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WAVEGUIDES WITH SYMMETRICALLY PLACED DOUBLE RIDGES

- E. V. JULL, W. J. BLEACKLEY AND M. M. STEEN -

OTTAWA

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ABSTRACT

The method of finite differences is used to obtain the cutoff wavelength and the field distribution for TE mode propagation in a rectangular waveguide with double ridges placed symmetrically about the guide axis. Design curves for this waveguide, which is potentially very useful in microwave heating and drying applications, are presented.

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FIGURES

WAVEGUIDES WITH SYMMETRICALLY PLACED DOUBLE RIDGES

– E.V. Jull, W.J. Bleackley and Margaret M. Steen –

INTRODUCTION

The introduction of ridges to a rectangular waveguide can provide a larger bandwidth and a greater degree of mode separation. These features have long been exploited in the single- and double-ridged waveguides [1, 2]. Waveguides with multiple ridges, which apparently have not been used previously, are, in addition, capable of producing a transversal and longitudinal field distribution suitable for microwave heating of lossy objects within the guide or for feeding a waveguide array of slots. Uniformity in microwave heating, for example, may be achieved by distributing the power density in the guide in accordance with the shape of the object and the power absorbed by it. This arrangement may be approximated in the waveguide described here.

A cross section of the waveguide with the electric field configuration for dominant mode excitation is shown in Fig. 1. Double rectangular ridges placed symmetrically about the center line of the rectangular waveguide produce an essentially uniform electric field of the desired intensity in a region about the waveguide axis. Power absorption in a long object placed along the guide axis can be compensated for by changing the position of the ridges or, more conveniently, the spacing between them. In this paper design curves for the waveguide, obtained by solving the wave equation inside the guide by the method of finite differences, are presented. A computer program was developed which automatically gives the cutoff wave number and cross sectional field distribution for various positions and sizes of the waveguide ridges.

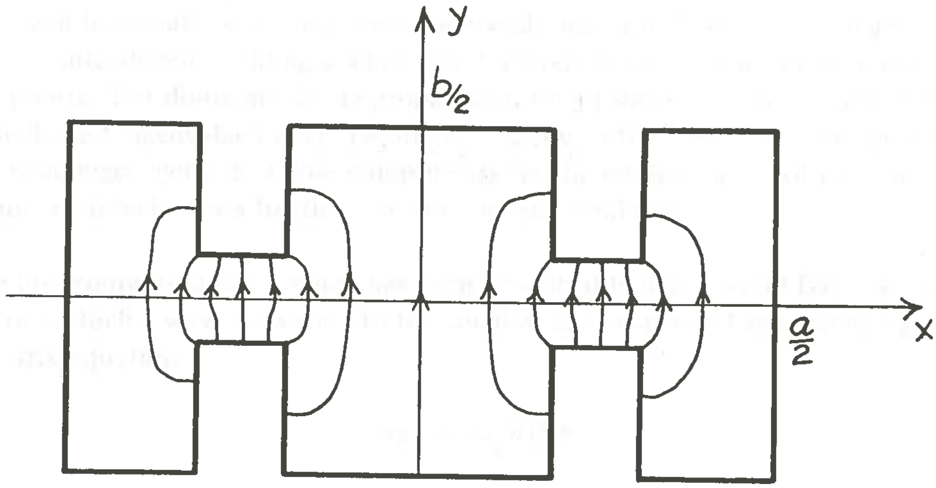

Figure 1 Cross section of a rectangular waveguide with symmetrically placed double ridges showing schematically the electric field distribution for dominant mode TE propagation

NUMERICAL SOLUTION OF THE WAVE EQUATION

It is difficult to obtain analytically the propagation characteristics of waveguides with multiple ridges. An analysis of a ridged rectangular waveguide by transverse resonance, for example, yields a transcendental equation for the cutoff wave number which is awkward to solve for more than a single or double ridge [1, 2]. With the help of an electronic computer, this difficulty can be avoided, by using instead the more direct method of finite differences which yields, in addition to the cutoff wave number, the field distribution in the waveguide cross section.

Finite differences solutions of the Helmholtz equation,

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2 \right] \phi(x,y) = 0, \tag{1}$$

in a cavity resonator were described more than 20 years ago by Motz [3]. As several authors have recently adapted the method to computer studies of waveguides [4, 5, 6, 7] there is little value in more than a brief outline of it here. For a waveguide with rectilinear boundaries, the cross section is subdivided most conveniently into a mesh of square regions with side h, and the longitudinal field $\phi_{j,k}$ at row j and column k of the mesh is related to that at adjacent mesh points by the difference equation

$$\phi_{j,k+1} + \phi_{j-1,k} + \phi_{j,k-1} + \phi_{j+1,k} + [(k_c h)^2 - 4]\, \phi_{j,k} = 0 \tag{2}$$

obtained by approximating a Taylor's series expansion of the Laplacian operator in (1). Boundary conditions at conducting waveguide walls and at field symmetry planes, if they are used, are introduced, yielding a set of simultaneous linear equations equal in number to the field points. For dominant mode propagation the problem is then one of solving this set for the lowest eigenvalue $(k_c h)^2$ (where $k_c = 2\pi/\lambda_c$ is the transverse wave number) and the associated eigenvector ϕ, whose components are the relative values of the longitudinal field. Numerical techniques for this solution are well established.

The procedure used here, which has been described by Collins and Daly [4], is an iterative one which always converges to the smallest eigenvalue and associated eigenvector of the matrix equation

$$A\phi = (k_c h)^2 \phi \tag{3}$$

whatever the trial value of $(k_c h)^2 \phi$ used initially. Iteration is stopped by the computer program when this convergence is judged to be essentially complete. The program required for this calculation is quite simple, as a standard subroutine for inversion of the matrix A may be employed.

The five-point finite difference equation (2)* was applied to the waveguide region in $x > 0$, $y > 0$ of Fig. 1. For the dominant TE mode, the boundary condition $\phi = H_z = 0$, where H_z is the longitudinal component of the magnetic field, was used on the field symmetry plane $x = 0$, and $\partial H_z/\partial n = 0$, where $\partial/\partial n$ is the normal derivative at the boundary, was used on the waveguide walls and on $y = 0$. Higher order modes were examined by changing the boundary conditions on the symmetry planes along the x and y axes.

The computer program fitted the mesh and generated the elements of the matrix from information on the mesh size h and the waveguide dimensions and ridge position. It was convenient to accommodate the entire matrix in computer storage in this process with all mesh points within or on the waveguide boundary, as this is both necessary for matrix inversion and desirable for storage economy. Storage space limitations, as well as the time required for matrix inversion, restricted the number of field points which could easily be handled to about 150 and the smallest mesh size used was $h/a = \frac{1}{32}$, where a is the broad waveguide dimension. This limited the accuracy that was directly obtainable in the final result. Successive mesh halving was used where possible, however, and the results were extrapolated to values approximating those for $h/a = 0$. When, as here, the mesh fitting is exact, this extrapolation procedure can be very accurate, even for results obtained from coarse meshes, as Davies and Muilwyk [5] have demonstrated.

NUMERICAL RESULTS

Dominant TE Mode Propagation Characteristics

Numerical values of the cutoff wavelength λ_c/a for dominant TE mode propagation in a symmetrically ridged rectangular waveguide with dimensions $b/a = \frac{1}{2}$ and ridge thickness $t/a = \frac{1}{8}$ are shown in Table I for several values of ridge spacing s/a, gap size g/b and mesh size h/a. The values $f = \lambda_c/a$ for $h/a = 0$ were extrapolated from those computed for f_1, f_2, f_3, where it was possible to fit three mesh sizes, h_1, h_2, h_3, into the waveguide, as illustrated in Fig. 2. With $h_1 = 2h_2 = 4h_3$, Richardson's deferred approach to the limit [9]† gives

$$f = \frac{Kf_2 - f_1}{K - 1}, \qquad (4)$$

where $K = (f_2 - f_1)/(f_3 - f_2)$. For a fixed s/a, the same value of K in (4) was assumed for all g/b when only two mesh sizes fitted, and interpolation was used to correct the values

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*Collins and Daly [4] observed that higher approximations to the Laplacian operator do not significantly affect the final results. This observation, based on numerical results for the ridged waveguide, may have been influenced by the reentrant corner. The five-point formula has been shown to yield results more accurate than the nine-point formula in solving Laplace's equation near a 90° corner [8].

†Aitken's δ^2 — process yielded essentially the same results.

TABLE I

Numerical values of λ_c/a for $b/a = \frac{1}{2}$, $t/a = \frac{1}{8}$

[ ] reference [1] ( ) measured values.

g/b	h/a	s/a 0	0.125	0.250	0.375	0.500	0.625	0.750
0.125	1/32	4.2083	4.1847	3.8653	3.4323	2.9045	2.2657	1.5885
	0	4.48	4.47	4.12	3.72	3.20	2.40	1.58
			(4.60)		(3.78)			
0.250	1/16	3.1471	3.1518	2.9648	2.6924	2.3668	2.0187	1.6769
	1/32	3.3157	3.3290	3.1235	2.8204	2.4512	2.0330	1.6581
	0	3.456	3.481	3.283	3.001	2.629	2.076	1.649
		[3.453]						
0.375	1/32	2.8459	2.8618	2.7178	2.4965	2.2307	1.9505	1.7247
	0	2.93	2.99	2.82	2.66	2.34	1.96	1.72
0.500	1/8	2.31103		2.2361		2.0353		1.8594
	1/16	2.4694	2.4736	2.3779	2.2370	2.0770	1.9280	1.8047
	1/32	2.5413	2.5526	2.4491	2.2904	2.1052	1.9235	1.7873
	0	2.6011	2.620	2.5208	2.366	2.1649	1.910	1.7792
		[2.604]			(2.36)			
0.625	1/32	2.3250	2.3301	2.2579	2.1520	2.0335	1.9242	1.8467
	0	2.37	2.39	2.32	2.22	2.06	1.91	1.84
0.750	1/16	2.1188	2.1144	2.0813	2.0383	1.9935	1.9556	1.9293
	1/32	2.1663	2.1652	2.1213	2.0618	1.9982	1.9422	1.9043
	0	2.206	2.209	2.162	2.095	2.008	1.901	1.893

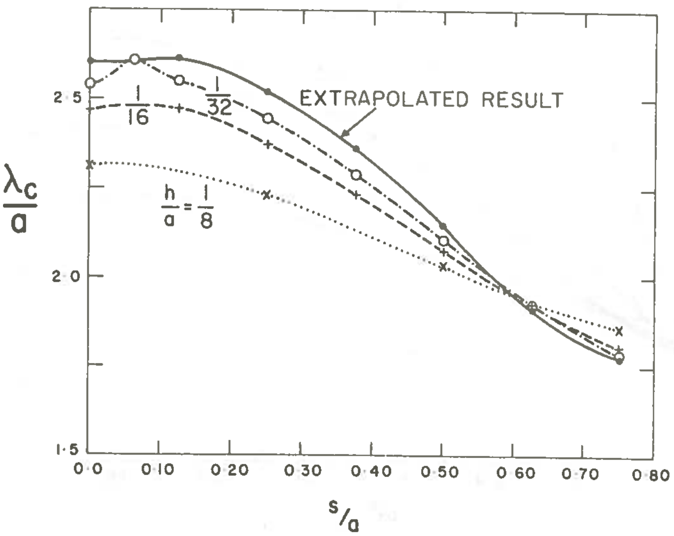

Figure 2 Illustrating extrapolation to an infinitely fine mesh size from coarse mesh sizes for dominant mode propagation in a waveguide with $b/a = \frac{1}{2}$, $t/a = \frac{1}{8}$, *and* $g/b = \frac{1}{4}$.

when only one mesh size was used. The number of figures retained in Table I is suggestive of the final accuracy. Numerical values for the double ridge waveguide ($s/a = 0$) obtained by a different method [1] are shown in square brackets in Table I. Some experimental values are also shown in parentheses.

The extrapolated values of Table I are shown graphically in Fig. 3, where it is evident that a reduction in either ridge or gap spacing increases λ_c/a. The bump which appears in the curves of Figs. 2 and 3 for small values of s/a is probably due to a neglect of the field singularities at the edges of the ridges. A much finer mesh size is needed to account adequately for these singularities.

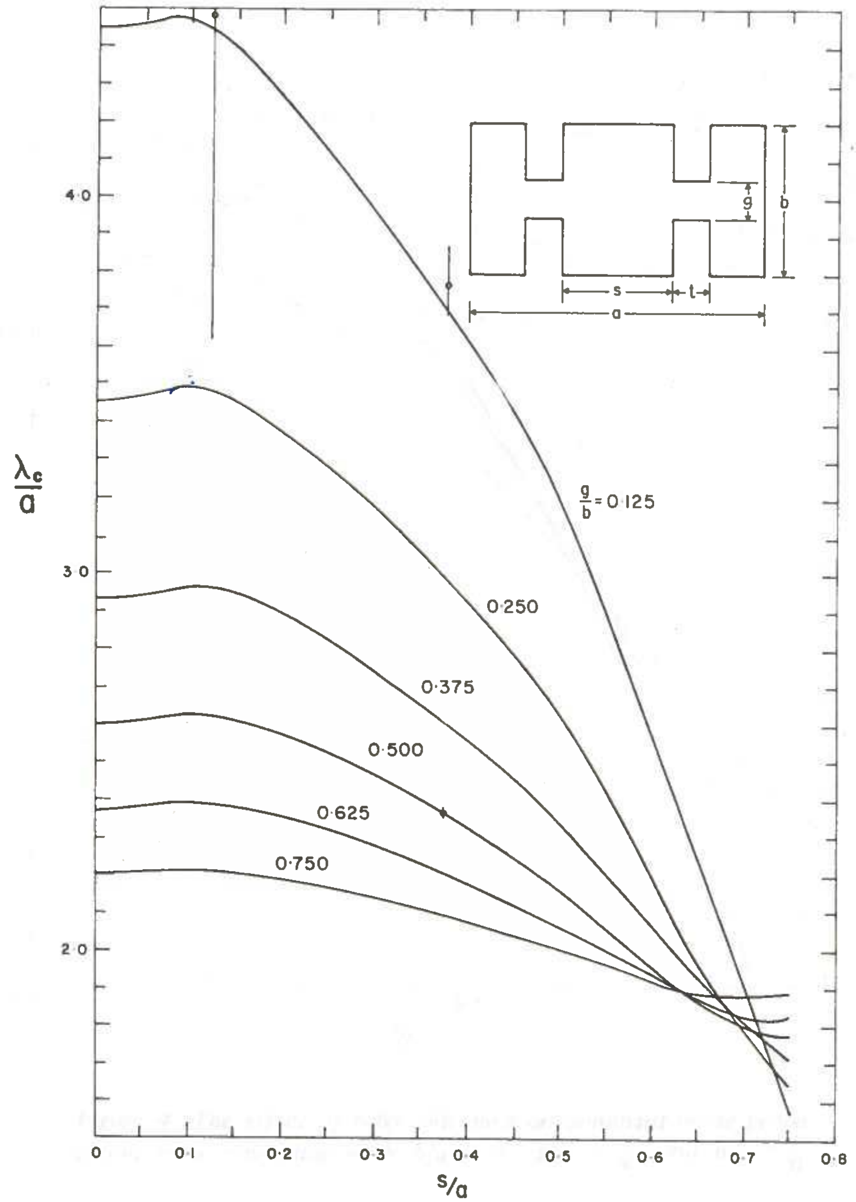

Figure 3 Extrapolated values of the cutoff wavelength for dominant mode propagation with $b/a = \frac{1}{2}$ and $t/a = \frac{1}{8}$. Circles are experimental values.

The effect of changes in ridge thickness t/a on λ_c/a when $g/b = \frac{1}{4}$ is shown in Fig. 4. Here the dimension s'/a is the spacing between the ridge center lines so that the smallest values of s'/a in the curves correspond to a doubly ridged waveguide and the largest values to a waveguide boundary in the shape of a cross. These unextrapolated values correspond to $h/a = \frac{1}{32}$ and so, using values in Table I as a guide, are approximately from $\frac{1}{2}$% above to 7% below the correct values, the largest error occurring near the middle of the range of s'/a. Figure 4 shows that wider ridges increase the cutoff wavelength if the ridge spacing is small, otherwise the cutoff wavelength is decreased.

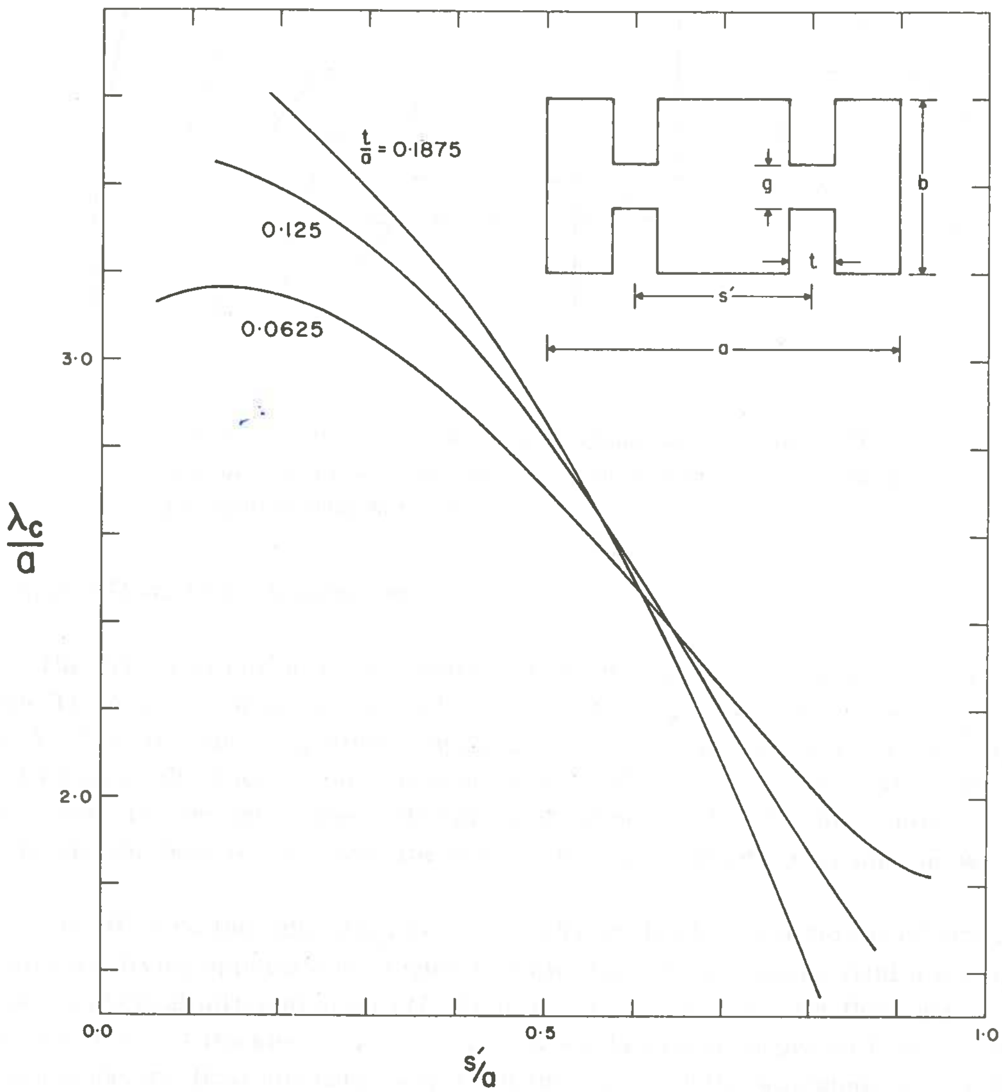

Figure 4 The effect of ridge thickness on dominant mode propagation in a waveguide with $b/a = \frac{1}{2}$, $g/b = \frac{1}{4}$, *and* $h/a = \frac{1}{32}$.

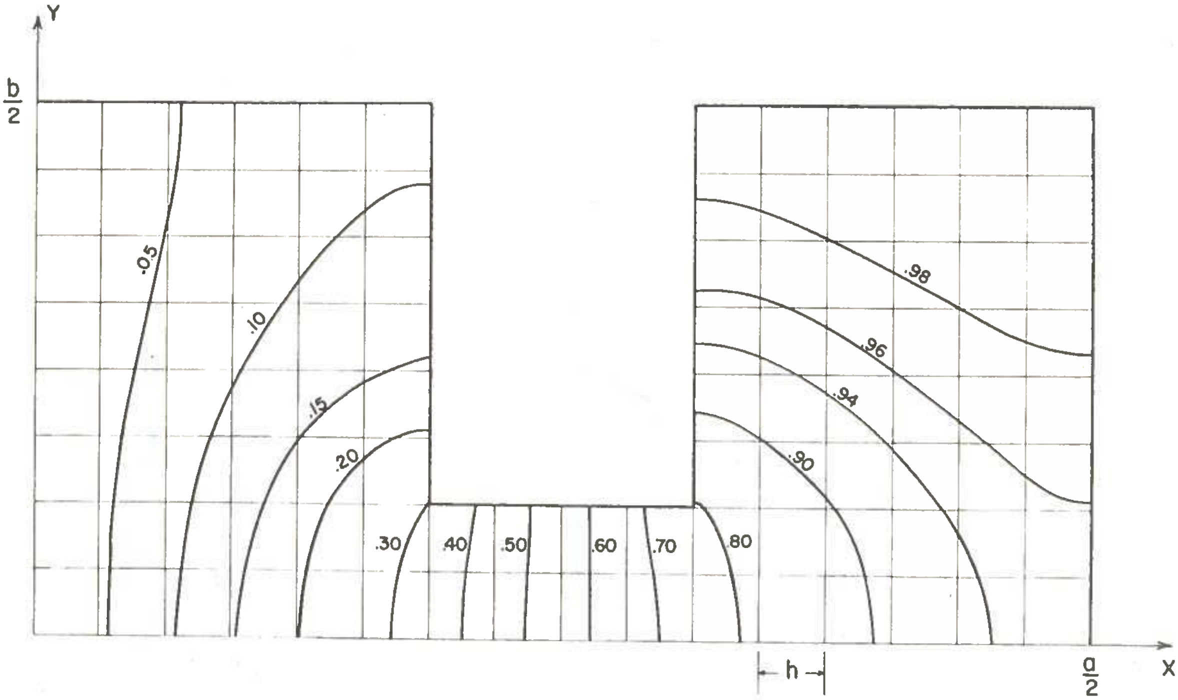

Figure 5 Contours of uniform longitudinal magnetic field in one quarter of the waveguide cross section for dominant mode propagation with $h/a = \frac{1}{32}$.

Dominant Mode Field Distribution

The field distribution in one quarter of the waveguide cross section for dominant mode TE propagation in a guide with $b/a = \frac{1}{2}$, $t/a = \frac{1}{8}$, $g/b = \frac{1}{4}$ and $s/a = \frac{3}{8}$ is shown in Fig. 5. The contours of uniform longitudinal magnetic field H_z were obtained from the field values at the intersection points of the grid; that is, the components of the eigenvector associated with the lowest eigenvalue in the solution of (3). The lines show the orientation of the electric field vector inside the waveguide, and their rate of change shows its intensity.

The effect of the ridge position on the electric field distribution is of interest in microwave drying applications. Figure 6 shows the relative electric field intensity Ey obtained from a numerical differentiation $\partial H_z/\partial x$ in $y = 0$, $0 < x < a/2$ for three gap spacings between ridges of thickness $t/a = \frac{1}{8}$. As one would expect, ridges with small gap spacings draw the electric field intensity away from the center of the waveguide most effectively. The optimum ridge spacing for this appears to be about $s/a = \frac{1}{2}$, as ridges at the edge of the waveguide are not very effective in controlling the field at the center.

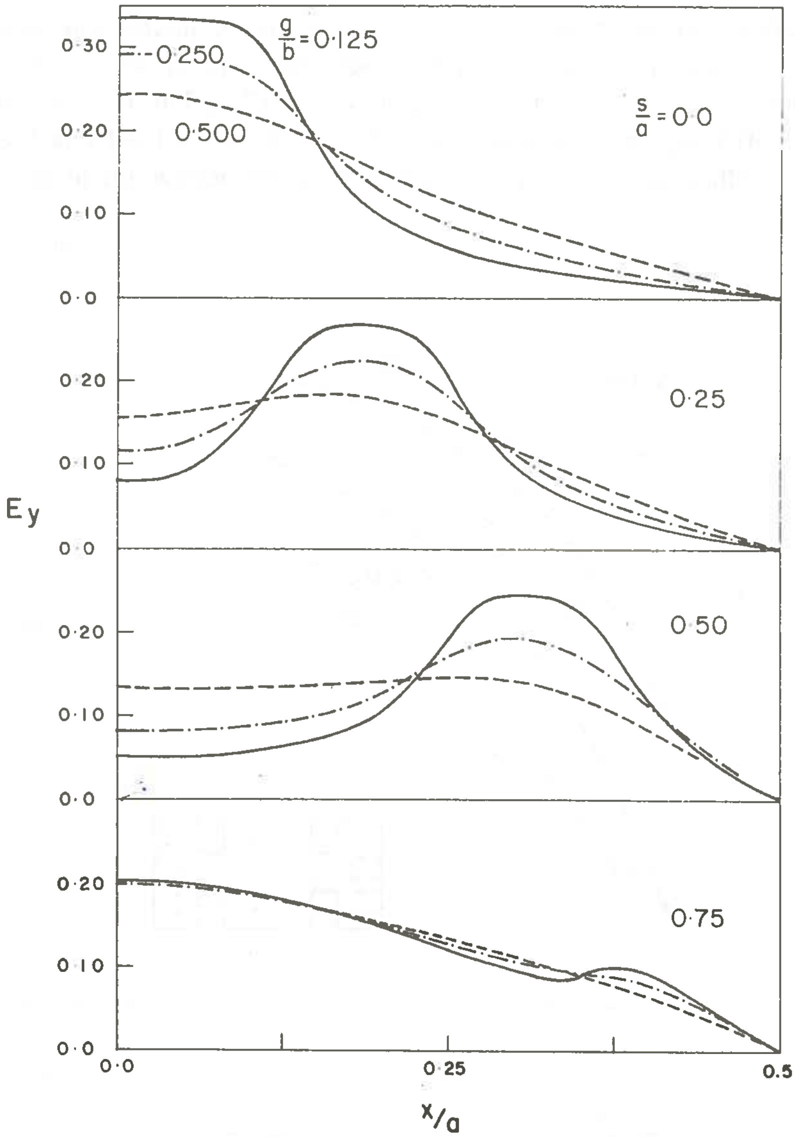

Figure 6 The effect of the ridge position on the relative electric field strength on y = 0, 0 < x < a/2 for dominant mode propagation in a waveguide with $b/a = \frac{1}{2}$, $t/a = \frac{1}{8}$ *and* $h/a = \frac{1}{32}$.

Higher Order TE Modes

The TE mode of next higher order to the dominant mode is that for which $\partial H_z/\partial n = 0$ on both symmetry planes $x = 0$ and $y = 0$. This corresponds to dominant mode propagation in an asymmetrically ridged waveguide for which some numerical values are already available [2, 6]. The largest value of λ_c/a occurs when the single ridge is symmetrically located, and for a waveguide with the parameters of Fig. 3 and $g/b = \frac{1}{8}$, λ_c/a for this mode is 2.5 [2]. Evidently this waveguide provides adequate bandwidth, as indicated by the ratio of the cutoff wavelengths of the fundamental mode to the next higher order mode, if the ridges are not close to the waveguide edge.

The propagation characteristics of some higher order TE modes in the waveguide are shown in Fig. 7. The waveguide dimensions of Fig. 3 apply here and the results are for a mesh size $h/a = \frac{1}{32}$. For Fig. 7*a* the boundary condition $H_z = 0$ on both symmetry planes was used and for Fig. 7*b*, $\partial H_z/\partial n = 0$ was used on $x = 0$, and $H_z = 0$ on $y = 0$. The electric field in the waveguide cross section is shown schematically.

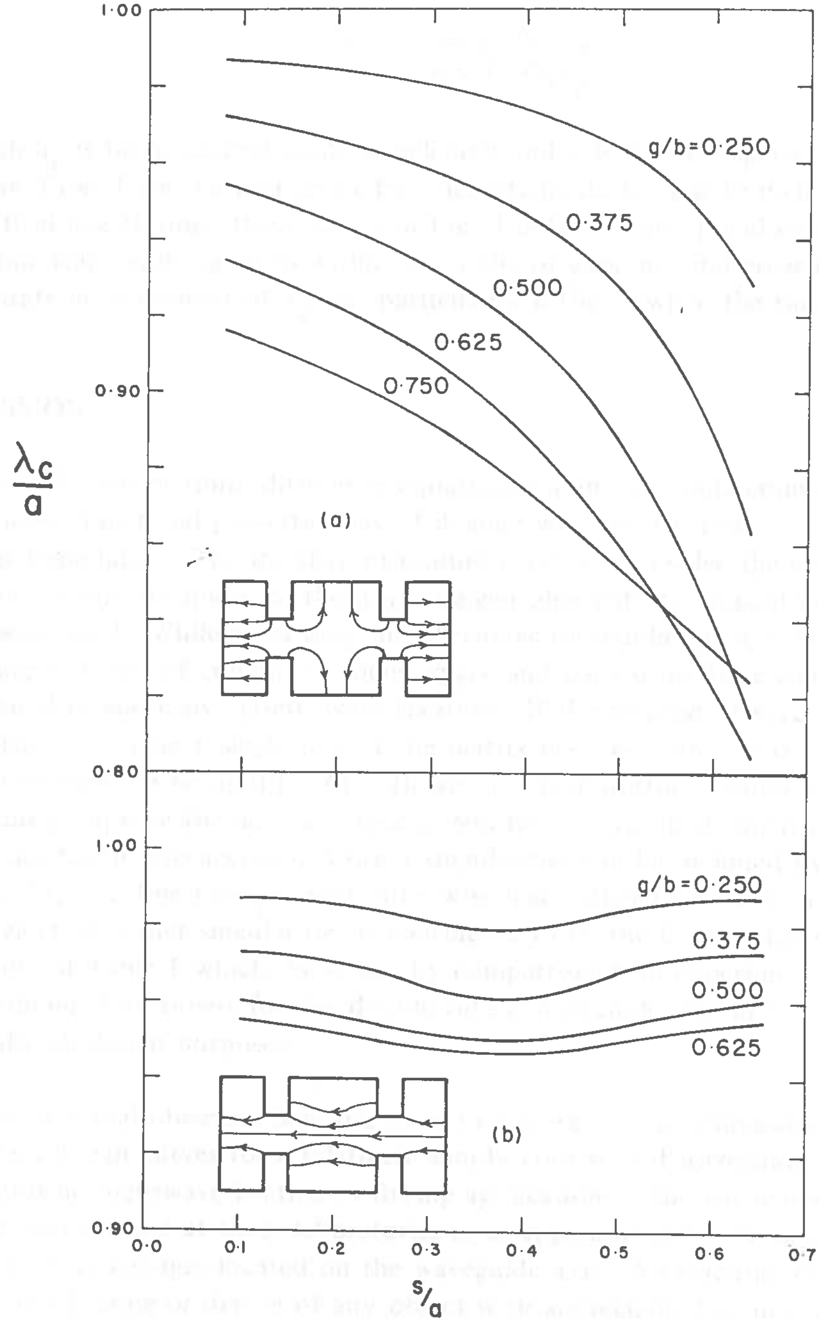

Figure 7 Cutoff wavelengths of some higher order modes in a waveguide with $b/a = \frac{1}{2}$, $t/a = \frac{1}{8}$ and $h/a = \frac{1}{32}$.

EXPERIMENTAL RESULTS

Some wavelength measurements were made at 2.45 GHz on a WR(340) waveguide with double ridges symmetrically placed and with dimensions corresponding to those in Fig. 3. Least accuracy is expected in the larger values of λ_c/a, whether they are obtained numerically or from

$$\frac{\lambda_c}{a} = \frac{\lambda}{a\sqrt{1-(\lambda/\lambda_g)^2}} \tag{5}$$

in which λ_g is the measured guide wavelength and λ is the free space wavelength. The values in Table I are the average of five measurements along a 15-inch ridged section, and the vertical line through these points in Fig. 3 indicates the spread of the measured values. The numerical results agree to within the limits of experimental error in each case, although an accurate measurement of λ_g was particularly difficult when the ridges were closely spaced.

DISCUSSION

The solution of finite difference equations on an electronic computer now appears to be the most direct and powerful way of dealing with propagation in a waveguide with irregular boundaries. The iterative procedure used here to solve the resulting matrix equation converged rapidly to the desired eigenvalue with the mixed boundary values which were used. While possessing the advantage of simplicity, it is an inefficient procedure in terms of computer storage space and computing time compared to, for example, the method of successive point overrelaxation. If the method of successive overrelaxation is used, little more than a single row of the matrix need be stored at one time, permitting a much finer mesh to be used [5, 6].. However it is doubtful whether a finer mesh would significantly improve the accuracy here unless the electric field singularities at the reentrant corners are taken into account. Corner singularities can be included by the method described by Motz [3], if a fine mesh is used, otherwise less, rather than more, accuracy may result. This neglect of corner singularities is considered to be the limiting factor in the accuracy of the results of Table 1 which, however, by comparison with experiment and with numerical results obtained by others for the double ridged waveguide, are sufficiently precise for practically all design purposes.

The principal objective here has been to investigate the propagation characteristics and obtain design curves for a relatively simply constructed waveguide which is potentially very useful in microwave heating or drying applications. The particular application which has been investigated at these laboratories is an apparatus for rapid uniform cooking of objects such as sausages located on the waveguide axis. An essential requirement is fulfilled in the heating or drying of any object with appreciable loss in this waveguide*

*Patents have been applied for in respect of this waveguide.

in that power absorption can be controlled by the gap spacing between the ridges. The numerical results presented show the effects of gap spacing changes, indicate optimum positions for the ridges and show that the waveguide has good bandwidth features. It is hoped that they will also be useful in other applications of this waveguide.

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