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Residual surface undulations in the OEB after the termination of wave generation in shallow water

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Residual surface undulations in the OEB after the termination of wave generation in shallow water

Technical Report - Unclassified OCRE-TR-2014-013

Hasanat Zaman Lawrence Mak Jim Millan Shane McKay

St. John's, NL

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Ocean, Coastal and River Engineering

Génie océanique, côtier et fluvial

Residual surface undulations in the OEB after the termination of wave generation in shallow water

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ABSTRACT

During the generation of any wave in the tank we get relevant primary wave component along with bounded wave components if the incident primary wave has more than one frequency. Inevitably we also get interacted wave components, natural frequency components of the tank and other free waves. In this report the natural frequency energy components in the OEB were investigated using several cases of mono- and bi-chromatic waves in the tank. These natural frequency components were compared with the total energy and bounded wave energy measured in the tank. The reason of this investigation is to understand the extent of the natural frequency energy components of the tank of different modes in the measured wave data and also the damping rate of the residual surface undulation components in the tank with time. Results are shown for both monoand bi-chromatic waves. It is found that the primary frequency components are interacting with each other in different forms and wave energies are transferred to the interacted frequency bands. Also from the data, we observed that the natural frequency components for both longitudinal and transverse directions of the tank were too small compare to the primary wave components at least in the cases that we considered here.

1. INTRODUCTION

We need to generate correct wave in the wave tank during any model test. So it is very important to know every wave component available in the wave tank before starting a "run" in the tank in any model test. We need to understand the characteristics and magnitude of the residual surface undulations in the tank over time after the wave making is terminated. This information will help the wave tank (OEB) users to reduce or increase the waiting-time in between consecutive "runs" in the wave tank. The model test results would be jeopardized due to the presence of the large amplitude residual surface undulations. So we have to be sure that the available surface undulation is small enough not to contaminate the next "run". When we generate wave in the tank we obtain primary wave components, bounded waves, interacted wave components, unwanted free wave components (Zaman et al, 2011) and some contribution of natural frequency energy components of the wave tank. Interactions of the wave components are numerous but not necessarily as important due to their small amplitudes. Low frequency bounded waves are obtained for frequency differences. Tank natural frequency energy components are obtained from the data measured after the wave making was stopped. We measured data for 20 minutes where the wave making was ended after first 3 minutes. The reason is to observe the residual surface undulations in different time segments over last or extra 17 minutes of measuring time and also to identify the natural frequency energy components of the tank. Several mono- and bi-chromatic waves were tested over three different water depths (04m, 0.5m and 0.6m) in this experiment.

2. EXPERIMENTS

The Offshore Engineering Basin (OEB) is a world class 3D wave tank in the National Research Council, St. John's Canada where the experiment was carried out. The top view of the basin is shown in Fig.1. The OEB is 75m long \times 32m wide and 56 independently

controlled segmented wave generators installed on the west wall generated the waves. Each segmented wave generator is 2m high and 0.5m wide. Passive absorbers, made of expanded metal sheets with varying porosities and spacing, are installed on the east wall. The water depths for the experiments were 0.4m, 0.5m and 0.6m.

2.1 Experimental setup

During the experiment, 14 wave probes installed as shown in Fig.1 and Table-1 measured the location of the wave probes throughout the basin. All the wave probes are capacitance type. All the data was acquired using GDAC (GEDAP Data Acquisition and Control) client-server acquisition system, developed by National Research Council Canada. The bottom of the basin was flat and the blanking plates were deployed to cover the north beach.



Fig.1: Top view of the experimental setup in the OEB



Photo 1: Setup of 14 wave probes in the OEB

Number of probe	Distance from the east wave	Distance from the
	paddle(<i>m</i>)	south wall (<i>m</i>)
1	26.891	13.475
2	27.221	13.475
3	27.731	13.475
4	27.731	12.955
5	27.731	12.635
6	27.731	14.825
7	27.731	18.365
8	29.081	13.475
9	32.621	13.475
10	41.621	13.475
11	2.0	12.635
12	2.0	13.475
13	2.0	18.365
14	10.744	13.475

Table 1 Location of the wave probe in the OEB

2.2 Natural frequency components in a closed basin

The OEB is a rectangular basin with significant length (= 72m), width (= 26m) and depth (= 3m). The natural periods and frequencies can be estimated using following equations (Mei, 1999):

$$T_{nm} = \frac{2}{\sqrt{gh}} \left[\left(\frac{n}{a} \right)^2 + \left(\frac{m}{b} \right)^2 \right]^{-\frac{1}{2}} \quad ; \quad \sqrt{gh} = c \quad ; \quad n, m = 0, 1, 2, 3.....$$
(1)

$$f_{nm} = \frac{1}{T_{nm}} \tag{2}$$

where T_{nm} and f_{nm} are natural period and frequency, h (= 3m) is the depth, a (= 72m) is the length and b (= 26m) is the width of the basin.

We can compute the transverse and longitudinal mode components as follows:

Transverse Ist mode component is

$$T_{01} = \frac{1}{f_{01}} = \frac{2}{\sqrt{gh}} \cdot b$$
; $n = 0$ and $m = 1$ (3)

Longitudinal Ist mode component is

$$T_{10} = \frac{1}{f_{10}} = \frac{2}{\sqrt{gh}} \cdot a \quad ; \quad n = 1 \text{ and } m = 0$$
(4)

Transverse 2nd mode component is

$$T_{02} = \frac{1}{f_{02}} = \frac{2}{\sqrt{gh}} \cdot \frac{b}{2} \quad ; \quad n = 0 \quad \text{and} \quad m = 2$$
(5)

Longitudinal 2nd mode component is

$$T_{20} = \frac{1}{f_{20}} = \frac{2}{\sqrt{gh}} \cdot \frac{a}{2}$$
; $n = 2$ and $m = 0$ (6)

Transverse 3rd mode component is

$$T_{03} = \frac{1}{f_{03}} = \frac{2}{\sqrt{gh}} \cdot \frac{b}{3} \quad ; \quad n = 0 \quad \text{and} \quad m = 3$$
(7)

Longitudinal 3rd mode component is

$$T_{30} = \frac{1}{f_{30}} = \frac{2}{\sqrt{gh}} \cdot \frac{a}{3}$$
; $n = 3$ and $m = 0$ (8)

The transverse and longitudinal natural frequency components for different modes can be computed using Eqs. 3 to 8:

$$T_{01} = \frac{1}{f_{01}} = 9.578s$$
 $T_{02} = \frac{1}{f_{02}} = 4.789s$ $T_{03} = \frac{1}{f_{03}} = 3.192s$ [Transverse] (9)

$$T_{10} = \frac{1}{f_{10}} = 26.524s$$
 $T_{20} = \frac{1}{f_{20}} = 13.262s$ $T_{30} = \frac{1}{f_{30}} = 8.841s$ [Longitudinal] (10)

2.3 Methodology, scope and limitations

This paper reports the first phase of the project to identify and quantify the natural frequency components in the OEB. The first phase is limited to the investigation of monochromatic and bichromatic waves. Irregular waves are planned for next phase. Wave splitting described Mansard et al. (1987) and Zaman et al. (2010, 2011) was used

to separate the different wave components – primary wave(s), bounded wave(s) and free wave(s). Natural frequency components in the longitudinal and transverse directions were identified using Equations 9 to 14. The amplitudes of the different components were estimated using spectral analysis. For typical regular and bichromatic wave tests, our clients are interested in the 5-10 repeat cycles, for zero-crossing analysis, spectral analysis, response amplitude operator analysis, while minimizing wave reflection. Thus, 3 minutes is adequate time to allow for 10s ramp up, evanescent wave component, 5-10 repeat cycles and 10s ramp down. As an example we can see Case-2, the longest monochromatic wave that we have used here where T=4.105s. In 3 minutes, we have 43 cycles. Δf =0.0055Hz.

3. EXPERIMENTAL CONDITIONS

In the experiments both mono- and bi-chromatic waves of varying wave periods, wave heights and water depths were used. Table 2 shows different incident wave conditions for bi-chromatic waves that we used in the experiments. On the other hand, Table 3 shows different conditions for mono-chromatic incident waves. In the tables below T_1 , T_2 and T are wave periods, H is the wave heights and h is the still water depth.

Cases	Water depth h (m)	Wave period T_1 (s)	Wave period T_2 (s)	Wave height $H(m)$
Case-1	0.4m	1.55s	1.45s	0.06m
Case-2	0.4m	2.22s	2.00s	0.06m
Case-3	0.5m	1.55s	1.45s	0.06m
Case-4	0.5m	2.22s	2.00s	0.06m
Case-5	0.6m	1.55s	1.45s	0.06m
Case-6	0.6m	2.22s	2.00s	0.06m

Table 2 shows incident wave conditions for bi-chromatic wave

Table 3 shows incident wave conditions for mono-chromatic waves

	Water depth	Wave period	Wave height
Cases	<i>h</i> (m)	<i>T</i> (s)	$H(\mathbf{m})$
Case-7	0.4m	2.145	0.08m
Case-8	0.4m	4.105	0.08m
Case-9	0.5m	1.977	0.08m

The bottom of the basin was flat and the blanking plates were deployed to cover the north beach.

4. DATA ANALYSES

We used the igor6 software routines to analyze the obtained experimental data. We have analyzed two sets of data, one for bi-chromatic (Tables 2) waves and the other one is for mono-chromatic waves (Table 3).

All the data are analyzed and the results are shown in terms of surface elevations and interacted wave components. Comparisons of the primary wave energies with bounded and natural frequency energy components along and across the wave tank at Probes 12-1-2-3-8 (P-12, P-1, P-2, P-3, P-8) and at Probes 6-3-4 (P-6, P-3, P-4) are also shown, see Fig.1.

4.1 COMPARISONS-1: Bi-chromatic waves

In this typical comparison six cases have been considered. Table 2 summarizes the bichromatic incident waves that we considered in this paper. Table 4, 5, 6, 7, 8 and 9 show the comparisons of the total energy (TE), bounded wave energy (BE) and natural frequency energy (NE) components (includes f01, f10, f02, f20, f03 and f30) at probes P-1, P-2, P-3, P-4, P-6, P-8 and P-12. The comparisons [%=bounded waves energy/total energy*100 or, natural frequency energy/ total energy * 100] are done with respect to the total energy at each probe location. The tables show results obtained from all 20 minutes experimental data. P-12 was located just 2m away from the face of the wave maker.

Probe	ТЕ		NE/TE*100%				
location	IE	DE/1E·100%	f01	f10			
	Longitudinal probes						
P-12	0.000082	0.48117%	0.02655%	0.05317%			
P-1	0.000107	1.02105%	0.03338%	0.01198%			
P-2	0.000098	1.19002%	0.03451%	0.01070%			
P-3	0.000070	2.23456%	0.03710%	0.01364%			
P-8	0.000072	2.49807%	0.02786%	0.01231%			
		Transverse	probes				
P-4	0.000076	2.13836%	0.04103%	0.02480%			
P-3	0.000070	2.23456%	0.03710%	0.01364%			
P-6	0.000076	2.17022%	0.03746%	0.03795%			

Table 4 Comparisons of bounded wave, natural frequency and total energy for Case-1

Probe	TE	BE/TE	NE/TE*100%				
location		*100%	f01	f10	f20	f30	
		L	ongitudinal	probes			
P-12	1.00E-04	1.06917%	0.01783%	0.02440%	0.01190%	0.05648%	
P-1	9.31E-05	1.23413%	0.02866%	0.00743%	0.00805%	0.12175%	
P-2	8.57E-05	1.57034%	0.03072%	0.00591%	0.00897%	0.13851%	
P-3	8.07E-05	2.04555%	0.02694%	0.01144%	0.01113%	0.13612%	
P-8	1.08E-04	2.41394%	0.01806%	0.00845%	0.01136%	0.09030%	
]	Transverse p	orobes			
P-4	8.72E-05	1.92386%	0.03129%	0.02386%	0.01562%	0.14171%	
P-3	8.07E-05	2.04555%	0.02694%	0.01144%	0.01113%	0.13612%	
P-6	7.10E-05	2.27652%	0.03484%	0.03246%	0.00915%	0.15946%	

Table 5 Comparisons of bounded wave, natural frequency and total energy for Case-2

From the comparisons it is observed that bounded wave energy is just 0.5% to 2.5% compare to the primary waves of different cases considered here. On the other hand, the energies of different natural frequency components are 0.01% to 0.15% compare to generated primary waves. In all cases it is also observed that the natural frequency energy components in the transverse direction are obtained negligible for higher modes ($f_{02} = f_{03} = ... = 0$). We did not show those values when the natural frequency components are exceptionally small.

Table 6 Comparisons of bounded wave, natural frequency and total energy for Case-3

Probe	ТЕ	DE/TE*1000/		NE/TE	*100%	
location	IE	DE/1E 100%		f10	f20	
	Longitudinal probes					
P12	1.47E-04	0.16246%		0.07720%	0.03273%	
P1	1.49E-04	0.76671%		0.01700%	0.02036%	
P2	1.48E-04	0.98104%		0.01279%	0.01878%	
P3	9.70E-05	1.69553%		0.02069%	0.03945%	
P8	8.68E-05	1.77684%		0.01899%	0.06007%	
Transverse probes						
P4	1.05E-04	1.55102%		0.01671%	0.04152%	
P3	9.70E-05	1.69553%		0.02069%	0.03945%	
P6	9.63E-05	1.40465%		0.02893%	0.03962%	

Probe	ΤE	DE/TE*1000/	NE	E/TE	*100%		
location	IE	DE/ 1E ¹ 100%	f01	f10	f20		
	Longitudinal probes						
P12	1.48E-04	0.65984%	0.07571%		0.03183%]	
P1	1.47E-04	1.64787%	0.12764%		0.01757%		
P2	1.52E-04	1.56269%	0.11487%		0.01347%		
P3	1.40E-04	1.55534%	0.11027%		0.02271%		
P8	1.52E-04	1.68559%	0.08304%		0.02667%		
	Transverse probes						
P4	1.49E-04	1.41643%	0.10241%		0.02193%		
P3	1.40E-04	1.55534%	0.11027%		0.02271%]	
P6	1.17E-04	1.68081%	0.11471%		0.02671%		

Table 7 Comparisons of bounded wave, natural frequency and total energy for Case-4

Table 8 Comparisons of bounded wave, natural frequency and total energy for Case-5

Probe	ТЕ	DE/TE *1000/		NE/TE	*100%	
location	IL	BE/1E*100%	f01	f10	f20	
	Longitudinal probes					
P12	1.09E-04	0.02604%		0.07159%	0.01148%	
P1	9.23E-05	0.44812%		0.01821%	0.00657%	
P2	8.77E-05	0.38763%		0.01689%	0.00514%	
P3	1.09E-04	0.30642%		0.01204%	0.00742%	
P8	1.05E-04	0.28397%		0.00949%	0.01004%	
Transverse probes						
P4	9.58E-05	0.30787%		0.01429%	0.00956%	
P3	1.09E-04	0.30642%		0.01204%	0.00742%	
P6	9.92E-05	0.31892%		0.01825%	0.00712%	

Probe	ТЕ			NE/TE*10	0%		
location	IL	BE/TE*100%	f01	f10	f20		
	Longitudinal probes						
P12	1.05E-04	0.80368%	0.01422%	0.06418%	0.01197%		
P1	1.05E-04	2.27455%	0.01573%	0.01232%	0.00620%		
P2	9.65E-05	2.09494%	0.01416%	0.01279%	0.00613%		
P3	1.07E-04	2.15163%	0.01443%	0.01130%	0.00762%		
P8	1.11E-04	1.65550%	0.00885%	0.00892%	0.01014%		
	Transverse probes						
P4	1.09E-04	1.87963%	0.01088%	0.01099%	0.00946%		
P3	1.07E-04	2.15163%	0.01443%	0.01130%	0.00762%		
P6	9.35E-05	2.24964%	0.01470%	0.01620%	0.00945%		

Table 9 Comparisons of bounded wave, natural frequency and total energy for Case-6

4.2 COMPARISONS-2: Mono-chromatic waves

In this typical comparison six cases have been considered. Table 3 summarizes the incident wave parameters for mono-chromatic wave.

Tables 10, 11 and 12 show the comparisons of natural frequency components and total energy components at probes P-1, P-2, P-3, P-4, P-6, P-8 and P-12. The comparisons [natural frequency energy/ total energy*100] is done with respect to the total energy at each probe location. In this case also natural frequency energy components are found very small compare to primary wave energies.

Table 10 Comparisons of natural frequency and total energy for Case-7

Probe	TE	NE/TE*100%							
location		f01	f10	f20	f30				
Longitudinal probes									
P12	8.00E-05		0.08939%	0.04948%	0.01305%				
P1	5.16E-05		0.02293%	0.06724%	0.03438%				
P2	6.14E-05		0.02390%	0.06392%	0.02999%				
P3	7.90E-05		0.01629%	0.04413%	0.01995%				
P8	6.80E-05		0.01743%	0.07412%	0.01956%				
Transverse probes									
P4	9.01E-05		0.02388%	0.04606%	0.02435%				
P3	7.90E-05		0.01629%	0.04413%	0.01995%				
P6	7.45E-05		0.03369%	0.05795%	0.02405%				

Probe location	TE	NE/TE*100%								
	IL	f01	f10	f20	f30					
Longitudinal probes										
P12	7.91E-05		0.06373%	0.03103%	0.01206%					
P1	1.01E-04		0.01190%	0.01952%	0.01328%					
P2	1.06E-04		0.01047%	0.01961%	0.01280%					
P3	1.04E-04		0.01029%	0.01964%	0.01135%					
P8	1.48E-04		0.00624%	0.01653%	0.00663%					
Transverse probes										
P4	1.02E-04		0.01117%	0.02183%	0.01351%					
P3	1.04E-04		0.01029%	0.01964%	0.01135%					
P6	1.02E-04		0.02265%	0.02897%	0.01340%					

Table 11 Comparisons of Natural frequency and total energy for Case-8

Table 12 Comparisons of Natural frequency and total energy for Case-9

Probe	TE	NE/TE*100%					
location	IL	f01	f10	f20			
Longitudinal probes							
P12	9.35E-05		0.11631%	0.02586%			
P1	6.11E-05		0.02562%	0.00322%			
P2	6.96E-05		0.01933%	0.02852%			
P3	8.03E-05		0.01471%	0.02705%			
Transverse probes							
P4	9.40E-05		0.01836%	0.02391%			
P3	8.03E-05		0.01471%	0.02705%			
P6	8.08E-05		0.03700%	0.03181%			

5. RESULTS

In this experiment 3 different water depths (h=0.4m, 0.5m and 0.6m) were used. Results of 3 different depths are shown and described here. The incident wave parameters are shown in Table 2 and Table 3 for bi- and mono-chromatic waves, respectively. From the measured data, we can find out the relationship between the primary frequency components and interacted frequency components. The natural frequency components are found very small.

5.1 Bi-chromatic waves

In Table-2, Case-2 represents the shallowest incident wave condition for bi-chromatic waves. We have chosen this case to comprehend the wave propagation and damping duration with the existing tank/beach condition after the wave making is terminated. Figs. 2a to 13h show the surface elevations for different propagation time segments (0-20*s*, 10-11.33*s*, 15-16.33*s* and 18.67-20*s* : each segment has 4096 data) and their corresponding energy spectra for Case-2 at probes P-1, P-2, P-3, P-4, P-5, P-6, P-7, P-8, P-9, P-10, P-12 and P-14. It is observed in the results that the residual surface undulations are dumped out over time naturally. It is also perceived that in 7 minutes after the wave making stopped, the surface undulation becomes negligibly small [$O(10^{-4})$ compare to the generated primary frequencies]. See **Appendix-I** for figures.

Figs. 14a to 19d show amplitude spectra of the incident primary wave frequencies, different interacted frequency components $(f_1+f_2, f_1+2f_2, 2f_1+2f_2, ...)$ and wave tank's natural frequency components $(f_{01}, f_{10}, f_{02}, ...)$ and are normalized by the incident wave height *H*. Figs. 14a to 19d show results for Case-2 at probes P-12, P-1, P-2, P-3 and P-8 along the longitudinal direction in the tank for different time segments (0-20s, 10-11.33s, 15-16.33s and 18.67-20s). **Appendix-II** shows these figures.

Figs. 20a to 20h show comparisons of the wave energies among Case-1, Case-3 and Case-5. In these three different cases wave periods and wave height remain constant but water depth varies (h=0.4m, 0.5m and 0.6m). The results are shown along the longitudinal probes P-12_P-1_P-2_P-3_P-8 and along the transverse probes P-4_P-3_P-6. Fig. 20a shows the comparisons between the Total Energy (TE) and Bounded Energy (BE) among above three cases along the longitudinal direction of the tank. On the other hand, Fig. 20b shows similar comparisons along the transverse direction. Figs. 20c and 20d show the comparisons among the BE of waves and natural frequency energy components (NE) in the longitudinal and transverse directions for Case-1. Figs. 20e and 20f show similar comparisons among the BE and NE in the longitudinal and transverse directions for Case-3. Figs. 20g and 20h show similar comparisons for Case-5. Figs. 21a to 21h show similar comparisons as described above but for Case-2, Cas-4 and Case-6. In these cases also a different set of constant wave period and wave height is used over varying water depths. Figs. 22a to 22r show similar comparisons of energies between Case-1 and Case-2, Case-3 and Case-4 and, Case-5 and Case-6. It may be observed from the Figs. 20a to 21h that BE is $O(10^{-4})$ compare to TE and NE is $O(10^{-6})$ compare to BE. That is the NE in the OEB is exceptionally small to be accounted for at least for the cases we considered here. Table 4 to Table 9 show the energies of different undulations in the OEB for different incident wave conditions mentioned in Table 2. Appendix-III illustrates all figures.

5.2 Mono-chromatic waves

In Table-3, Case-8 represents the shallowest mono-chromatic wave. This case is adopted to analyze and understand the wave propagation and required damping time after

the wave making is ended in the tank. Figs. 23a to 34j show the surface elevations for different propagation time segments (0-20*s*, 8-9.33*s*, 10-11.33*s*, 15-16.33*s* and 18.67-20*s*) and their corresponding energy spectra for Case-8. The results are shown at probes P-1, P-2, P-3, P-4, P-5, P-6, P-7, P-8, P-9, P-10, P-12 and P-14. **Appendix-IV** demonstrates above figures.

Figs. 35a to 39d show amplitude spectra of the incident frequencies, different interacted frequency components (f, 2f, 3f, ...) and wave tank's natural frequency components $(f_{01}, f_{10}, f_{02},...)$ and are normalized by the incident wave height *H*. Figures show results for Case-8 at probes P-12_P-1_P-2_P-3_P-8 along the longitudinal direction of the tank for different time segments (0-20s, 10-11.33s, 15-16.33s and 18.67-20s). **Appendix-V** shows these figures.

Figs. 40a to 43d show comparisons of the wave energies among cases, Case-7, Case-8 and Case-9. Here Case-7 and Case-8 have similar water depth and similar wave height but different waver periods (*T*=2.145*s* and *T*=4.105*s*). The results are shown along longitudinal probes P-12_P-1_P-2_P-3_P-8and along the transverse probes P-4_P-3_P-6. Fig. 40a shows the comparisons between the Total Energy (TE) and Bounded Energy (BE) among above three cases along the longitudinal direction of the tank. On the other hand, Fig. 40b shows similar comparisons along the transverse direction. Fig. 40c shows the comparisons among measured natural frequency energy components of the OEB for Case-7, Case-8 and Case-9 at wave probes P-12_P-1_P-2_P-3_P-8. On the other hand, Fig. 40d describes the comparisons among measured natural frequency energy components of the OEB for Case-7, Case-8 and Case-7, Case-8 and Case-7, Case-8 and Case-9 at wave probes P-12_P-1_P-2_P-3_P-8.

Fig. 41a shows measured total wave energy (TE) and natural frequency energy (NE) components of the OEB for Case-7 at wave probes P-12_P-1_P-2_P-3_P-8. On the other hand, Fig. 41b shows the measured total wave energy (TE) and natural frequency energy (NE) components of the OEB for Case-7 at wave probes P-4_P-3_P-6. Figs. 41c and 41d describe only the measured natural frequency energy components of the OEB for Case-7 at wave probes P-12_P-3_P-6. Figs. 41c and 41d describe only the measured natural frequency energy components of the OEB for Case-7 at wave probes P-12_P-3_P-6. Figs. 41c and 41d describe only the measured natural frequency energy components of the OEB for Case-7 at wave probes P-12_P-1_P-2_P-3_P-8 and P-4_P-3_P-6, respectively.

Figs. 42a to 42d show similar comparisons as above for Case-8 and Figs. 43a to 43d show comparisons for Case-9.

It may be observed from the Figs. 40a to 43d that NE is $O(10^{-4})$ compare to TE. So to speak the NE in the OEB is exceptionally small to be accounted for at least for the cases we considered here. Table 10 to Table 12 show the energies of different undulations in the OEB for different incident wave conditions mentioned in Table 3. Appendix-VI displays all these figures.

6. CONCLUSIONS

Several cases of measured wave data in the OEB for mono- and bi-chromatic shallow water waves are analyzed. For bi-chromatic waves, it is found that the primary frequency components are interacting with each other in different forms. From both the mono- and bi-chromatic waves, we have observed that the natural frequency components are very small regardless of varying incident wave conditions. In the analyses Case-2 [h=0.4m, H=0.06m, T_1 =2.22s and T_2 =2.0s] and Case-8 [h=0.4m, H=0.08m and T=4.105s] are watched more closely than any other cases as these two cases are most shallow water cases that we considered here.

It is observed that for most of the mono- and bi-chromatic cases the residual wave energies are of $O(10^{-4})$ of the primary wave energies after 8 or 10 minutes the wave making is terminated. So the traditional waiting times, 20 minutes in between two runs is not necessary for all cases. So NRC can save and earn more money by accommodating more commercial projects. According to FY2013-2014, the savings could be 190.4 hours of OEB time with wave makers (in average of 20% of the total OEB usage, 952 hours). In terms of money this saving has a value of \$55K for OGD projects or \$100K for external client projects.

From the comparisons it is observed that bounded wave energy is just 0.5% to 2.5% compare to the respective primary wave energy. On the other hand, the energies of different natural frequency components are 0.01% to 0.15% to the generated primary waves. It is also found that the natural frequency energy components in the transverse direction are negligible compare to the longitudinal direction for higher modes. However, the natural frequency energy components in both directions are found too small in magnitude to be taken into account for shallow water waves that we considered here.

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Appendix-1

Surface elevations for different time segments for bi-chromatic waves and their corresponding energy specta for Case-2.

[*h*=0.4m, *H*=0.06m, *T*₁=2.22s and *T*₂=2.0s]





Fig. 2d Energy for 10-11.33 minutes data (4096 data used) at P-1.



Fig. 2e Surface elevation (15-16.33 minutes) data at P-1.

















Fig. 3d Energy for 10-11.33 minutes data (4096 data used) at P-2.



 $[h=0.4\text{m}, H=0.06\text{m}, T_1=2.22\text{s and } T_2=2.0\text{s}]$





Fig. 3h Energy for 18.67-20 minutes data (4096 data used) at P-2.







Fig. 4d Energy for 10-11.33 minutes data (4096 data used) at P-3.



Fig. 4e Surface elevation (15-16.33 minutes) data at P-3.









Fig. 4h Energy for 18.67-20 minutes data (4096 data used) at P-3.







[*h*=0.4m, *H*=0.06m, *T*₁=2.22s and *T*₂=2.0s]





Fig. 5d Energy for 10-11.33 minutes data (4096 data used) at P-4.



Fig. 5e Surface elevation (15-16.33 minutes) data at P-4.

Fig. 5f Energy for 15-16.33 minutes data (4096 data used) at P-4.







Fig. 5h Energy for 18.67-20 minutes data (4096 data used) at P-4.



[*h*=0.4m, *H*=0.06m, *T*₁=2.22s and *T*₂=2.0s]





Fig. 6d Energy for 10-11.33 minutes data (4096 data used) at P-5.





Fig. 6h Energy for 18.67-20 minutes data (4096 data used) at P-5.







Fig. 7d Energy for 10-11.33 minutes data (4096 data used) at P-6.



Fig. 7e Surface elevation (15-16.33 minutes) data at P-6.



[*h*=0.4m, *H*=0.06m, *T*₁=2.22s and *T*₂=2.0s]





Fig. 7h Energy for 18.67-20 minutes data (4096 data used) at P-6.





Fig. 8d Energy for 10-11.33 minutes data (4096 data used) at P-7.

27

10Hz

8



Fig. 8e Surface elevation (15-16.33 minutes) data at P-7.

Fig. 8f Energy for 15-16.33 minutes data (4096 data used) at P-7.







Fig. 8h Energy for 18.67-20 minutes data (4096 data used) at P-7.





Fig. 9d Energy for 10-11.33 minutes data (4096 data used) at P-8.

29



Fig. 9e Surface elevation (15-16.33 minutes) data at P-8.



[*h*=0.4m, *H*=0.06m, *T*₁=2.22s and *T*₂=2.0s]





Fig. 9h Energy for 18.67-20 minutes data (4096 data used) at P-8.




Fig. 10d Energy for 10-11.33 minutes data (4096 data used) at P-9.











Fig. 11d Energy for 10-11.33 minutes data (4096 data used) at P-10.



Fig. 11e Surface elevation (15-16.33 minutes) data at P-10.



```
[h=0.4m, H=0.06m, T<sub>1</sub>=2.22s and T<sub>2</sub>=2.0s]
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Fig. 11h Energy for 18.67-20 minutes data (4096 data used) at P-10.









Fig. 12h Energy for 18.67-20 minutes data (4096 data used) at P-12.

NRC · CNRC



(Measured leftover surface undulation data when wave making stopped after 3 minutes)



Fig. 13e Surface elevation (15-16.33 minutes) data at P-14.

Fig. 13f Energy for 15-16.33 minutes data (4096 data used) at P-14.

```
[h=0.4m, H=0.06m, T<sub>1</sub>=2.22s and T<sub>2</sub>=2.0s]
```



(Measured leftover surface undulation data when wave making stopped after 3 minutes)

Appendix-II

Interaction of the primary wave components and natural frequency energy components for Case-2.



Fig. 14a: Probe-12: Full normalized amplitude spectrum, 20 minutes wave data



Fig. 14b: Probe-12: Normalized amplitude spectrum after (10-11.320 minutes of) wave making stopped



Fig. 14c: Probe-12: Normalized amplitude spectrum after [15-16.320 minutes of] wave making stopped



Fig. 14d: Probe-12: Normalized amplitude spectrum after [18.67-20 minutes of] wave making stopped



Fig. 15a: Probe-1: Full spectrum, 20 minutes wave data



Fig. 15b: Probe-1: Spectrum after [10-11.320 minutes of] wave making stopped



Fig. 15c: Probe-1: Spectrum after (15-16.33 minutes of) wave making stopped



Fig. 15d: Probe-1: Spectrum after (18.67-20 minutes of) wave making stopped



Fig. 16a: Probe-1: Full Normalized amplitude spectrum, 20 minutes wave data



Fig. 16b: Probe-1: Normalized amplitude spectrum after (10-11.33 minutes of) wave making stopped



Fig. 16c: Probe-1: Normalized amplitude spectrum after (15-16.33 minutes of) wave making stopped



Fig. 16d: Probe-1: Normalized amplitude spectrum after (18.67-20 minutes of) wave making stopped



Fig. 17a: Probe-2: Normalized amplitude spectrum, 20 minutes wave data



Fig. 17b: Probe-2: Normalized amplitude spectrum after (10-11.33 minutes of) wave making stopped



Fig. 17c: Probe-2: Normalized amplitude spectrum after (15-16.33 minutes of) wave making stopped



Fig. 17d: Probe-2: Normalized amplitude spectrum after (18.67-20 minutes of) wave making stopped





Fig. 18a: Probe-3: Normalized amplitude spectrum, 20 minutes wave data



Fig. 18b: Probe-3: Normalized amplitude spectrum after (10-11.33 minutes of) wave making stopped



Fig. 18d: Probe-3: Normalized amplitude spectrum after (18.67-20 minutes of) wave making stopped



Fig. 19a: Probe-8: Normalized amplitude spectrum, 20 minutes wave data



Fig. 19b: Probe-8: Normalized amplitude spectrum after (10-11.33 minutes of) wave making stopped



Fig. 19c: Probe-8: Normalized amplitude spectrum after (15-16.33 minutes of) wave making stopped



Fig. 19d: Probe-8: Normalized amplitude spectrum after (18.67-20 minutes of) wave making stopped

Appendix-III

Comparisons of Total Energy (TE), Bounded Energy (BE) and Natural frequency Energy components in the tank for Bi-chromatic wave cases (Case-1 to Case-6).

$[h=0.4m, H=0.06m, T_1=1.55s \text{ and } T_2=1.45s]$	Case-1
$[h=0.5m, H=0.06m, T_1=1.55s \text{ and } T_2=1.45s]$	Case-3
$[h=0.6m, H=0.06m, T_1=1.55s \text{ and } T_2=1.45s]$	Case-5
$[h=0.4m, H=0.06m, T_1=2.22s \text{ and } T_2=2.0s]$	Case-2
$[h=0.5m, H=0.06m, T_1=2.22s \text{ and } T_2=2.0s]$	Case-4
$[h=0.6m, H=0.06m, T_1=2.22s \text{ and } T_2=2.0s]$	Case-6

 $\begin{bmatrix} h=0.4m, H=0.06m, T_1=1.55s \text{ and } T_2=1.45s \end{bmatrix}$ Case-1 [h=0.5m, H=0.06m, T_1=1.55s and T_2=1.45s] [h=0.6m, H=0.06m, T_1=1.55s and T_2=1.45s]Case-3 Case-5



Fig. 20a Measured total (TE) and bounded wave energies (BE) for Case-1, Case-3 and Case-5 at wave probes 12-1-2-3-8.



Fig. 20b Measured total (TE) and bounded wave energies (BE) for Case-1, 3 and 5 at wave probes 4-3-6.



Fig. 20c Measured bounded wave (BE) and natural frequency energy components of the OEB for Case-1 at wave probes 12-1-2-3-8.



Fig. 20d Measured bounded wave (BE) and natural frequency energy components (f_{01} , f_{10}) of the OEB for Case-1 at wave probes 4-3-6.



Fig. 20e Measured bounded wave (BE) and natural frequency energy components (f_{10} , f_{20}) of the OEB for Case-3 at wave probes 12-1-2-3-8.



Fig. 20f Measured bounded wave (BE) and natural frequency energy components (f_{10} , f_{20}) of the OEB for Case-3 at wave probes 4-3-6.


Fig. 20g Measured bounded wave (BE) and natural frequency energy components (f_{10} , f_{20}) of the OEB for Case-5 at wave probes 12-1-2-3-8.



Fig. 20h Measured bounded wave (BE) and natural frequency energy components (f_{10} , f_{20}) of the OEB for Case-5 at wave probes 4-3-6.

$[h=0.4m, H=0.06m, T_1=2.22s \text{ and } T_2=2.0s]$	Case-2
$[h=0.5m, H=0.06m, T_1=2.22s \text{ and } T_2=2.0s]$	Case-4
$[h=0.6m, H=0.06m, T_1=2.22s \text{ and } T_2=2.0s]$	Case-6



Fig. 21a Measured total (TE) and bounded wave energies (BE) for Case-2, Case-4 and Case-6 at wave probes 12-1-2-3-8.



Fig. 21b Measured total (TE) and bounded wave energies (BE) for Case-2, 4 and 6 at wave probes 4-3-6.



Fig. 21c Measured bounded wave (BE) and natural frequency energy components $(f_{01}, f_{10}, f_{20}, f_{30})$ of the OEB for Case-2 at wave probes 12-1-2-3-8.



Fig. 21d Measured bounded wave (BE) and natural frequency energy components $(f_{01}, f_{10}, f_{20}, f_{30})$ of the OEB for Case-2 at wave probes 4-3-6.



Fig. 21e Measured bounded wave (BE) and natural frequency energy components of the OEB for Case-4 at wave probes 12-1-2-3-8.



Fig. 21f Measured bounded wave (BE) and natural frequency energy components of the OEB for Case-4 at wave probes 4-3-6.



Fig. 21g Measured bounded wave (BE) and natural frequency energy components (f_{10} , f_{20}) of the OEB for Case-6 at wave probes 12-1-2-3-8.



Fig. 21h Measured bounded wave (BE) and natural frequency energy components (f_{01} , f_{10} , f_{20}) of the OEB for Case-6 at wave probes 4-3-6.



Fig. 22a Comparisons of measured total (TE) and bounded wave energies (BE) for Case-1 and Case-2 at wave probes 12-1-2-3-8.



Fig. 22b Comparisons of measured total (TE) and bounded wave energies (BE) for Case-1 and Case-2 at wave probes 4-3-6.



Fig. 22c Comparisons of measured total (TE) and bounded wave energies (BE) for Case-3 and Case-4 at wave probes 12-1-2-3-8.



Fig. 22d Comparisons of measured total (TE) and bounded wave energies (BE) for Case-3 and Case-4 at wave probes 4-3-6.



Fig. 22e Comparisons of measured total (TE) and bounded wave energies (BE) for Case-5 and Case-6 at wave probes 12-1-2-3-8.



Fig. 22f Comparisons of measured total (TE) and bounded wave energies (BE) for Case-5 and Case-6 at wave probes 4-3-6.



Fig. 22g Comparisons of bounded wave energies (BE) with OEB natural frequency components $(f_{01}, f_{10}, f_{20}, f_{30})$ for Case-1 and Case-2 at wave probes 12-1-2-3-8.



Fig. 22h Comparisons of bounded wave energies (BE) with OEB natural frequency components $(f_{01}, f_{10}, f_{20}, f_{30})$ for Case-1 and Case-2 at wave probes 4-3-6.



Fig. 22i Comparisons of bounded wave energies (BE) with OEB natural frequency components (f_{01}, f_{10}, f_{20}) for Case-3 and Case-4 at wave probes 12-1-2-3-8.



Fig. 22j Comparisons of bounded wave energies (BE) with OEB natural frequency components (f_{10}, f_{20}) for Case-3 and Case-4 at wave probes 4-3-6.



Fig. 22k Comparisons of bounded wave energies (BE) with OEB natural frequency components (f_{10}, f_{20}) for Case-5 and Case-6 at wave probes 12-1-2-3-8.



Fig. 221 Comparisons of bounded wave energies (BE) with OEB natural frequency components (f_{10}, f_{20}) for Case-5 and Case-6 at wave probes 4-3-6.



Fig. 22m Comparisons of OEB natural frequency components $(f_{01}, f_{10}, f_{20}, f_{30})$ for Case-1 and Case-2 at wave probes 12-1-2-3-8.



Fig. 22n Comparisons of OEB natural frequency components $(f_{01}, f_{10}, f_{20}, f_{30})$ for Case-1 and Case-2 at wave probes 4-3-6.



Fig. 220 Comparisons of OEB natural frequency components (f_{01} , f_{10} , f_{20}) for Case-3 and Case-4 at wave probes 12-1-2-3-8.



Fig. 22p Comparisons of OEB natural frequency components (f_{01}, f_{10}, f_{20}) for Case-3 and Case-4 at wave probes 4-3-6.



Fig. 22q Comparisons of OEB natural frequency components (f_{01} , f_{10} , f_{20}) for Case-5 and Case-6 at wave probes 12-1-2-3-8.



Fig. 22r Comparisons of OEB natural frequency components (f_{01}, f_{10}, f_{20}) for Case-5 and Case-6 at wave probes 4-3-6.

Appendix-IV

Surface elevations for different time segments for bi-chromatic waves and their corresponding energy specta for Case-8.

[*h*=0.4*m*, *H*=0.08*m* and *T*=4.105*s*]





520

time(seconds)

540

500

Fig. 23d Energy for 8-9.33 minutes data (4096 data used) at P-1.

1.0 Hz 1.5

2.0

0.5

(Measured leftover surface undulation data when wave making stopped after 3 minutes)

560s

0

0.0

-4

T

480



Fig. 23e Surface elevation (10-11.33 minutes) data at P-1.



Fig. 23f Energy for 10-11.33 minutes data (4096 data used) at P-1.





Fig. 23h Energy for 15-16.33 minutes data (4096 data used) at P-1.



Fig. 23i Surface elevation (18.67-20 minutes) data at P-1.

Fig. 23j Energy for 18.67-20 minutes data (4096 data used) at P-1.









Fig. 24c Surface elevation (8-9.33 minutes) data at P-2.



Fig. 24d Energy for 8-9.33 minutes data (4096 data used) at P-2







Fig. 24f Energy for 10-11.33 minutes data (4096 data used) at P-2.

2.0



Fig. 24g Surface elevation (15-16.33 minutes) data at P-2.

Fig. 24h Energy for 15-16.33 minutes data (4096 data used) at P-2.

[*h*=0.4*m*, *H*=0.08*m* and *T*=4.105*s*]





Fig. 24j Energy for 18.67-20 minutes data (4096 data used) at P-2.



[*h*=0.4*m*, *H*=0.08*m* and *T*=4.105*s*]





Fig. 25d Energy for 8-9.33 minutes data (4096 data used) at P-3



Fig. 25e Surface elevation (10-11.33 minutes) data at P-3.

Fig. 25f Energy for 10-11.33 minutes data (4096 data used) at P-3.





Fig. 25h Energy for 15-16.33 minutes data (4096 data used) at P-3.



Fig. 25i Surface elevation (18.67-20 minutes) data at P-3.

Fig. 25j Energy for 18.67-20 minutes data (4096 data used) at P-3.









Fig. 26c Surface elevation (8-9.33 minutes) data at P-4.



Fig. 26d Energy for 8-9.33 minutes data (4096 data used) at P-4





Fig. 26f Energy for 10-11.33 minutes data (4096 data used) at P-4.



Fig. 26g Surface elevation (15-16.33 minutes) data at P-4.

Fig. 26h Energy for 15-16.33 minutes data (4096 data used) at P-4.





Fig. 26j Energy for 18.67-20 minutes data (4096 data used) at P-4.





Fig. 27d Energy for 8-9.33 minutes data (4096 data used) at P-5.



Fig. 27e Surface elevation (10-11.33 minutes) data at P-5.

Fig. 27f Energy for 10-11.33 minutes data (4096 data used) at P-5.





Fig. 27h Energy for 15-16.33 minutes data (4096 data used) at P-5.



Fig. 27i Surface elevation (18.67-20 minutes) data at P-5.

Fig. 27j Energy for 18.67-20 minutes data (4096 data used) at P-5.









Fig. 28c Surface elevation (8-9.33 minutes) data at P-6.



Fig. 28d Energy for 8-9.33 minutes data (4096 data used) at P-6





Fig. 28f Energy for 10-11.33 minutes data (4096 data used) at P-6.





Fig. 28j Energy for 18.67-20 minutes data (4096 data used) at P-6.

1.0

Hz

Т

1.5

T 2.0

Τ

0.5

(Measured leftover surface undulation data when wave making stopped after 3 minutes)

0

0.0

-4 -





Fig. 29d Energy for 8-9.33 minutes data (4096 data used) at P-7.

1.0 Hz 1.5

2.0

0.5

(Measured leftover surface undulation data when wave making stopped after 3 minutes)

0

0.0



Fig. 29e Surface elevation (10-11.33 minutes) data at P-7.

Fig. 29f Energy for 10-11.33 minutes data (4096 data used) at P-7.





Fig. 29h Energy for 15-16.33 minutes data (4096 data used) at P-7.



Fig. 29i Surface elevation (18.67-20 minutes) data at P-7.

Fig. 29j Energy for 18.67-20 minutes data (4096 data used) at P-7.









Fig. 30c Surface elevation (8-9.33 minutes) data at P-8.



Fig. 30d Energy for 8-9.33 minutes data (4096 data used) at P-8.





Fig. 30f Energy for 10-11.33 minutes data (4096 data used) at P-8.



Fig. 30g Surface elevation (15-16.33 minutes) data at P-8.

Fig. 30h Energy for 15-16.33 minutes data (4096 data used) at P-8.





Fig. 30j Energy for 18.67-20 minutes data (4096 data used) at P-8.





Fig. 31d Energy for 8-9.33 minutes data (4096 data used) at P-9.


Fig. 31e Surface elevation (10-11.33 minutes) data at P-9.

Fig. 31f Energy for 10-11.33 minutes data (4096 data used) at P-9.





Fig. 31h Energy for 15-16.33 minutes data (4096 data used) at P-9.



Fig. 31i Surface elevation (18.67-20 minutes) data at P-9.

Fig. 31j Energy for 18.67-20 minutes data (4096 data used) at P-9.









Fig. 32c Surface elevation (8-9.33 minutes) data at P-10.



Fig. 32d Energy for 8-9.33 minutes data (4096 data used) at P-10.







Fig. 32f Energy for 10-11.33 minutes data (4096 data used) at P-10.



Fig. 32g Surface elevation (15-16.33 minutes) data at P-10.

Fig. 32h Energy for 15-16.33 minutes data (4096 data used) at P-10.





Fig. 32j Energy for 18.67-20 minutes data (4096 data used) at P-10.



Fig. 33a Measured total surface elevation at P-12.



[*h*=0.4*m*, *H*=0.08*m* and *T*=4.105*s*]





Fig. 33e Surface elevation (10-11.33 minutes) data at P-12.

Fig. 33f Energy for 10-11.33 minutes data (4096 data used) at P-12.





Fig. 33h Energy for 15-16.33 minutes data (4096 data used) at P-12.



Fig. 33i Surface elevation (18.67-20 minutes) data at P-12.

Fig. 33j Energy for 18.67-20 minutes data (4096 data used) at P-12.









Fig. 34c Surface elevation (8-9.33 minutes) data at P-14.

Fig. 34d Energy for 8-9.33 minutes data (4096 data used) at P-14.





Fig. 34f Energy for 10-11.33 minutes data (4096 data used) at P-14.



Fig. 34g Surface elevation (15-16.33 minutes) data at P-14.

Fig. 34h Energy for 15-16.33 minutes data (4096 data used) at P-14.





Fig. 34j Energy for 18.67-20 minutes data (4096 data used) at P-14.

Appendix-V

Interaction of the primary wave component and natural frequency energy components for Case-8.



Fig. 35a: Probe-12: Full normalized amplitude spectrum, 20 minutes wave data



Fig. 35b: Probe-12: Normalized amplitude spectrum after [10-11.33 minutes of] wave making stopped [h=0.4m, H=0.08m and T=4.105s]



Fig. 35c: Probe-12: Normalized amplitude spectrum after [15-16.33 minutes of] wave making stopped



Fig. 35d: Probe-12: Normalized amplitude spectrum after [18.67-20 minutes of] wave making stopped



Fig. 36a: Probe-1: Full normalized amplitude spectrum, 20 minutes wave data



Fig. 36b: Probe-1: Normalized amplitude spectrum after [10-11.33 minutes of] wave making stopped [*h*=0.4*m*, *H*=0.08*m* and *T*=4.105*s*]



Fig. 36c: Probe-1: Normalized amplitude spectrum after [15-16.33 minutes of] wave making stopped [h=0.4m, H=0.08m and T=4.105s]



Fig. 36d: Probe-1: Normalized amplitude spectrum after [18.67-20 minutes of] wave making stopped [*h*=0.4*m*, *H*=0.08*m* and *T*=4.105*s*]



Fig. 37a: Probe-2: Full Normalized amplitude spectrum, 20 minutes wave data



Fig. 37b: Probe-2: Normalized amplitude spectrum after [10-11.33 minutes of] wave making stopped [h=0.4m, H=0.08m and T=4.105s]



Fig. 37c: Probe-2: Normalized amplitude spectrum after [15-16.33 minutes of] wave making stopped [*h*=0.4*m*, *H*=0.08*m* and *T*=4.105*s*]



Fig. 37d: Probe-2: Normalized amplitude spectrum after [18.67-20 minutes of] wave making stopped [*h*=0.4*m*, *H*=0.08*m* and *T*=4.105*s*]



Fig. 38a: Probe-3: Normalized amplitude spectrum, 20 minutes wave data



Fig. 38b: Probe-3: Normalized amplitude spectrum after [10-11.33 minutes of] wave making stopped [*h*=0.4*m*, *H*=0.08*m* and *T*=4.105*s*]



Fig. 38c: Probe-3: Normalized amplitude spectrum after [15-16.33 minutes of] wave making stopped [h=0.4m, H=0.08m and T=4.105s]



Fig. 38d: Probe-3: Normalized amplitude spectrum after [18.67-20 minutes of] wave making stopped



Fig. 39a: Probe-8: Normalized amplitude spectrum, 20 minutes wave data



Fig. 39b: Probe-8: Normalized amplitude spectrum after [10-11.33 minutes of] wave making stopped



Fig. 39c: Probe-8: Normalized amplitude spectrum after [15-16.33 minutes of] wave making stopped



Fig. 39d: Probe-8: Normalized amplitude spectrum after [18.67-20 minutes of] wave making stopped

Appendix-VI

Comparisons of Total Energy (TE), Bounded Energy (BE) and Natural frequency Energy components in the tank for Mono-chromatic wave cases (Case-7 to Case-9).

[<i>h</i> =0.4m, <i>H</i> =0.08m and <i>T</i> =2.145s]	Case-7
[<i>h</i> =0.4m, <i>H</i> =0.08m and <i>T</i> =4.105s]	Case-8
[<i>h</i> =0.5m, <i>H</i> =0.08m and <i>T</i> =1.977s]	Case-9

[<i>h</i> =0.4m, <i>H</i> =0.08m and <i>T</i> =2.145s]	Case-7
[<i>h</i> =0.4m, <i>H</i> =0.08m and <i>T</i> =4.105s]	Case-8
[<i>h</i> =0.5m, <i>H</i> =0.08m and <i>T</i> =1.977s]	Case-9



Fig. 40a Comparisons among measured total energy (TE) energies for Case-7, Case-8 and Case-9 at wave probes 12-1-2-3-8.



Fig. 40b Comparisons among measured total energy (TE) energies for Case-7, Case-8 and Case-9 at wave probes 4-3-6.



Fig. 40c Comparisons among measured natural frequency energy components of the OEB for Case-7, Case-8 and Case-9 at wave probes 12-1-2-3-8.



Fig. 40d Comparisons among measured natural frequency energy components of the OEB for Case-7, Case-8 and Case-9 at wave probes 4-3-6.



Fig. 41a Measured total wave energy (TE) and natural frequency energy components of the OEB for Case-7 at wave probes 12-1-2-3-8.



Fig. 41b Measured total wave energy (TE) and natural frequency energy components of the OEB for Case-7 at wave probes 4-3-6.



Fig. 41c Measured natural frequency energy components of the OEB for Case-7 at wave probes 12-1-2-3-8.



Fig. 41d Measured natural frequency energy components of the OEB for Case-7 at wave probes 4-3-6.



Fig. 42a Measured total wave energy (TE) and natural frequency energy components of the OEB for Case-8 at wave probes 12-1-2-3-8.



Fig. 42b Measured total wave energy (TE) and natural frequency energy components of the OEB for Case-8 at wave probes 4-3-6.


Fig. 42c Measured natural frequency energy components of the OEB for Case-8 at wave probes 12-1-2-3-8.



Fig. 42d Measured natural frequency energy components of the OEB for Case-8 at wave probes 4-3-6.



Fig. 43a Measured total wave energy (TE) and natural frequency energy components of the OEB for Case-9 at wave probes 12-1-2-3-8.



Fig. 43b Measured total wave energy (TE) and natural frequency energy components of the OEB for Case-9 at wave probes 4-3-6.



Fig. 43c Measured natural frequency energy components of the OEB for Case-9 at wave probes 12-1-2-3-8.



Fig. 43d Measured natural frequency energy components of the OEB for Case-9 at wave probes 4-3-6.