$$

Since the residuals at nodes (1) and (3) are unaffected by the proximity of the boundary the resulting relaxation pattern is as shown in Figure 5 (b). If only one grid arm is intercepted by a boundary, m or n becomes unity for the other arm. If more than two arms are intercepted a finer grid should be used.

(d) Solution Procedure.

The solution procedure consists of:

(i) establishing stream function values for the boundaries of the flow field.

(ii) assuming values for all interior nodes.

(iii) evaluating residuals at these nodes via equations (5) and (6).

(iv) relaxing the residuals by adjusting function and residual values throughout the grid via the relaxation patterns until all residuals are within a predetermined tolerance.

Since boundary values are constant, boundary residuals are not evaluated except as discussed in the subsequent section.

Block relaxation and over or under relaxation techniques are outlined in Reference 1. Since the over-all residual is reduced only by relaxing nodes adjacent to the boundaries, the solution may be expedited by pushing large residuals towards the boundaries. Frequent review of the residuals during the solution may reduce computation time by avoiding the propagation of errors.

(e) Internal Boundaries.

Laplace's equation will yield a unique solution for each set of boundary values, but it will not differentiate between upstream and downstream nor indicate the validity of the boundary values. For the case of boundaries immersed in the flow field (splitters), it becomes necessary to verify assumed boundary values as they are dependent on the flow field.

With midpoint of the splitter leading edge as one node a grid is defined with its axes parallel and normal to the splitter centreline. The grid will be sufficiently fine that the leading edge node and the node immediately upstream will have essentially the same values of stream function. If such a node immediately upstream of the leading edge satisfies Laplace's equation it may be assumed that the local flow field is aligned with the splitter and that its assumed boundary stream function is correct.

Essentially the same results are obtained by considering the leading edge node as the one of interest, with one arm of the relaxation pattern projected along the splitter centreline, as shown in Figure 6. Then since $\psi_3 = \psi_0$ the residual equation becomes:

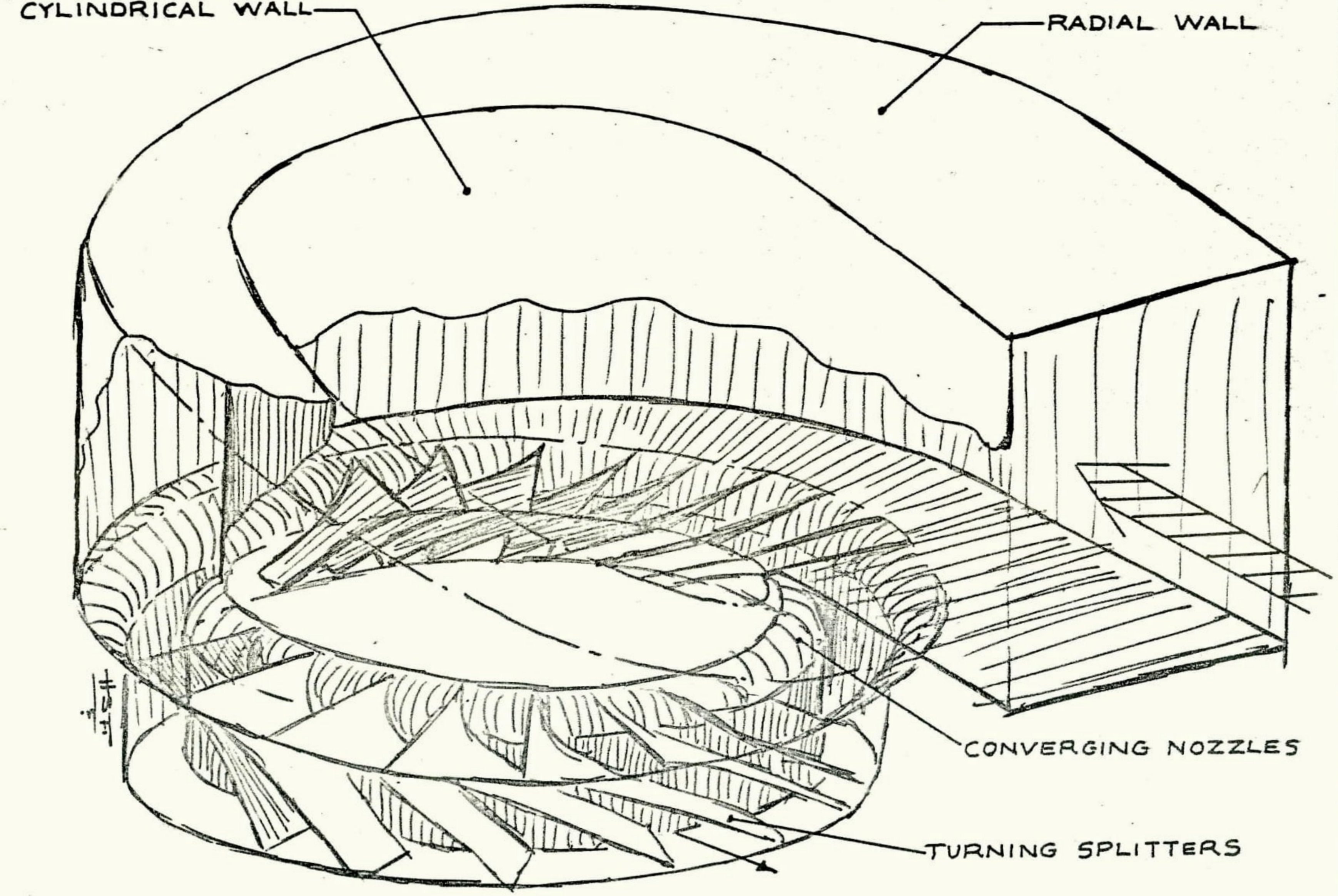
$$R_0 = \psi_1 + \psi_2 + \psi_4 - 3\psi_0 \qquad (7)$$

The leading edge relaxation pattern is set up with the same spacing as the flow field grid since the stream function is assumed linear over such distances.

Subsequent to initial relaxation of the flow field residuals to within tolerance, the splitter stream function values are verified by evaluating the leading edge residuals via equation (7). Then the splitter stream function values are adjusted to reduce these residuals, which necessitates further relaxation of the field nodes. This process is iterative until the splitter leading edge residual is within the tolerance.

The foregoing technique was verified through application to a straight, infinitesimally thick, splitter in a straight passage. The splitter was initially assigned an obviously erroneous stream function and the result of applying the above technique was to bring the splitter function to within the residual tolerance of the correct value.

FIG. 1 PROPOSED VOLUTE AND NOZZLE SECTION SCHEME



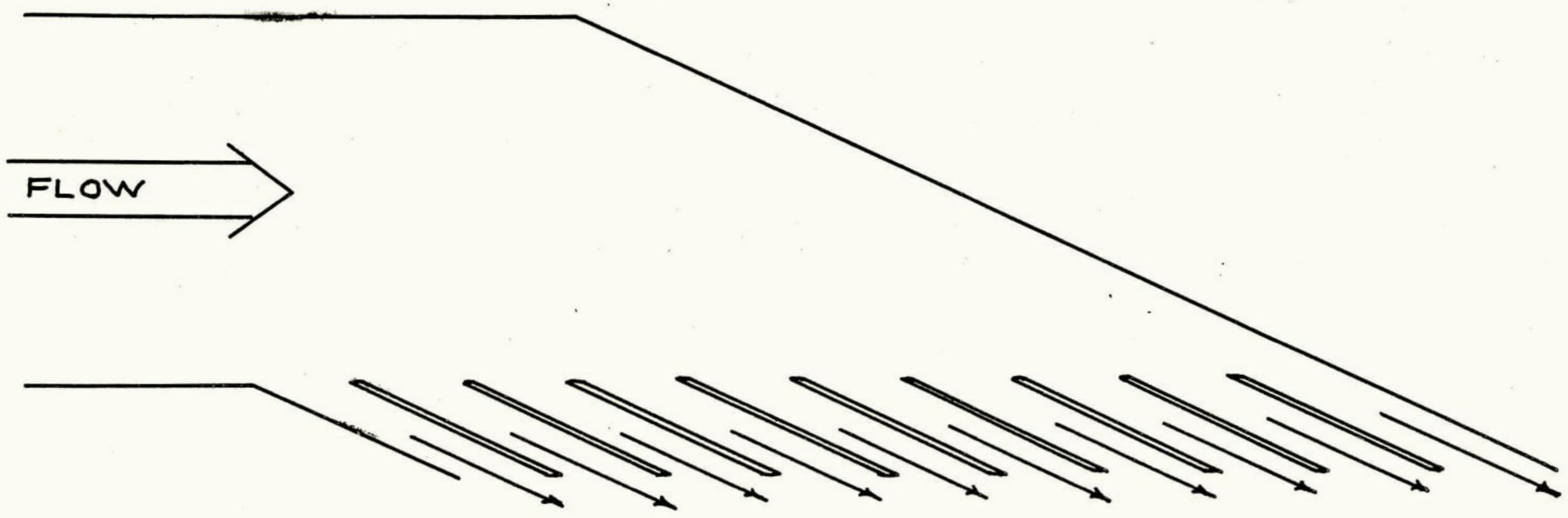


FIG. 2 SIMPLIFIED SECTION THROUGH NOZZLE PASSAGE OF THE UNROLLED VOLUTE AND NOZZLES

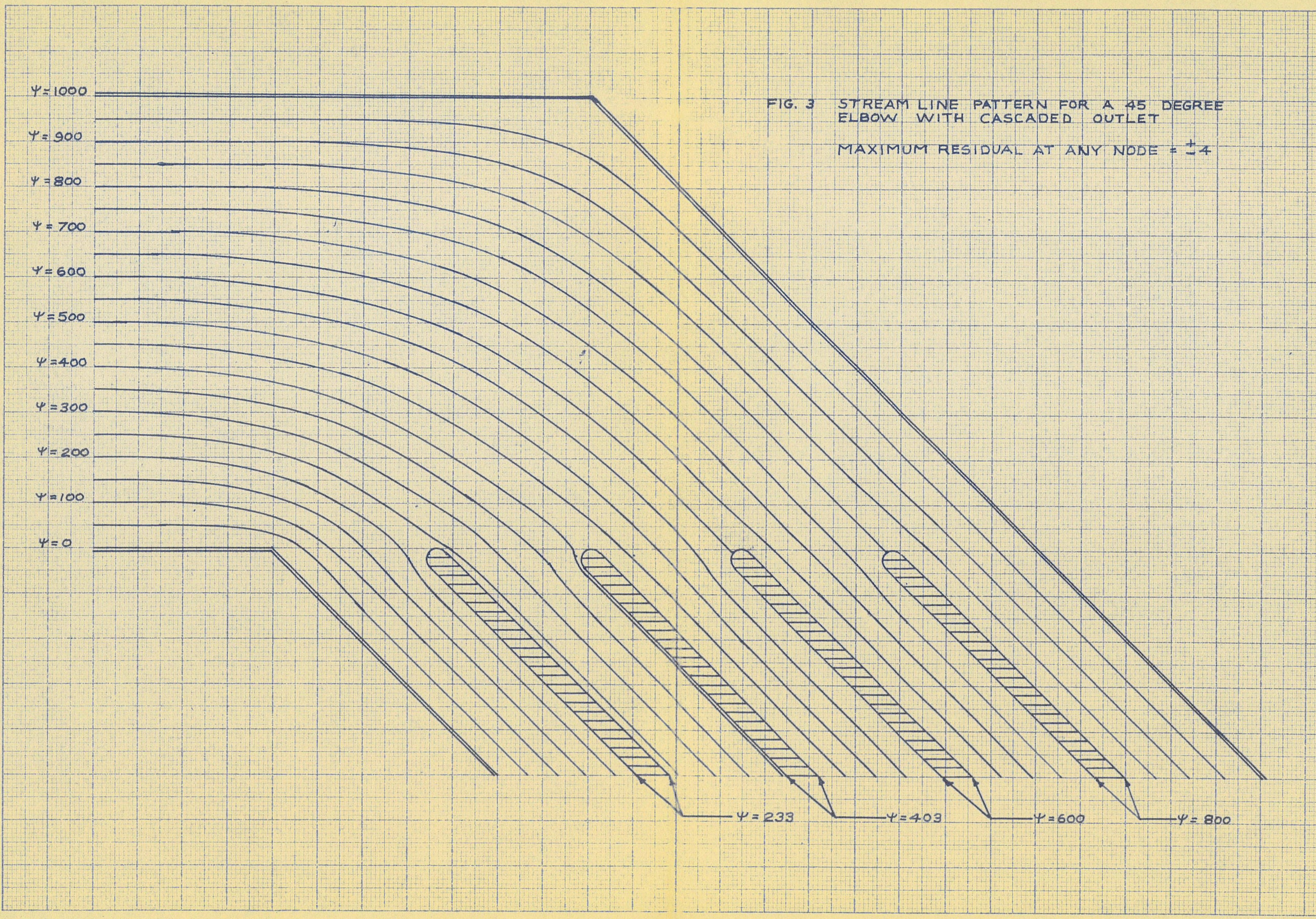


FIG. 3 STREAM LINE PATTERN FOR A 45 DEGREE ELBOW WITH CASCADED OUTLET

MAXIMUM RESIDUAL AT ANY NODE = ±4

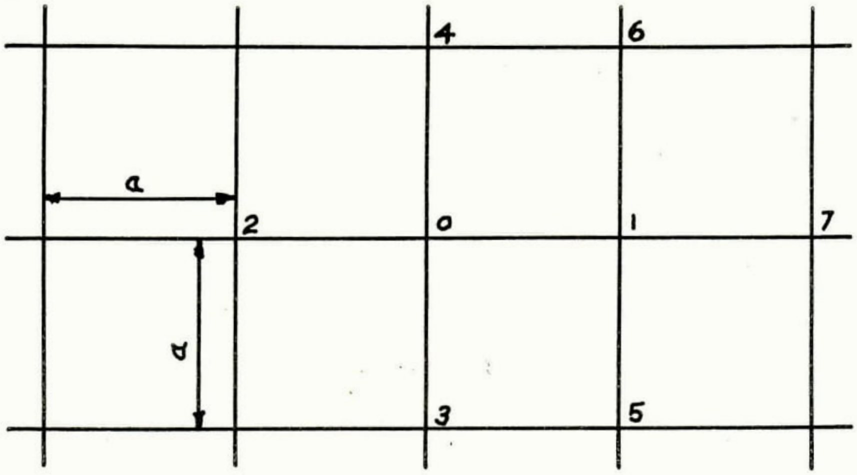
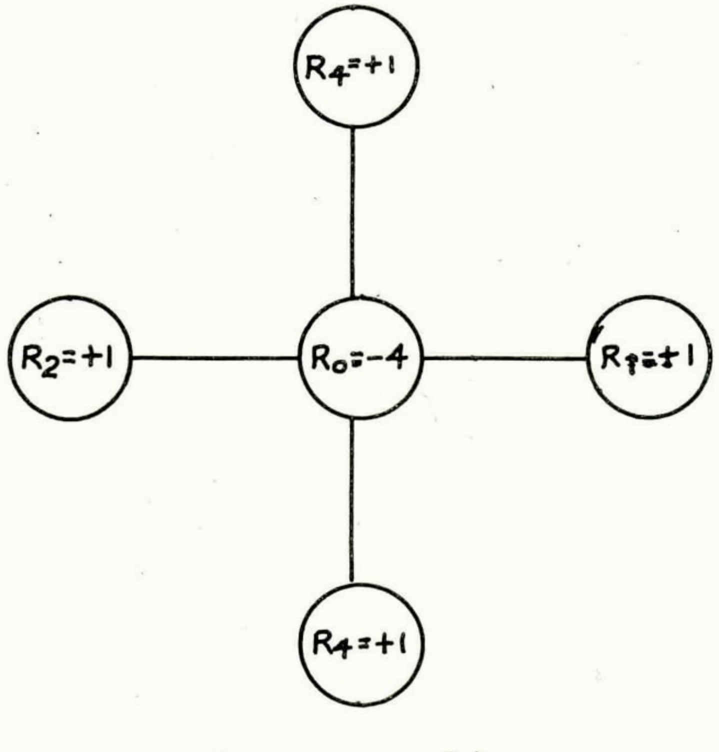


FIG. 4a INTERIOR FLOW FIELD GRID



ΔΨ AT NODE "O" = +1

FIG. 4b INTERIOR RELAXATION PATTERN

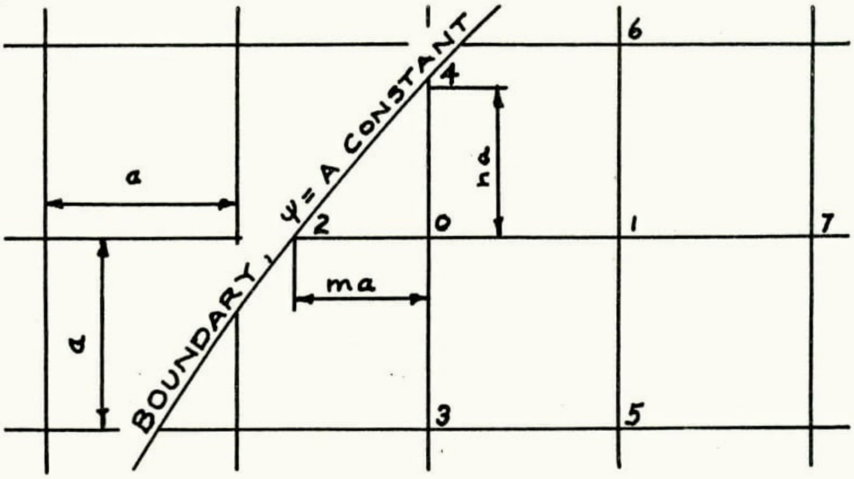
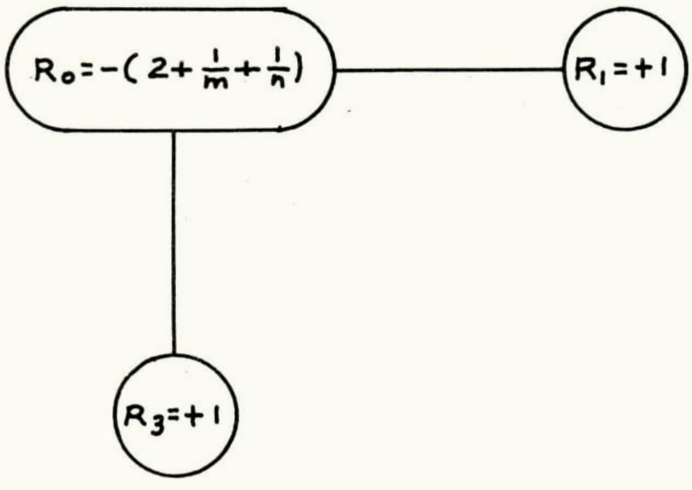


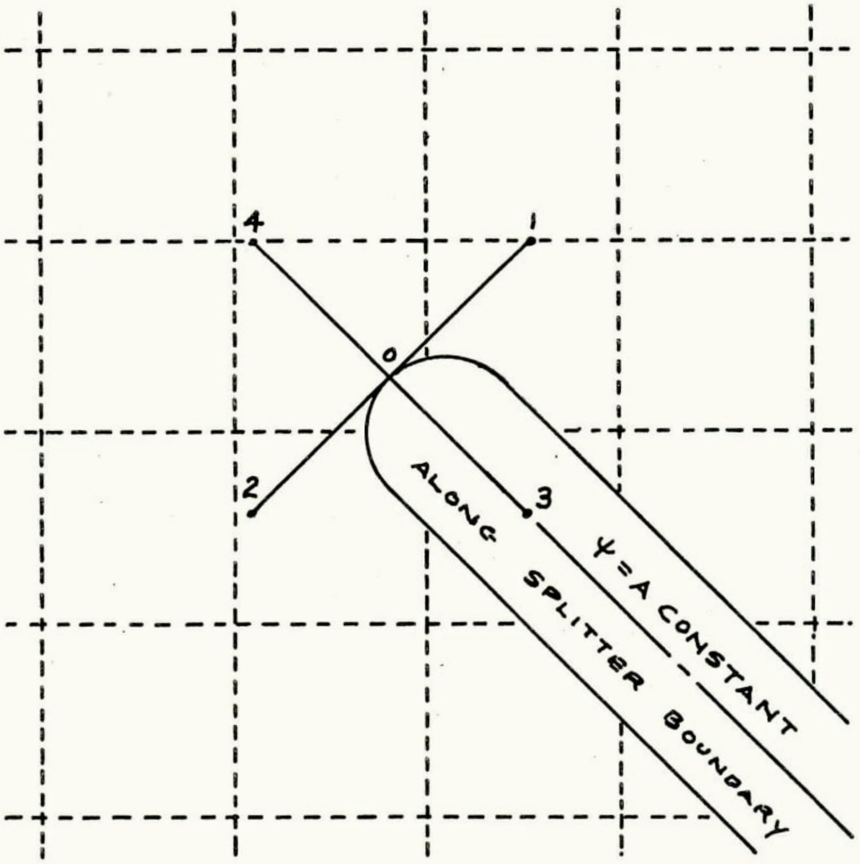
FIG. 5a FLOWFIELD GRID ADJACENT TO BOUNDARY



$\Delta\psi$ AT NODE "0" = +1

FIG. 5b RELAXATION PATTERN ADJACENT TO BOUNDARY

FIG. 6 GRID FOR FINDING SPLITTER RESIDUALS



RESIDUAL EQUATION

$$R_0 = \psi_1 + \psi_2 + \psi_4 - 3\psi_0$$