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Revision 1

Methodology for Designing an MCDV Antenna Support System

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Introduction

The goal of the project was to design a support system for a mast and payload such that it could survive in Sea State 4 while deployed and Sea State 6 while nested.

The goal of this paper is to relay the process by which the various forces and torques imposed on the design system were computed, and how the design was then sized to absorb them.

Requirement

A Support System for an Antenna and RF Distribution Unit was requested by the Canadian Forces Electronic Warfare Centre (CFEWC) for the Canadian Forces Marine Coastal Defense Vessels (MCDV) such that the supporting mechanism can be installed and removed from a 20’ Special Equipment Vehicle (SEV) without the need for modification of the SEV. The system needed to be designed and built such that it could operate during Sea State 4 while deployed (mast extended) and survive Sea State 6 while stowed (mast nested).

A concept was proposed that constrains the numerous degrees of freedom. An analysis was then conducted to size the various design elements such that any forces and torques can be managed.

The design consists of three main elements: a bridge that can be set onto the SEV and supports the mast, four ISO corner adapters which allow straps to be anchored, and a payload adapter plate that allows the antenna and RFD to be installed and which provides locations for guy lines.

Design

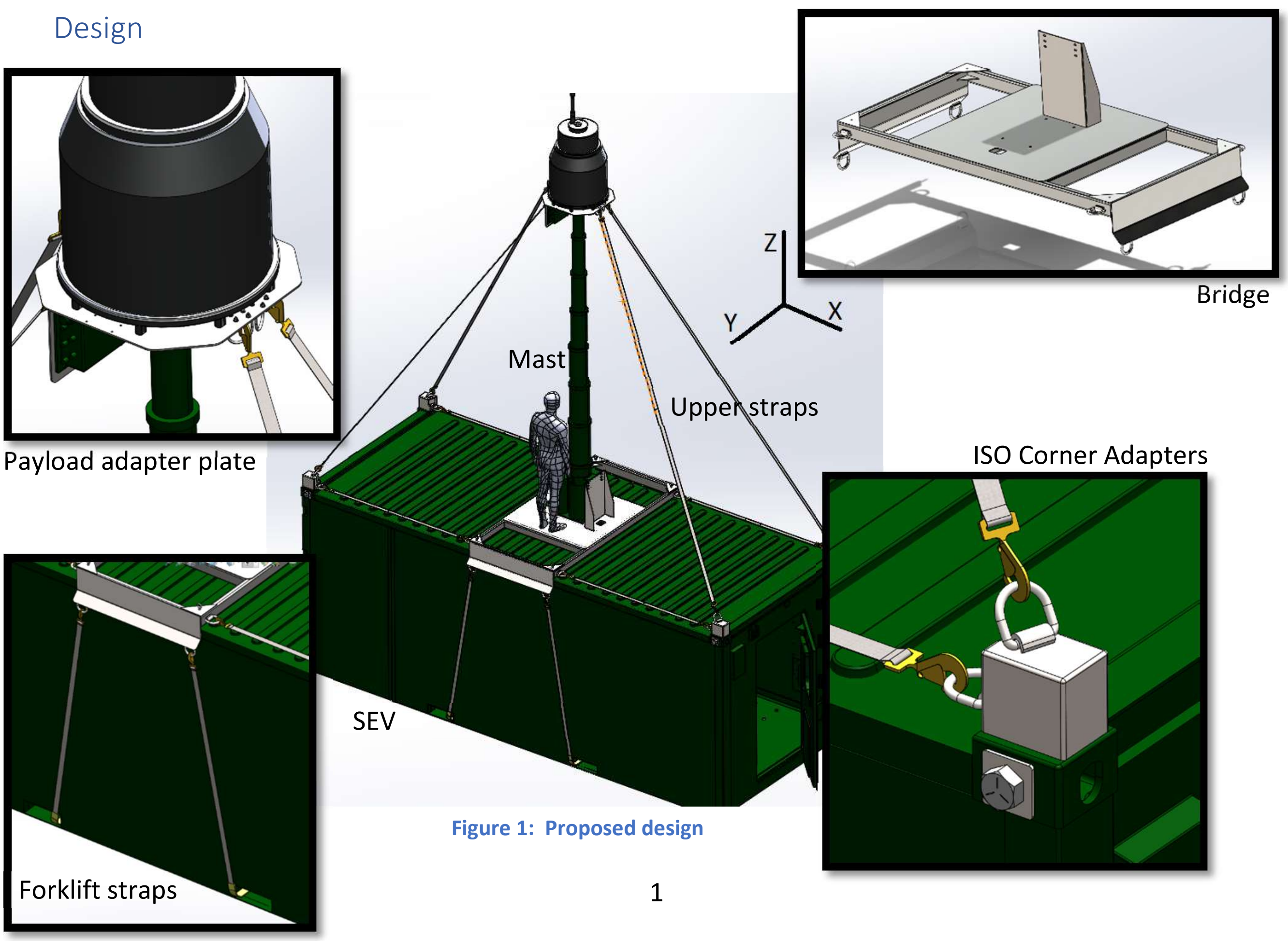

Figure 1: Proposed design

Background

The design conditions imposed on the Antenna support system were defined by the conditions in which the antenna would be deployed or nested (the sea state) and approximations for the maximum roll angle during these conditions. These conditions considered the worst-case scenarios for both the nested and deployed states – up to **Sea State 4 (SS4) deployed**, and **Sea State 6 (SS6) nested**, a maximum roll of **25° and 40° respectively**. The roll angle will be the main motion of consideration throughout these calculations as the acceleration about the roll axis (a product of the maximum roll angle) is larger than that of the pitch axis, this can be justified intuitively.

The following table, supplied by DND, provides the wave period at the varying states, as well the wind speed, which will be used to determine the drag force.

STANAG 4194
TABLE D-1 – NATO SEA STATE NUMERAL TABLE FOR THE OPEN OCEAN NORTH ATLANTIC

Sea State Number	Significant Wave Height (m)		Sustained Wind Speed (knots) *		Percentage Probability of Sea State	Modal Wave Period (sec)	
	Range	Mean	Range	Mean		Range **	Most Probable ***
0 - 1	0 - 0.1	0.05	0 - 6	3	0.70	-	-
2	0.1 - 0.5	0.3	7 - 10	8.5	6.80	3.3 - 12.8	7.5
3	0.5 - 1.25	0.88	11 - 16	13.5	23.70	5.0 - 14.8	7.5
4	1.25 - 2.5	1.88	17 - 21	19	27.80	6.1 - 15.2	8.8
5	2.5 - 4	3.25	22 - 27	24.5	20.64	8.3 - 15.5	9.7
6	4 - 6	5	28 - 47	37.5	13.15	9.8 - 16.2	12.4
7	6 - 9	7.5	48 - 55	51.5	6.05	11.8 - 18.5	15.0
8	9 - 14	11.5	56 - 63	59.5	1.11	14.2 - 18.6	16.4
> 8	> 14	>14	>63	>63	0.05	18.0 - 23.7	20.0

* Ambient wind sustained at 19.5 m above surface to generate full-developed seas. To convert to another altitude, H_2, apply $V_2 = V_1(H_2/19.5)^{1/7}$
** Minimum is 5 percentile and maximum is 95 percentile for periods given wave height range.
*** Based on periods associated with central frequencies included in Hindcast Climatology.
REVISED MARCH 1964

Figure 2: Sea State Table, provided by DND

Given these operating conditions and the following assumptions, it is possible to determine the nominal stresses induced on the design and therefore validate the support system.

Assumptions

Simple Harmonic Motion is assumed, which models the behaviour of the ship *to have periodic motion where the restoring force is directly proportional to the displacement and acts in the direction opposite to that of the displacement.* In the case of the roll of the ship, the displacement considered is the angular displacement about the roll axis. A conservative assumption would be to take the maximum angular displacement of the ship equal to that of the waves, as in reality some of the rotational inertia would be dampened by non-trivial adjustments in buoyancy. However, for this analysis the maximum angular displacement was given to be 25 and 40 degrees during roll, for SS4 and SS6 respectively (10 and 15 degrees during pitch) per empirical Navy input, based on personnel experience. The maximum angle of rotation is not as extensive as that of the waves as the ship tends to ‘sit’, or lean into the wave during changes in directions. The ship was also assumed to be oriented such that the waves were impacting it side-on during roll, an unlikely worst-case scenario.

Another conservative assumption is made when calculating the torque about the base of the mast – the wind gusts are considered to act perpendicular to the mast even during extreme rolls. This is unlikely as the ocean winds tend to gust parallel to the surface, and not on a downward angle toward the ocean.

The mast's centre of mass is conservatively taken as half its height. In reality, the telescoping nature of the mast means the centre of the mass lies closer to the base. The placement of the centre of mass controls where the inertial and drag forces act, which in turn determines the magnitude of the torques experienced about the base – as the centre of mass gets higher, the lever arm gets longer, and therefore the torque increases. In addition the ship's centre of mass was lowered to the waterline, which is yet another conservative estimate.

For all the strap calculations, it is assumed that each set of straps (horizontal, upper, and forklift) all act independently during operation, which is an extremely conservative estimate. In reality, these support systems all work together to balance the stress and strain within the system. The following force magnitudes can be considered worst-case scenarios as if all other support systems had failed.

Procedure

1.0 Drag Forces

First off, it is necessary to identify which forces affect the support system, particularly those which act against the upright position of the mast. These include the wind drag force and the inertial force caused by the change in direction of motion during rolls. The following details the procedure for calculating the **Drag Force** on the antenna and mast, in SS4 and SS6;

$$Antenna\ Drag\ Force = \boldsymbol{F_{D-Ant}} = \frac{1}{2} \times C_D \times \rho \times v^2 \times A$$

Where;

C_D = Drag Coefficient (estimated for a cylinder) = <u>1.2</u>

ρ = Air Density at STP = <u>0.0023769</u> [slug/ft^3]

v = Wind Speed = <u>35.43</u> (SS4) = <u>79.29</u> (SS6) [ft/s]

A = Cross-sectional Area of Antenna = <u>899.30</u> [in^2]

$$\boldsymbol{F_{D-Ant}} = \frac{1}{2} \times 1.2 \times 0.0023769 \times 35.43^2 \times \frac{899.30}{12^2}$$

$$\boldsymbol{F_{D-Ant} = 11.18\ [lbf]\ [SS4]}$$

The drag forces for the Antenna and Mast in both Sea States can be found in the table below;

Table 1: Drag Forces of Antenna and Mast

Component	Sea State 4	Sea State 6
Antenna	11.18 lbf	56.00 lbf
Mast	18.07 lbf	22.63 lbf

It can be seen that SS6 imposes the largest force on the antenna. This is due to the controlling wind speed variable (force increases exponentially in relation to wind speed), which is more than twice as large in SS6. The dramatic increase in drag force on the antenna between SS4 and SS6 is not seen in the mast since the mast is retracted during SS6 and therefore represents a much smaller area for the wind to act on.

Torque is simply the force multiplied by the lever arm, which changes between SS4 and SS6. While it is understood that only the *total* torque about the base is relevant (which includes both drag and inertia forces) it is valuable to look at drag torque alone here, as it offers an important insight into the rationale of designing for SS4 rather than SS6:

Table 2: Drag Torques created about the Mast base

Component	Sea State 4	Sea State 6
Antenna	162.57 lbf*ft	226.33 lbf*ft
Mast	124.22 lbf*ft	36.77 lbf*ft
Total Drag Torque =	286.79 lbf*ft	263.10 lbf*ft

When the mast is nested it loses 80% of its height, not only dramatically reducing the associated lever arm but also reducing its wind profile. Table 2 shows that the total drag torques are comparable across sea states despite the wind speed more than doubling. Though the torques associated with the antenna alone are greater in SS6, the system's total torque is higher in SS4 because the drag force is a function of the cross-sectional area. When the mast is nested this area reduces significantly, such that the wind speed's influence is significantly reduced.

As mentioned, these forces will later be summed with the inertial forces and multiplied by their respective lever arms (distance to the centre of rotation) to find the total torque about the base of the mast - the torque that will be reacted by the design of the support system. However, before the reaction loads can be computed it is necessary to determine and quantify the inertial forces imposed on the antenna and mast system.

2.0 Inertial Forces

Inertial forces are created by the acceleration (or deceleration) of the antenna and mast mass, which in this case arises from the angular acceleration of the ship as it rolls about its axis. To determine these **Angular Accelerations**, a conservative assumption of **Simple Harmonic Motion** is made, which allows us to determine the inertial accelerations as follows;

$$Angular\ Acceleration = \boldsymbol{\alpha} = -4 \times \pi^2 \times f^2 \times \theta$$

Where;

f = frequency of wave oscillation = 1/Wave Period (obtained from the Sea State Table above, again using the worst-case shortest periods, 6.1 s and 9.8 s)
= 0.164 (SS4) = 0.102 (SS6) [1/s]

θ = Max Roll Angle = 0.44 (SS4) = 0.70 (SS6) [rads] (source is 25° and 40°, a worst case estimate from experienced Navy staff)

$$\boldsymbol{\alpha} = -4 \times \pi^2 \times 0.164^2 \times 0.44$$

$$\boldsymbol{\alpha} = -0.46 = \mathbf{0.46} \left[\frac{\boldsymbol{rad}}{\boldsymbol{s^2}}\right] [\boldsymbol{SS4}]$$

$$= -0.29 = \mathbf{0.29} \left[\frac{\boldsymbol{rad}}{\boldsymbol{s^2}}\right] [\boldsymbol{SS6}]$$

It can be seen from the results above that the angular acceleration of the ship is largest in SS4, which might seem counter-intuitive. This outcome is largely a result of the wave period at SS4 being smaller than that of the larger waves in SS6. The smaller period means a larger frequency of waves, and even though the roll angle of the ship at SS4 is smaller than SS6, squaring the larger frequency has the effect of producing a larger angular acceleration at the calmer sea state.

Having determined the angular accelerations about the centre of rotation of the ship, it is now possible to calculate the resultant **Linear Acceleration** of the antenna and mast masses by multiplying in their respective distance to the ship's centre of rotation;

$$Linear\ Acceleration\ of\ Antenna = \boldsymbol{a_{Ant}} = \alpha \times l_{T-A}$$

Where;

α = Angular Acceleration = 0.46 (SS4) = 0.29 (SS6) [rad/s^2]

$l_{T\text{-}Ant}$ = Total length of Antenna lever arm with respect to the ships centre of rotation

= Antenna CofG height from top of Mast + Extended Mast Height + SEV Container Height + Height from Deck to Waterline (which has been taken to approximate the centre of the ship's rotation)

= 0.79 + 13.75 + 8.00 + 9.84 = 32.38 (SS4-Extended) [ft]

= 0.79 + 3.25 + 8.00 + 9.84 = 21.88 (SS6-Nested) [ft]

$$\boldsymbol{a_{Ant}} = 0.46 \times 32.38$$

$$\boldsymbol{a_{Ant}} = \mathbf{14.8948} \left[\frac{\boldsymbol{ft}}{\boldsymbol{s^2}}\right] [\boldsymbol{SS4}]$$

The remaining linear acceleration values for both the Antenna and Mast at the different sea states can be found in the following table;

Table 3: Linear Acceleration of Antenna and Mast

Component	Sea State 4	Sea State 6
Antenna	14.89 ft/s^2	6.35 ft/s^2
Mast	11.37 ft/s^2	5.64 ft/s^2

The values confirm what is to be expected: the larger angular acceleration in SS4 results in larger linear acceleration values experienced locally by the two major components of the structure. In the interest of brevity, the following example inertia calculations focus on SS4, with the equivalent results at SS6 available in the tables.

These linear accelerations must next be applied to the mass of the corresponding components to get their induced inertial forces. The calculated inertial accelerations will be added to the respective component gravity for SS4 and SS6. These components are again calculated at the maximum roll angle, and the force is determined through Newton's 2nd law;

$$Inertial\ Force\ of\ Antenna = \boldsymbol{F_{Inertia-Ant}} = m_{Ant} \times (a_{Ant} + g sin\theta)$$

Where;

m_{Ant} = Mass of the Antenna = <u>4.41</u> [lbm] = 4.41 [slugs] (Antenna is 142 lbs)

m_{Mast} = Mass of the Mast = <u>5.90</u> [lbm] = 5.90 [slugs] (Mast is 190 lbs)

a_{Ant} = Linear Acceleration of Antenna = <u>14.8948</u> [ft/s^2] [see table 3 for add. values]

g = Acceleration due to Gravity = <u>32.2</u> [ft/s^2]

$sin\theta$ = The Gravity Component acting perpendicular to Mast, based on roll angle $\theta = 25°$

$$\boldsymbol{F_{Inertia-Ant}} = 4.41 \times (14.8948 + 32.2\sin(25°))$$

$$\boldsymbol{F_{Inertia-Ant}} = \mathbf{126.1165}\ [\boldsymbol{lbf}]\ [\boldsymbol{SS4}]$$

The following table gives the inertial forces created by the antenna and mast during both SS4 and SS6;

Table 4: Inertial Forces of Antenna and Mast

Component	Sea State 4	Sea State 6
Antenna	126.12 lbf	118.97 lbf
Mast	147.81 lbf	155.09 lbf

Note that even though the linear acceleration at SS6 is about half the value at SS4, the larger mass of the mast means its gravitational component of the total acceleration becomes more significant to the overall force during the 40° roll. The inertial torques about the base are then calculated and tabulated below – the steps for calculating the torque are given in section 3.0, where both drag and inertial forces are considered.

Table 5: Inertial Torques created about the Mast base

Component	Sea State 4	Sea State 6
Antenna	1833.94 lbf*ft	480.83 lbf*ft
Mast	1016.17 lbf*ft	252.02 lbf*ft
Total Inertial Torque =	2850.12 lbf*ft	732.85 lbf*ft

It can be seen clearly that the inertial component of the torques are far more significant than the drag forces, though the drag forces should still be considered as they are non-negligible. It should also be noted that, as previously discussed, the sea state 4 conditions produce a much larger torque about the base – almost exclusively since the *mast is extended* in these conditions, which provides a much longer lever arm for forces to act about.

3.0 Total Torque

With the Drag and Inertial Forces determined, they are then applied as to act perpendicular to the mast at the respective centre of gravities, giving a **Total Torque** value about the base of the Antenna installation – this will be the design torque on which the support loads will be determined.

Considering the SS4 Antenna for calculation purposes, the following formula represents the torque created about the base;

$$Antenna\ Torque\ about\ Base = \boldsymbol{\tau_{T-Ant}} = (F_{D-Ant} + F_{Inertia-Ant}) \times (l_{Base-Ant})$$

Where;

$F_{D\text{-}Ant}$ = Drag Force experienced by the Antenna = <u>11.18</u> [lbf] [SS4]

$F_{Inertia\text{-}Ant}$ = Inertial Force generated by the Antenna's mass = <u>126.12</u> [lbf] [SS4]

$l_{Base\text{-}Ant}$ = Antenna Lever Arm with respect to Base

= Antenna CofG height from top of Mast + Extended Mast Height

= 0.79 + 13.75 = <u>14.54</u> [ft] [SS4]

$$\boldsymbol{\tau_{T-Ant}} = (11.18 + 126.12) \times (14.54)$$

$$\boldsymbol{\tau_{T-Ant}} = \mathbf{1996.5129}\ [\boldsymbol{lbf * ft}]$$

The total Torque for each component at the respective sea states is finally combined to give the total torque acting about the base of the mast. The remaining component torques, as well as the Total Torques, are calculated and tabulated as follows;

Table 6: Total Torque about the Mast base

Component	Sea State 4	Sea State 6
Antenna	1,997 lbf*ft	707 lbf*ft
Mast	1,140 lbf*ft	289 lbf*ft
Total Torque =	$\tau_{T\text{-}SS4}$ = 3,137 lbf*ft	$\tau_{T\text{-}SS6}$ = 996 lbf*ft

The total torque can now be used to determine the required strength of the various supports. The next step in determining the feasibility of the system is to determine the resultant reaction tension in the upper support straps attached to the antenna plate (used as guy wires). This is the most important support system as it reduces the stresses on the mast and its base, and translates it to ISO corners.

4.0 Support Tension

4.1 Upper Support Straps

To determine the tension in the upper support straps, the notion of the **Unit Vector** is introduced, a concept which can dictate the proportionality of the tension by treating the support like a vector. It works by finding the direction that opposes the force (torque about the base in roll) and using the unit length in that direction, along with the unit length in the other directions, to find the total resultant reaction force experienced by the strap.

The unit vector is made up of its component lengths of the strap, divided by their magnitude. The component lengths are given in the following figure, and can be written as; x = 96.8 in, y = 41.3 in (the unit length opposing the roll torque), and z = 165.0 in (the height of the mast, in SS4), derived from the SolidWorks model of the system.

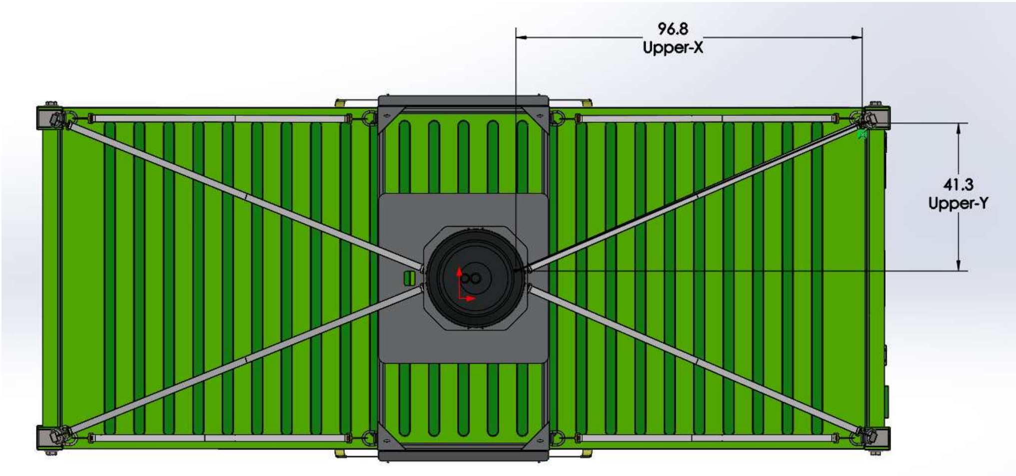

Figure 3: Top View of ISO container, showing dimensions of Upper Support Strap

As can be seen from the above image, the unit vector required to counter the roll forces acting against the broad face of the container is the length in the **y-direction**, **41.3 inches**. The following outlines the process necessary to determine resultant tension along the length of the *strap vector*.

First, the component length is used to find the unit vector in the y-direction. This is done by dividing the y-component length by the magnitude of the strap vector, as follows;

$$y - Direction\ Unit\ Vector = \boldsymbol{y_{unit}} = \frac{y_{length}}{\sqrt{x_{lengt}\ ^2 + y_{length}{}^2 + z_{lengt}\ ^2}}$$

Where;

X_{length} = Length of the strap in the x-direction = 96.8 [in]

y_{length} = Length of the strap in the y-direction = 41.3 [in]

z_{length} = Length of the strap in the z-direction, the height of the extended Mast = 165.0 [in]

$$\boldsymbol{y_{unit}} = \frac{41.3}{\sqrt{96.8^2 + 41.3^2 + 165.0^2}}$$

$$\boldsymbol{y_{unit}} = \mathbf{0.21103}$$

This is the only necessary unit length to determine the reaction forces during roll, but the other unit lengths, as well as the unit vectors for the horizontal and forklift straps, can be calculated similarly and are given below.

Table 7: Unit Lengths of Various Support Vectors

Support Strap	x-unit length	y-unit length	z-unit length
Upper Straps	0.49	0.21	0.84
Horizontal Straps	1.00	0.00	0.00
Forklift Straps	0.18	0.00	0.98

With the y-unit vector now known, the required tensile reaction force needs to be calculated to find the y-component of the resultant strap tension. This is simply done by taking the torque experienced and dividing it by the lever arm created by the strap support, the z-direction length of the strap;

$$y - Component\ Tension\ in\ Support\ Strap = \boldsymbol{T_{y-strap}} = \frac{\tau_{T-SS4}}{z_{length}}$$

Where;

$\boldsymbol{\tau_{T-SS4}}$ = The Total Torque about the Mast Base in SS4 = 3,137 [lbf*ft]

$\boldsymbol{z_{length}}$ = The z-component of the Strap Length, also the Height of the Mast = 13.75 [ft]

$$\boldsymbol{T}_{\boldsymbol{y-strap}} = \frac{3{,}137}{13.75}$$

$$\boldsymbol{T}_{\boldsymbol{y-strap}} = \mathbf{228.145}\ [\boldsymbol{lbf}]$$

Finally, with the y-component tension calculated, the Total Resultant Strap Tension can be found using the y-unit vector, based on the following understanding;

$$y - Component\ Tension\ in\ Support\ Strap = \boldsymbol{T}_{\boldsymbol{y-strap}} = y_{unit} \times T_{Total}$$

$$\rightarrow Total\ Resultant\ Strap\ Tension = \boldsymbol{T}_{\boldsymbol{Total}} = \frac{T_{y-strap}}{y_{unit}}$$

$$\rightarrow \boldsymbol{T}_{\boldsymbol{Total}} = \frac{228.145}{0.21103} = \mathbf{1{,}081.10}\ [\boldsymbol{lbf}]$$

Considering that there are two support straps symmetric about the roll axis, this tensile term should be halved as each strap will equally share the load. The **Max Resultant Tensile Load** in the each of the guy wire straps becomes **T_{Total} = 540.552 lbf.** The tension experienced by the support straps during pitch, as well as in SS6, can be found as;

Table 8: Resultant Support Tension of upper straps under Various Operating Conditions

Axis of Rotation	SS4	SS6
Roll	541 lbf	416 lbf
Pitch	106 lbf	99 lbf

As can be seen from the above results, the roll axis produces the largest reaction forces within the strap. Sea State 4 has remained the larger concern, mainly due to the reduced wave period and the longer lever arm of the extended mast.

It should also be noted that an additional bolted support at the base is used to help stiffen the mast, and allows the system to behave more similarly to a rigid body – the ideal system. In other words, it reduces flexural stresses in the mast and helps to efficiently and predictably translate the stresses to support areas, such as at the end of the bridge and the top of the mast. This rigid support has been left out of this analysis to continue to provide conservative figures.

The main focus for this report has been on the upper support straps, as preliminary calculations in Excel proved them to be the critical design concern. There are, however, other important support features to the design. The following sections break down the other two support systems and their expected tensile requirements;

4.2 Forklift Support Straps

To find the tension in the forklift straps a process similar to the upper strap calculation in section **3.1** must be used. The opposing force to the roll torque can be seen in **Figure 4** to be in the **z-direction** of the ratchet strap, which has a component length of **81.6 inches**. It should also be noted that any length in the y-direction is small enough to be neglected in the unit vector calculations; such that the vector is made up of only two directions.

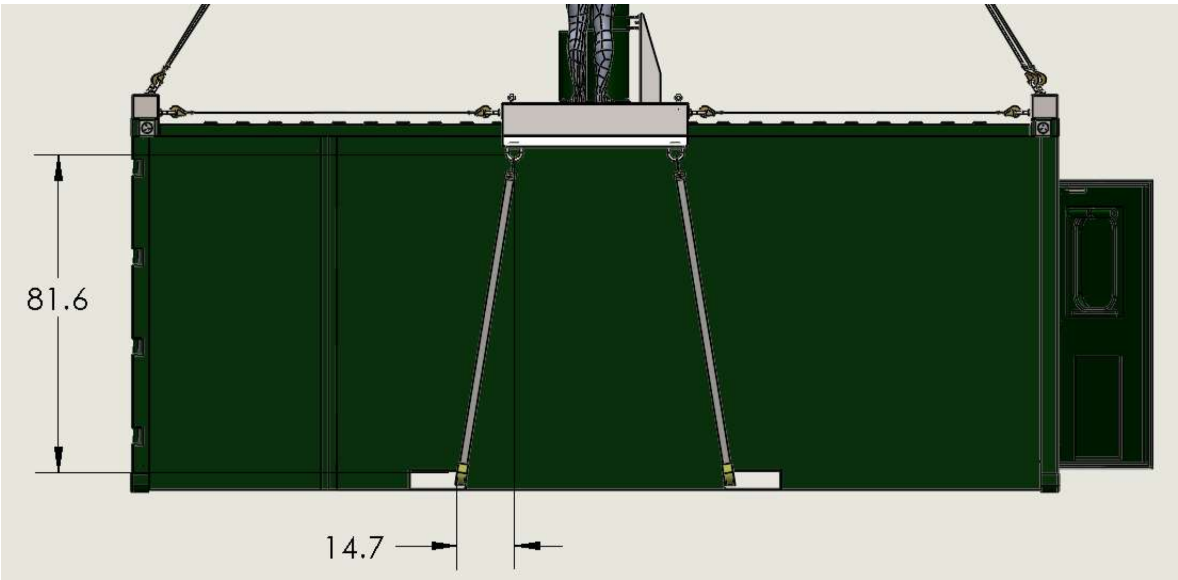

Figure 4: Side of ISO container, showing the Forklift Support Strap

From the figure above, the z-direction unit vector calculation, necessary to find the resultant tension along the strap, can be performed as follows;

$$z - Direction\ Unit\ Vector = \boldsymbol{z_{unit}} = \frac{z_{length}}{\sqrt{x_{leng}{}^2 + y_{length}{}^2 + z_{leng}{}^2}}$$

Where;

X_{length} = Length of the strap in the x-direction = <u>14.70</u> [in]

y_{length} = Length of the strap in the y-direction = <u>0.00</u> [in]

z_{length} = Length of the strap in the z-direction, most of the height of the SEV = <u>81.60</u> [in]

$$\boldsymbol{z_{unit}} = \frac{81.60}{\sqrt{14.70^2 + 0.00^2 + 81.60^2}}$$

$$\boldsymbol{z_{unit} = 0.98416}$$

This value can also be seen in **Table 7** from section **3.1**. Next, the required resultant force acting against the roll torque, which acts in the same direction of the z-unit vector, must be found by multiplying the total torque about the mast base by the lever arm distance to the reacting strap. This distance is roughly equal to half the width of the container and the calculation for this resultant force/component tension can then be found as follows;

$$z - Component\ Tension\ in\ Support\ Strap = \boldsymbol{T_{z-strap}} = \frac{\tau_{T-SS}}{y_{leng}}$$

Where;

$\boldsymbol{\tau_{T-SS4}}$ **=** The Total Torque about the Mast Base in SS4 = 3,136.9 [lbf*ft]

$\boldsymbol{y_{length}}$ **=** The lever arm length for the resultant force, half the width of the Container = 3.98 [ft]

$$\boldsymbol{T_{z-strap}} = \frac{3{,}136.9}{3.98}$$

$$\boldsymbol{T_{z-strap}} = 788.33\ [\boldsymbol{lbf}]$$

With the z-component tension of the strap found, it is then divided by the z-unit vector for the forklift strap to get the total resultant tension in the support straps;

$$z - Component\ Tension\ in\ Support\ Strap = \boldsymbol{T_{z-strap}} = z_{unit} \times T_{Total}$$

$$\rightarrow Total\ Resultant\ Strap\ Tension = \boldsymbol{T_{Total}} = \frac{T_{z-strap}}{z_{unit}}$$

$$\rightarrow \boldsymbol{T_{Total}} = \frac{788.33}{0.98416} = \boldsymbol{801.02}\ [\boldsymbol{lbf}]$$

This tension, as with that of the upper supports, must be divided by two to account for the two symmetrical straps. The **Max Resultant Tensile Load** in the forklift straps (in SS4 roll) is therefore given as **T_{Total} = 400.51 lbf**. The similarly computed loads for SS6 and pitch are all less than half of this.

As shown previously, roll generates the largest torque that reacts about the mast base, and therefore it is the driving design consideration for the support system. The tension in the forklift straps is shown to be smaller than the tension experienced in the upper supports, **400.51 lbf < 540.53 lbf**. Finally, the tension in the horizontal straps must now be found. It should be noted that they are only meant to eliminate translation of the support system along the container, rather than reacting to the torque about the mast base.

4.3 Horizontal Support Straps

To calculate the expected tensile force experienced by the horizontal supports, a completely different approach to that taken in sections **3.1 and 3.2** must be considered. These support straps restrict translational motion along the length of the container, and therefore need only consider the linear forces - the torques won't be needed. Similarities to **3.1** and **3.2** include the consideration of the inertial force generated from the gravitational and oscillatory accelerations and the drag forces generated by the wind. **Figure 5** below shows the location and length of the horizontal support straps. It can be seen that they act along only the x-axis, and therefore have only one component length – the length in the **x-direction = 82.5 inches**;

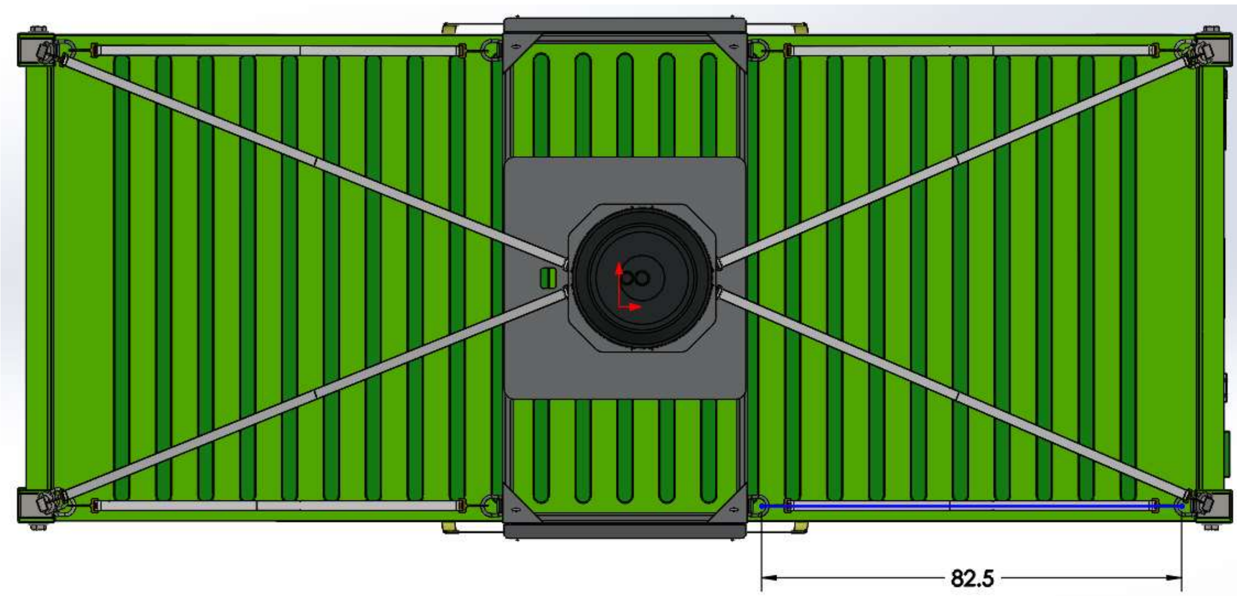

Figure 5: Top of ISO container, showing the Horizontal Support Strap

The drag force calculated in section 1.0 is used here. For simplicity, **SS6** calculations are presented in their entirety while **SS4** calculations are given but not elaborated on, as it's expected that the greater pitch angle will result in larger tensile forces due to the gravitational component being the driving force. This means the total drag force considered for these supports is as follows;

$$\boldsymbol{F_{D-Total}} = F_{D-Mast} + F_{D-Antenna} = 22.63 + 56.0 = \mathbf{78.63}\ [\boldsymbol{lbf}][\boldsymbol{SS6}]$$

$$= 18.07 + 11.18 = \mathbf{29.25}\ [\boldsymbol{lbf}]\ [\boldsymbol{SS4}]$$

The inertial forces for section **3.3** are calculated the same way as those in **2.0**, but consider motion about the pitch axis, due to the orientation of the horizontal supports. This means the driving variable behind the gravitational component and the angular acceleration, the angle of tilt, will change in this section; a maximum pitch angle of **15°** at **SS6** about the pitch axis (per Navy advice).

As mentioned previously, the calculations throughout this section are evaluated in further detail at **SS6, since the rationale to explore **SS4** in the previous sections is no longer valid: the extended mast length has little influence on these values when torque isn't relevant.

First off the angular acceleration due to the pitch oscillation of the ship must be calculated, as in section **2.0**, but with different angles;

$$Angular\ Acceleration = \boldsymbol{\alpha} = -4 \times \pi^2 \times f^2 \times \theta$$

Where;

f = frequency of wave oscillation = 1/Wave Period (obtained from the Sea State Table above, again using the worst-case shortest periods, 6.1 s and 9.8 s)
= 0.164 (SS4) = 0.102 (SS6) [1/s]

θ = Max Pitch Angle = 0.17 (SS4) = 0.26 (SS6) [rads] (source is 10° and 15°, a worst case estimate from experienced Navy staff)

$$\boldsymbol{\alpha} = -4 \times \pi^2 \times 0.102^2 \times 0.26$$

$$= -0.1067 = \mathbf{0.11}\ \left[\frac{\boldsymbol{rad}}{\boldsymbol{s^2}}\right] [\boldsymbol{SS6}]$$

$$= \mathbf{0.19}\ \left[\frac{\boldsymbol{rad}}{\boldsymbol{s^2}}\right] [\boldsymbol{SS4}]$$

This angular acceleration is again used to find the linear acceleration of the various components during pitch. It's also important to note that even though the axis of rotation is closer to the front of the ship, rather than below the SEV, this additional length to the lever arm is not a concern for the analysis. This is because we only need the vertical component of the lever arm to calculate the horizontal linear acceleration, which is the translated tangential acceleration from the point of rotation. The changes in the linear acceleration value are as follows;

$$Linear\ Acceleration\ of\ Antenna = \boldsymbol{a_{Ant}} = \alpha \times l_{T-Ant}$$

Where;

α = Angular Acceleration = 0.19 (SS4) = 0.11 (SS6) [rad/s^2]

$l_{T\text{-}Ant}$ = Total z-length of Antenna lever arm with respect to the ships centre of rotation

= Antenna CofG height from top of Mast + Nested Mast Height + SEV Container Height + Height from Deck to Waterline

= 0.79 + 13.75 + 8.00 + 9.84 = 32.38 (SS4-Extended) [ft]

= 0.79 + 3.25 + 8.00 + 9.84 = 21.88 (SS6-Nested) [ft]

$$\boldsymbol{a_{Ant}} = 0.11 \times 21.88$$

$$\boldsymbol{a_{Ant}} = \mathbf{2.3552}\ \left[\frac{\boldsymbol{ft}}{\boldsymbol{s^2}}\right][\boldsymbol{SS6}]$$

$$\boldsymbol{a_{Ant}} = \mathbf{5.9957}\ \left[\frac{\boldsymbol{ft}}{\boldsymbol{s^2}}\right][\boldsymbol{SS4}]$$

This is the linear acceleration of the antenna during **SS6**, and the remaining linear acceleration values are listed below in **Table 9**. Though the lin. acc. values are higher in **SS4**, (driven by the extended length of the mast), it will be shown that the increased gravitational component at **SS6**, due to the larger pitch angle, is the controlling variable when finding the inertial force.

Table 9: Linear Acceleration about the Pitch Axis

Component	Sea State 4	Sea State 6
Antenna	6.00 ft/s^2	2.36 ft/s^2
Mast	4.58 ft/s^2	2.10 ft/s^2
Bridge	3.30 ft/s^2	1.92 ft/ s^2

Next, the gravitational acceleration component is added to the linear acceleration values created due to the motion of the ship and multiplied by the total mass of the system. This is identical to the process in section **1.0**, though the pitch angle changes the gravitational component's effect. The final inertial force calculation is as follows;

$$Inertial\ Force\ of\ Antenna = \boldsymbol{F_{Inertia-Ant}} = m_{Ant} \times (a_{Ant} + gsin\theta)$$

Where;

m_{Ant} = Mass of the Antenna = <u>4.41</u> [lbm] = <u>4.41</u> [slugs] (Antenna is 142 lbs)

m_{Mast} = Mass of the Mast = <u>5.90</u> [lbm] = <u>5.90</u> [slugs] (Mast is 190 lbs)

m_{System} = Mass of the Support Bridge =20.19 [slugs](Bridge is 650 lbs)

a_{Ant} = Linear Acceleration of Antenna = <u>2.36</u> [ft/s^2] [see table 9 for add. Values]

g = Acceleration due to Gravity = <u>32.2</u> [ft/s^2]

$sin\theta$ = The Gravity Component acting perpendicular to Mast base, based on pitch angle $\theta = 15°$

$$\boldsymbol{F_{Inertia-Ant}} = 4.41 \times (2.36 + 32.2\sin(15°))$$

$$\boldsymbol{F_{Inertia-Ant}} = \mathbf{47.1385}\ [\boldsymbol{lbf}]\ [\boldsymbol{SS6}]$$

$$\boldsymbol{F_{Inertia-Ant}} = \mathbf{51.0940}\ [\boldsymbol{lbf}]\ [\boldsymbol{SS4}]$$

The following table gives the inertial forces created by the antenna, mast and steel bridge during both **SS4** and **SS6**;

Table 10: Inertial Forces in the x-direction

Component	Sea State 4	Sea State 6
Antenna	51.09 lbf	47.14 lbf
Mast	59.99 lbf	61.54 lbf
Support System	179.53 lbf	206.99 lbf
Total Inertial Force =	290.62 lbf	315.67 lbf

It can be seen from the results in **Table 10** that **SS6** does, in fact, produce the larger total linear inertial force when compared to **SS4**, but the difference is so small that it can be considered negligible.

As a final step, the forces acting in the x-direction are summed, that is the mast/antenna drag and inertial forces.

$$\boldsymbol{T_{Total}} = \boldsymbol{F_{D-Total}} + \boldsymbol{F_{Inertia-Total}}$$

$$= \mathbf{78.63 + 315.67 = 394.30}\ [\boldsymbol{lbf}]\ [\boldsymbol{SS6}]$$

$$= \mathbf{29.25 + 290.62 = 319.87}\ [\boldsymbol{lbf}]\ [\boldsymbol{SS4}]$$

As can be seen from the above results, the **SS6** conditions do produce a somewhat larger total translational force. This is largely due to the shallow angles and small differences in pitch magnitudes, which means the lin. acc. term is small in comparison to the gravitational term.

To remain conservative the above process assumes that the bridge/mast system acts as a rigid body and ignores the non-trivial resistance to slip (friction) that would be present from the system mass and ratchet straps pulling the bridge down onto the SEV.

It should also be noted that the tensile force must be divided by two to get the tensile force in the individual straps; $\boldsymbol{T_{Total}} = \mathbf{197.15}\ [\boldsymbol{lbf}][\boldsymbol{SS6}]$**.**

Table 11: Forces in the system's straps

Component	Sea State 4	Sea State 6
Upper Support (Guy Wire) Straps	**541 lbf**	416 lbf
Forklift Support Straps	394 lbf	320 lbf
Horizontal Support Straps	160 lbf	197 lbf

Conclusion

Based on the findings above, it can be seen that the upper support straps (guy wires) will experience the largest tensile loads, at ≈ **541 lbf**. This load occurs during **SS4**, which is expected as the extended mast generates far more torque to be reacted entirely by the two straps alone (in the design scenario). Pre-tension can then be applied to this value, as well as a suitable safety factor, to determine the required strength of the support members in the bridge.

Design Validation

The proposed design handles all degrees of freedom without requiring modification to the SEV. While many of the design elements work together to secure the system, below is a summary of the main methods by which the degrees of freedom are handled:

Bridge Translation in X is prevented by the horizontal straps
Bridge Translation in Y is prevented by the bridge geometry (downward-facing wings)
Bridge Translation in Z is prevented by the forklift straps

Bridge Rotation in X is prevented by the forklift straps
Bridge Rotation in Y is prevented by the horizontal and forklift straps
Bridge Rotation in Z is prevented by the bridge geometry (downward-facing wings)

Payload Translation in X and Y are prevented by the guy wire straps (which are pre-tensioned)
Payload Translation in Z is locked to the bridge (via the mast, which is in compression)

Payload Rotation in X, Y and Z are unlikely but prevented by the guy wire straps

With the external inputs worked out it is now possible to size design elements to adequately absorb them, including proper selection of straps and choice of raw materials.

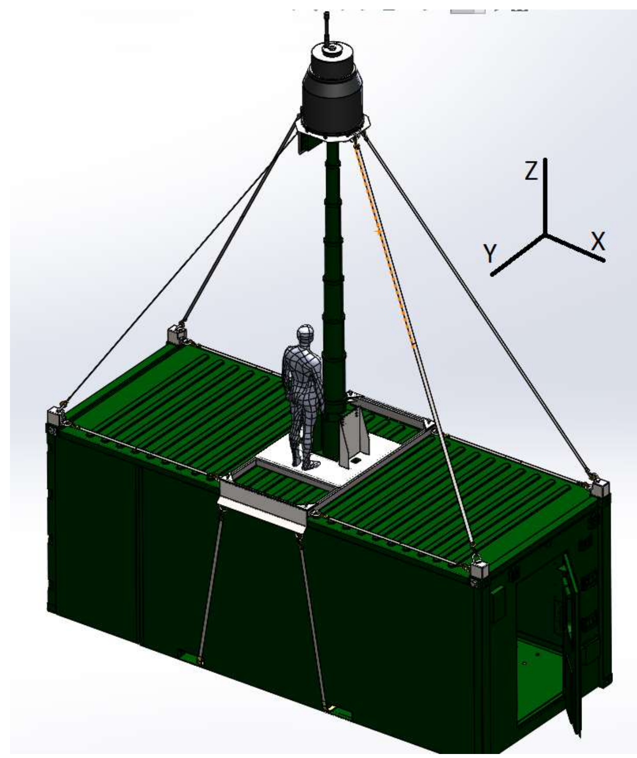

Figure 6: Final Design with Coordinate Reference

Given that the highest strap tension was shown to be 541 lbs, Kinedyne Rhino straps were selected as they have a 3335 lbs working load limit, providing a 6X Factor of Safety. These straps are inexpensive and durable, and custom part numbers from Kinedyne allow them to be ordered at the required lengths with end-configurations suited to the design geometry.

Finally, Finite Element Analysis is used to confirm which materials and stock sizes should be selected for the bridge (steel) and payload support (aluminum), based on the loads they will see due to the various forces and torques (from system mass and from straps tugging locally). FEA is employed to determine material stresses and conduct modal analysis, confirming that the proposed system geometry and material choices properly absorb and dissipate all forces in both SS4 and SS6. As a final check, the mast OEM was consulted and confirms that the mast will perform as expected given our anticipated vectors and loads.

Stress analysis on the bridge shows that the maximum stress is less than 2 ksi, well under our 44 ksi steel yield strength. This result assumes that the guy wires are in place, but the bridge has been designed to withstand the worst-case scenario where multiple failures occur. Similarly, the payload platform's maximum stress is shown to be less than 4 ksi, well under the 35 ksi attributed to 6061-T6 aluminum. Given the somewhat mass-insensitive nature of the requirement, the design has been oversized to account for extreme cases at minimal disadvantage.

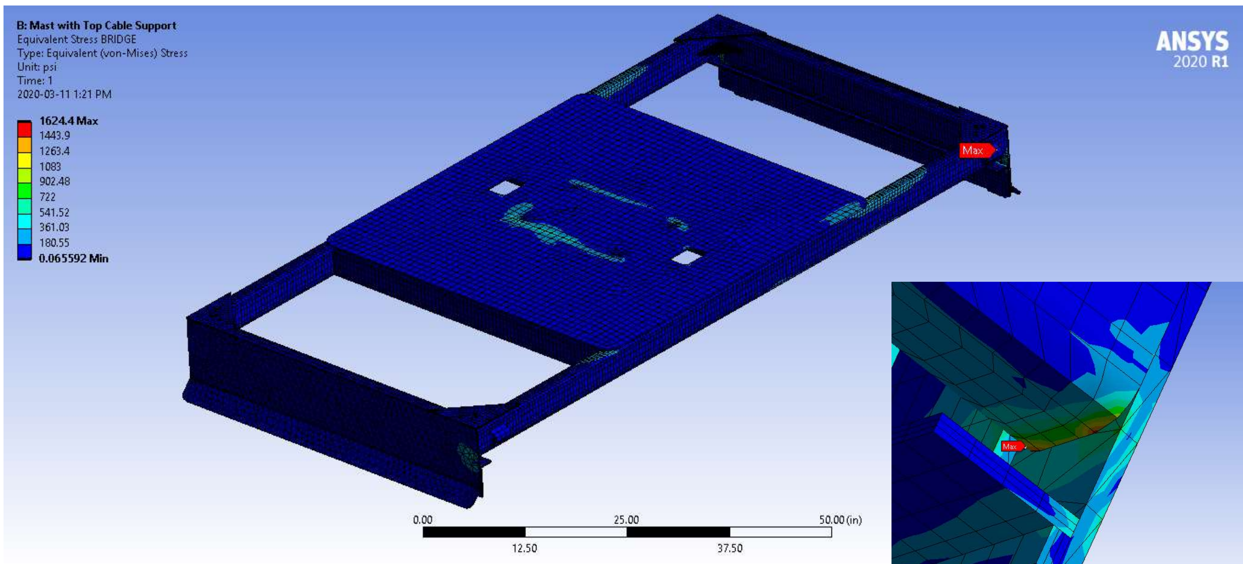

Figure 7: Stress Analysis on the Bridge

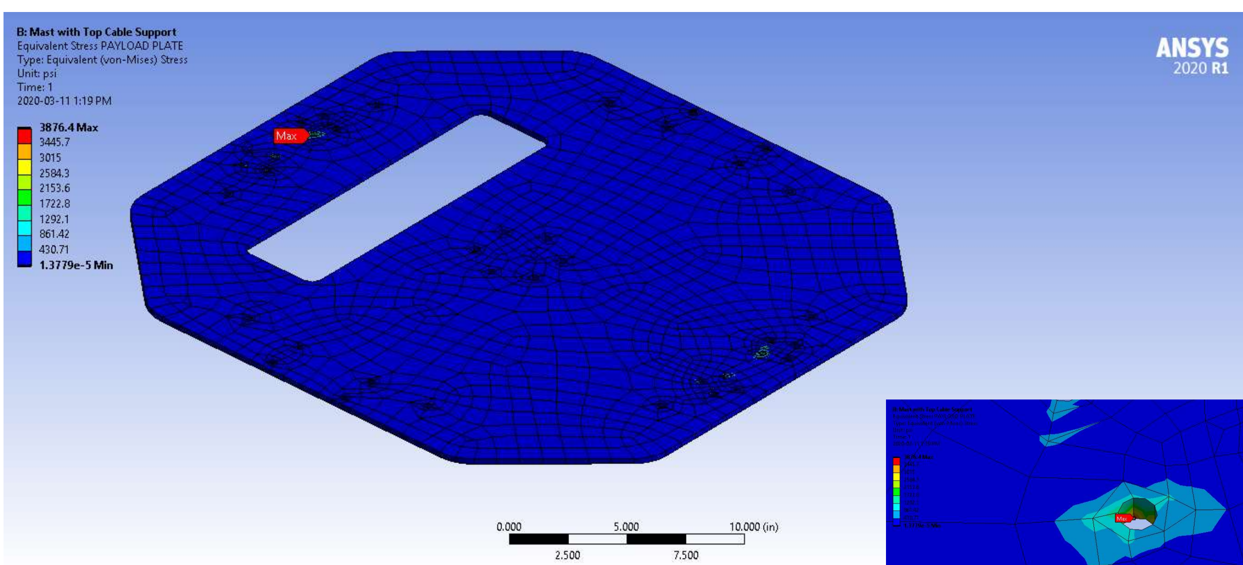

Figure 8: Stress Analysis on the Payload Adapter

Finally, a modal analysis is conducted to determine the system's natural frequencies. The first 6 modes are shown to range from 25 to 95 Hz, which is safely 2 orders of magnitude away from the SS4 and SS6 wave frequencies of 0.16 and 0.10 Hz respectively. Note that the wave frequencies are simply the inverse of their worst case periodicity, 6.1 and 9.8 seconds respectively.

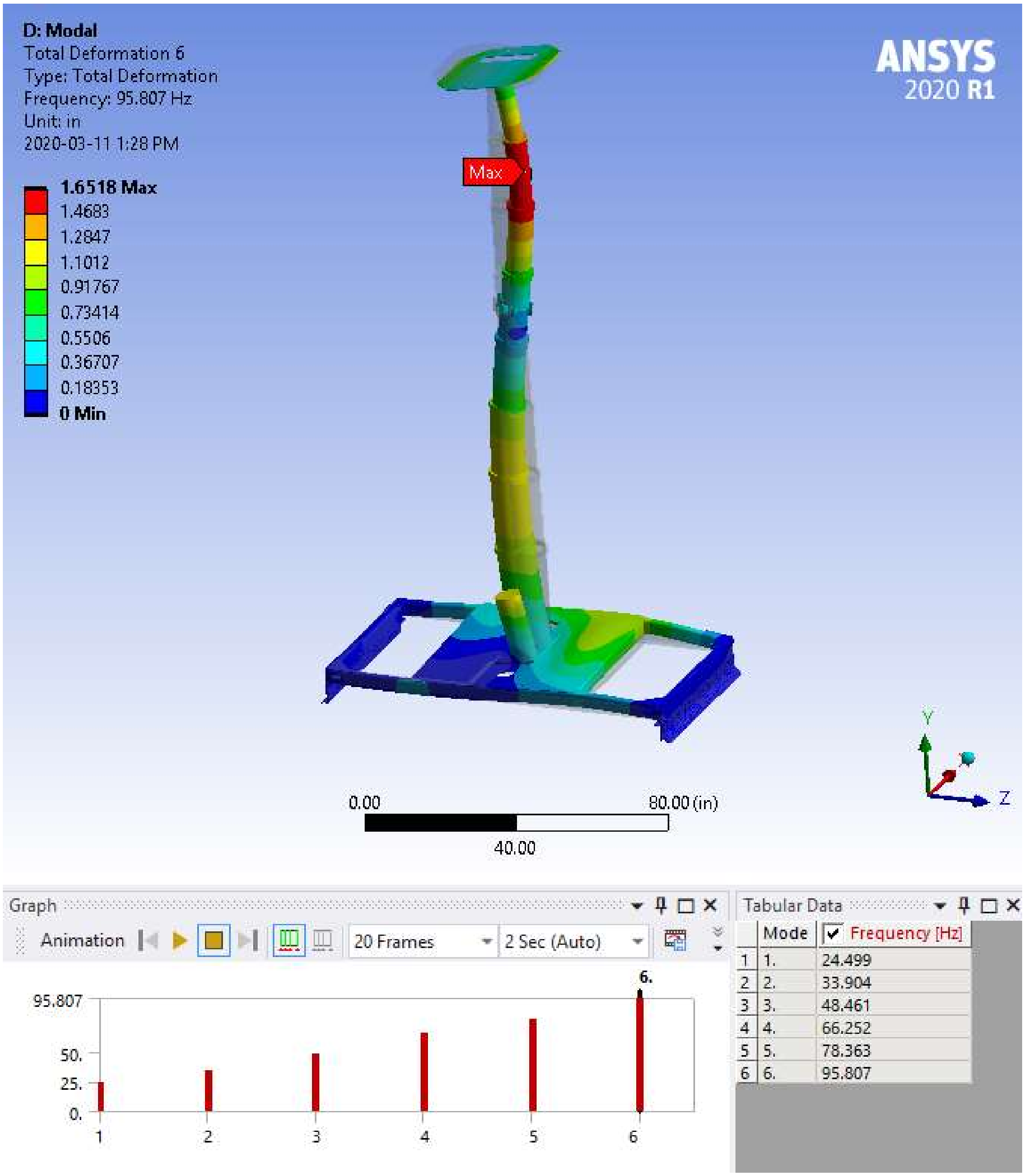

Figure 9: Nodal Analysis on the Full Design