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Pathfinding and optimization for vessels in ice: A literature review

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ABSTRACTS

Voyages through ice-covered waters must maintain safety by adhering to maritime regulations. It is also important to optimize maritime shipping in terms of both economic and environmental factors. There has been much research on this topic. However, a systematic review has not been executed. Hence, this work summarizes systematically what has been done and indicates the current gaps. The present research aims to provide a comprehensive investigation of the following questions: (1) What are the objectives of route optimization in ice? (2) What are the ship performance models for vessels in ice operation? (3) What are the operational constraints in ice? (4) What kind of optimization techniques are used in the routing model? (5) Where do the ice data come from? (6) Is the dynamic changing ice environment considered in the model? (7) Is route validation executed? A review of 32 articles in the literature is performed. The results show that main objectives typically include travelled distance, voyage time, and/or fuel consumption, while wide ranges of ship performance models, constraints, optimization methods, and ice data are used. A few studies consider dynamic ice conditions and route validation. This review article is limited to online sources. Results of the current review suggest that future research in the area of pathfinding for vessels in ice should explore more operational constraints and solve the pathfinding in ice problem under uncertainties. It is also recommended that future work consider validation techniques to enhance the reliability and practicality of these routing tools.

1. Introduction

Route optimization for vessels in ice has been researched for years, during which time many approaches have been used to solve the pathfinding problem for ships from different viewpoints. There is a need to synthesize these works in order to draw a big picture of this problem and point out what could be improved. This study aims to bridge this gap.

Route selection is an optimization problem when all possibilities are considered to find the optimal route. It usually comprises essential components, including an informative map of the region, hydrographical information, sea ice, land, and open water with two ends of the voyage, a list of objectives of the routing problem, and the algorithm to search for the best route. The ice data are either from a meteorological institute (e.g. Canadian Ice Service, Finish Meteorology Institute) or an ice forecast model. Depending on the sophistication of the routing models, these solutions consider more constraints, such as the operational regulations to ensure safety in ice (e.g. Browne et al., 2022; Lee et al., 2021) and the uncertainty of the models (e.g. Choi et al., 2015). Some research provides details about ship performance models to evaluate the total resistance of vessels in open water and ice for a specific situation so that the required power of the vessel is estimated. Such models result in an optimal speed for the ship given the ice conditions (Browne, 2022; Choi et al., 2015; Nam et al., 2013).

Literature reviews for route optimization for ice navigation have been done. However, they are limited to brief review sections as part of larger studies, typically reviewing the current status and background of the ice routing system. To the best of our knowledge, there is no systematic literature review on this topic. The review section of Lehtola et al. (2019) is outstanding in that it provides a review of twelve papers about ice navigation in terms of routing algorithms, spatial and temporal resolution, ship performance models, and the source of ice data. Additionally, they also categorize ship performance models in the literature, such as semi-empirical models, data-driven models, or a hybrid of both approaches. More detailed reviews of ship performance modelling can be found in the literature (Erceg and Ehlers, 2017; Fu et al., 2016; Lehtola et al., 2019; Li et al., 2018; Montewka et al., 2015; Montewka et al., 2019; Sun et al., 2022).

The purpose of this study is to conduct a systematic literature review on route selection for ice-class vessels. Research questions are: (1) What

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Nomenclature

AARI	Arctic and Antarctic Research Institute
AIRSS	Arctic Ice Regime Shipping System
AIS	Automatic Identification System
CIS	Canadian Ice Service
CRREL	Cold Regions Research and Engineering Laboratory
HELMI	Helsinki Multi-category sea-Ice model
HIROMB	High-Resolution Operational Model for the Baltic Sea
Ice-POM	Ice–Princeton Ocean Model
IMO	International Maritime Organization
LASSO	Least Absolute Shrinkage and Selection Operator
NMEFC	National Marine Environmental Forecasting Center
NSIDC	National Snow and Ice Data Center
POLARIS	Polar Operational Limit Assessment Risk Index System
PRISMA	Preferred Reporting Items for Systematic Reviews and Meta-Analyses
TOPAZ4	Towards an Operational Prediction system for the North Atlantic European coastal Zones version 4

are the objectives of the route optimization in ice? (2) What are the ship performance models for vessels in ice operation? (3) What are the operational constraints in ice? (4) What kind of optimization techniques are used in existing routing models? (5) Where do the ice data come from? (6) Is the dynamic changing ice environment considered in the model? (7) Is route validation executed? There are other factors that should be considered in voyage planning and navigation for vessels in ice. Only ice-related constraints and maritime regulations are reviewed in the current study. Assessing literature based on other constraints, such as hydrographical information, is not in the scope of the current study.

The structure of this study is as follows. Section 1 shares the introduction. Section 2 describes the method of how this study retrieves and processes data. Section 3 reveals the results of the data collection process. Section 4 discusses the research questions as aforementioned. Section 5 concludes the findings for this review on route selection in ice.

2. Method

This section introduces the general process of how the search for relevant literature is conducted. The current study applied the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) approach (Page et al., 2021). The purpose of searching is to identify all relevant articles to route optimization in ice. The information sources are online databases, including Scopus, Science Direct, and Web of Science. The queries do not limit time; the latest update was on Jan 01, 2022.

The chosen studies for this review are based on the following eligibility criteria: they have to be related to route optimization in ice-covered waters or winter navigation, and the source is published in English. Table 1 shows the search string on selected databases. Besides the search results, some other works, which were mentioned by Lehtola et al. (2019), are included in this set. Fig. 1 illustrates the selection process.

Table 1
Search strings used to collect publications about route selection for vessels in ice.

Databases	Search string
Scopus	ALL (“route” OR “path”) AND (“ice-covered waters” OR “winter navigation”) AND “optimization” AND (LIMIT-TO (LANGUAGE, “English”))
Science Direct	(“route” OR “path”) AND (“ice-covered waters” OR “winter navigation”) AND “optimization”
Web of Science	(route OR path) AND (ice-covered waters OR winter navigation) AND (optimization OR best)

There are two stages of the screening process. The titles and abstracts are scanned in the first stage to filter out the irrelevant articles. The second stage is a full-text screening of potential articles. After determining the shortlist, the data extraction of eligible papers is implemented. The extracted information includes operational objectives of the route optimization, ship performance models, operational constraints in ice, optimization methodology, ice data source, dynamic environment, and type of route validation.

3. Results

There were 265 articles returned from the queries, where 126 papers were from Scopus, 84 papers belonged to Science Direct, and the 55 remaining ones were from Web of Science. The results had 31 duplicate items, so 234 individual articles were examined. The screening based on title and abstract helped exclude 122 articles because they were irrelevant to ice navigation. Their topics are oil spills, fishing decision support systems, and dynamic positioning operations. Subsequently, the remaining 112 articles were reviewed carefully, and only 28 papers were identified as relevant to the current study. The 85 additional works were removed because their main research focus is on ice classification, transportation in open waters or in-land waters, ship performance models, or risk analysis of voyages in ice. Of the twelve papers mentioned by Lehtola et al. (2019), eight already appear in the 28-article set, and the remaining four were added. Therefore, this study will review 32 articles in total, where the number of journal articles is 18 while there are 14 conference papers. Fig. 1 describes the process of data collection. Fig. 2 summarizes the number of articles published by year on the route finding for vessels in ice.

Three main operational objectives applied in the research literature for the route selection problem are optimizing voyage distance, voyage time, and fuel consumption. Table 2 describes the combination of objectives used in the reviewed articles. Browne et al. (2022), Frederking (2003), and Topaj et al. (2019) combined all three objectives for the optimization function. Some other works selected two objectives in the optimization function. For instance, distance and time were used by Choi et al. (2013), Choi et al. (2015), and Reimer (2015). In comparison, Jeong et al. (2018), Nam et al. (2013), and Wang et al. (2018) optimized voyage time and fuel consumption. Interestingly, no study chose the combination of voyage distance and fuel consumption. Other studies included only one indicator for their system. Specifically, Lee et al. (2019), Lee et al. (2021), Li et al. (2019), Li et al. (2020), Piehl et al. (2017), and Zhang et al. (2019) chose minimizing fuel consumption. Koyama et al. (2021), Kotovirta et al. (2009), Lehtola et al. (2019), May et al. (2018), May et al. (2020), Mishra et al. (2021), Schütz (2014), Smith and Stephenson (2013), Voitkunskaia et al. (2019), and Wang et al. (2021) optimized voyage time. Minimizing voyage distance was the sole objective in the articles of Aksakalli et al. (2017), Guinness et al. (2014), Hsieh et al. (2021), Liu et al. (2016), Zhang et al. (2017), Zvyagin and Voitkunskaia (2016), Zvyagina and Zvyagin (2022).

Table 3 summarizes what kind of ship performance model the reviewed studies apply to their route optimization. In this research, a performance model includes any model that estimates the ice resistance and power of the vessels in both open water and ice conditions. Nearly half of them (13 out of 32) described a ship performance model. From Table 3, some work applied Keinonen's method (Keinonen et al., 1991) (e.g. Browne et al., 2022; Frederking, 2003). Others used Riska's approach (Riska, 1997) (e.g. Guinness, 2014; Kotovirta et al., 2009; Mishra et al., 2021), and Tillig et al. (2017) (Li et al., 2019; Li et al., 2020). Data-driven techniques by regression from ship log data and Automatic Identification System (AIS) data have been used in several cases (Wang et al., 2021; Zhang et al., 2019). Lehtola et al. (2019) applied a hybrid model where both semi-empirical and data-driven approaches were combined. Jeong et al. (2018), Lee et al. (2021), and

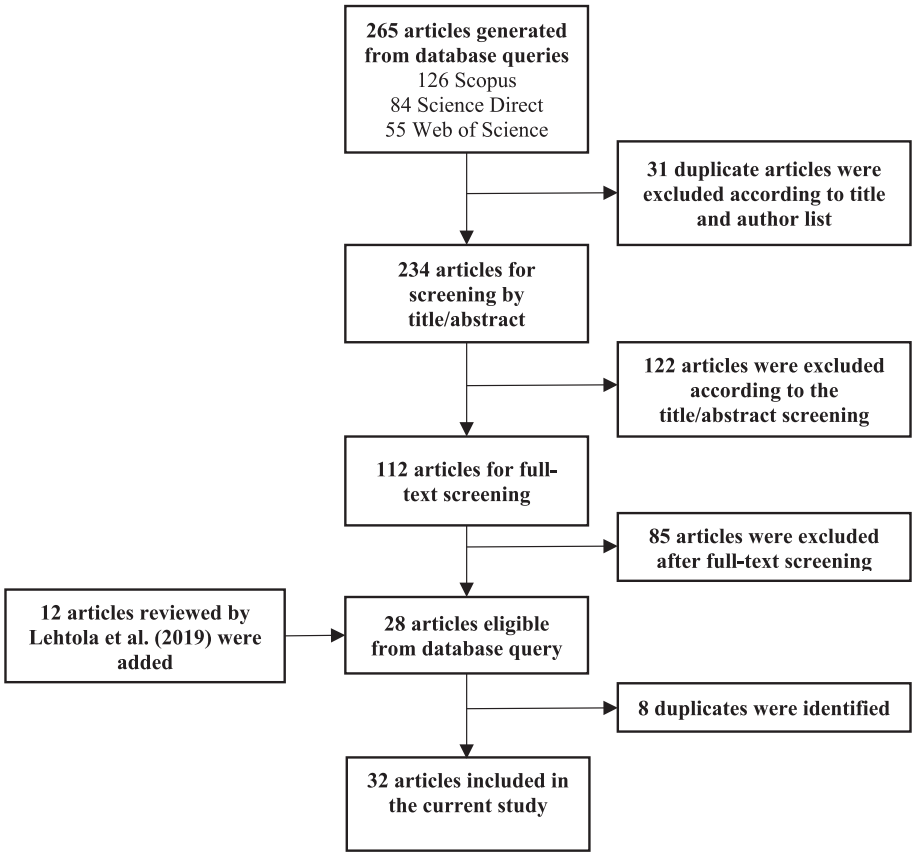

Fig. 1. The dataset selection process.

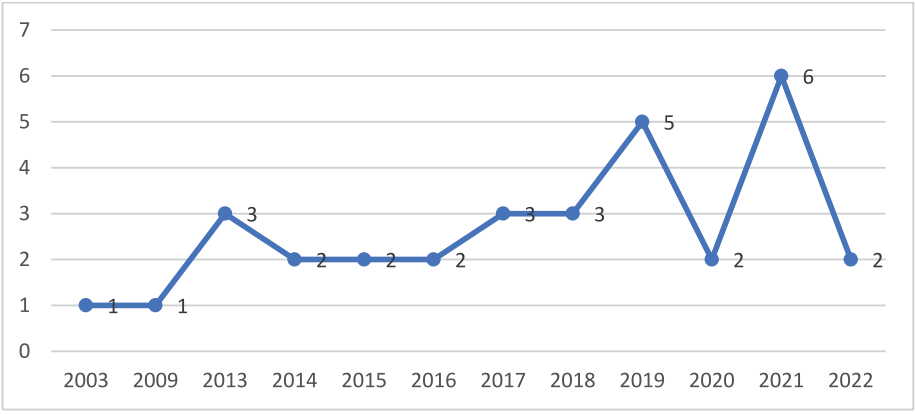

Fig. 2. The number of publications by year from 2003 to January 2022.

Reimer (2015) customized their performance models by model tests to estimate the relationship between sea ice and ship speed. Several studies did not estimate fuel consumption because it did not play any role in their pathfinding and optimization. They just found the attainable speeds in certain ice conditions instead. For instance, Choi et al. (2015) and Nam et al. (2013) used the model of the Cold Regions Research and Engineering Laboratory (CRREL) to find the optimal speed of the vessels in certain ice conditions.

Operational constraints are summarized in Table 4. Polar Operational Limit Assessment Risk Index System (POLARIS), which was introduced under the International Maritime Organization (IMO) Polar Code (IMO, 2016), was the most popular constraint (7 out of 32) in the literature, used by Browne et al. (2022), Jeong et al. (2018), Lee et al. (2019), Lee et al. (2021), Li et al. (2020), Wang et al. (2021), and Zhang et al. (2017). The second most common restriction was the Canadian Arctic Ice Regime Shipping System (AIRSS), which was applied by Browne et al. (2022), Liu et al. (2016), Smith and Stephenson (2013), and Wang et al. (2018). The AIRSS manual and its pictorial guide are provided by Transport Canada (2018a) and Transport Canada (2018b). Other studies introduced safety conditions such as safe speed from ship speed reduction models in ice (Browne et al., 2022; Choi et al., 2015; Nam et al., 2013). These models helped find the maximum speeds that vessels can attain given a particular ice condition. Hsieh et al. (2021) proposed a radar sea ice risk index to estimate the potential risk of ice to the vessel. The risk index was determined by the ratio of the area of sea ice to the distance from it to the vessel. Preference rules were used by Lehtola et al. (2019). The rules preferred thinner ice over thick ice, lower probability of besetting in ice, and no ship grounding. They were

Table 2
Multiple objectives in ice navigation.

Objectives			Quantity	Studies
Distance	Time	Fuel consumption		
✓	✓	✓	3	Browne et al. (2022), Frederking (2003), and Topaj et al. (2019)
✓	✓		3	Choi et al. (2013), Choi et al. (2015), and Reimer (2015)
✓		✓	0	
✓			7	Aksakalli et al. (2017), Guinness et al. (2014), Hsieh et al. (2021), Liu et al. (2016), Zhang et al. (2017), Zvyagin and Voitkunskaia (2016), Zvyagina and Zvyagin (2022)
	✓	✓	3	Jeong et al. (2018), Nam et al. (2013), and Wang et al. (2018)
	✓		10	Koyama et al. (2021), Kotovirta et al. (2009), Lehtola et al. (2019), May et al. (2018), May et al. (2020), Mishra et al. (2021), Schütz (2014), Smith and Stephenson (2013), Voitkunskaia et al. (2019), and Wang et al. (2021)
		✓	6	Lee et al. (2019), Lee et al. (2021), Li et al. (2019), Li et al. (2020), Piehl et al. (2017), and Zhang et al. (2019)

Table 3
The summary of ship performance models used in the literature.

Ship performance model	Quantity	Studies
Keinonen et al. (1991)	2	Browne et al. (2022) and Frederking (2003)
Riska (1997)	3	Guinness et al. (2014), Kotovirta et al. (2009), and Mishra et al. (2021)
Tillig et al. (2017)	2	Li et al. (2019), Li et al. (2020)
Regression from log data	2	Wang et al. (2021), Zhang et al. (2019)
Hybrid model	1	Lehtola et al. (2019)
Customized model tests	3	Jeong et al. (2018), Lee et al. (2021), and Reimer (2015)
N/A	19	Others

Table 4
The summary of operational constraints in ice used in the literature.

Constraints in ice	Quantity	Studies
AIRSS	4	Browne et al. (2022), Liu et al. (2016), Smith and Stephenson (2013), and Wang et al. (2018)
POLARIS	7	Browne et al. (2022), Jeong et al. (2018), Lee et al. (2019), Lee et al. (2021), Li et al. (2020), Wang et al. (2021), and Zhang et al. (2017)
Ship speed reduction model in ice	3	Browne et al. (2022), Choi et al. (2015), and Nam et al. (2013)
Radar sea ice risk index	1	Hsieh et al. (2021)
Preference rules	1	Lehtola et al. (2019)
Minimum ice thickness	1	Zvyagin and Voitkunskaia (2016)
Iceberg avoidance	2	Li et al. (2019), Voitkunskaia et al. (2019)
N/A	15	Others

integrated into the model by a speed map for implementation. In addition, some other constraints were used, such as minimum ice thickness (Zvyagin and Voitkunskaia, 2016) and iceberg avoidance (Li et al., 2019; Voitkunskaia et al., 2019). Fifteen articles did not report any specific ice-related operational constraints. Note that the total quantity is more than 32 in Table 4 because the work by Browne et al. (2022) is counted three times. They applied three constraints (POLARIS, AIRSS, and safe speed) individually to compare the effects.

The summary of optimization techniques used in the literature is shown in Table 5. Graph-based methods, including A* and Dijkstra's algorithm, were dominant, with more than half of the studies choosing this approach (18 out of 32). Other techniques, such as genetic-based method, wave-based algorithm, the ant colony algorithm, stochastic dynamic programming, finite element method, linear regression, and Powell's method were also used in the reviewed research.

Ice data sources are reported in Table 6. The ice input for the navigation system was used from many sources. Canadian Ice Service (CIS) provided ice charts for specific areas of Canadian waters (Browne et al., 2022; Frederking, 2003; Liu et al., 2016). Sea ice of the Baltic Sea could be found in the Helsinki Multi-category sea-Ice model (HELMI) of the Finnish Meteorological Institute (Guinness, 2014; Lehtola et al., 2019) and the High-Resolution Operational Model for the Baltic Sea (HIROMB) of the Swedish Meteorological and Hydrological Institute (Kotovirta et al., 2009). There were other models used in ice forecasting, such as the Ice–Princeton Ocean Model (Ice-POM) (Choi et al., 2013; Choi et al., 2015; De Silva et al., 2015), the Towards an Operational Prediction system for the North Atlantic European coastal Zones version 4 (TOPAZ4) (Jeong et al., 2018; Koyama et al., 2021; Sakov et al., 2012), National Snow and Ice Data Center (NSIDC) (Li et al., 2019; Mishra et al., 2021; National Snow and Ice Data Center, n.d), National Marine Environmental Forecasting Center of China (NMEFC) (Zhang et al., 2019), UK Met Office (Li et al., 2020), and the Arctic and Antarctic Research Institute (AARI) in Russia (Arctic and Antarctic Research Institute, n.d; May et al., 2018; May et al., 2020). Other researchers just mentioned that they use ice data from customized general models. Eight out of 32 studies did not report the source of ice data.

Table 7 shows how many studies consider the dynamics of the ice environment. There are four studies working on it, including May et al. (2020), Schütz (2014), Voitkunskaia et al. (2019), and Zvyagina and Zvyagin (2022).

The validation summary of the ice navigation system is shown in Table 8. A work is considered without validation when it simply proposes the route in a particular ice condition and concludes it is optimal without further reference. In contrast, the validated work compares the

Table 5
The summary of optimization techniques used in the literature.

Algorithm	Quantity	Studies
Dijkstra, A* algorithms	18	Aksakalli et al. (2017), Browne et al. (2022), Choi et al. (2015), Guinness et al. (2014), Jeong et al. (2018), Lehtola et al. (2019), Li et al. (2019), Li et al. (2020), Liu et al. (2016), Mishra et al. (2021), Nam et al. (2013), and Smith and Stephenson (2013), Topaj et al. (2019), Voitkunskaia et al. (2019), Wang et al. (2018), Wang et al. (2021), Zhang et al. (2017), and Zvyagin and Voitkunskaia (2016)
Genetic algorithm	3	Choi et al. (2013), Lee et al. (2019), and Lee et al. (2021)
Wave-based algorithm	4	May et al. (2018), May et al. (2020), Topaj et al. (2019), Zvyagina and Zvyagin (2022)
Ant colony algorithm	1	Zhang et al. (2019)
Stochastic dynamic programming	1	Schütz (2014)
Finite element method	1	Piehl et al. (2017)
Linear Regression	1	Koyama et al. (2021)
Manual	1	Frederking (2003)
Powell's method	1	Kotovirta et al. (2009)
Rapidly-exploring random tree B-RRT	1	Hsieh et al. (2021)
N/A	1	Reimer (2015)

Table 6
Ice sources.

Ice estimation model	Quantity	Studies
CIS	3	Browne et al. (2022), Frederking (2003), and Liu et al. (2016)
HELMI	2	Guinness et al. (2014), Lehtola et al. (2019)
Ice-POM	2	Choi et al. (2013), Choi et al. (2015)
NSIDC	2	Li et al. (2019), Mishra et al. (2021)
TOPAZ4	2	Jeong et al. (2018), Koyama et al. (2021)
HIROMB	1	Kotovirta et al. (2009)
NMEFC	1	Zhang et al. (2019)
UK Met Office models	1	Li et al. (2020)
AARI	2	May et al. (2018), May et al. (2020)
Customized models	8	Hsieh et al. (2021), Lee et al. (2019), Lee et al. (2021), Nam et al. (2013), Reimer (2015), Smith and Stephenson (2013), Wang et al. (2018), Wang et al. (2021)
N/A	8	Others

Table 7
Dynamics ice environment.

Is the ice environment changing or not?	Quantity	Studies
Yes	4	May et al. (2020), Schütz (2014), Voitkunskaia et al. (2019), Zvyagina and Zvyagin (2022).
No	28	Others

Table 8
Validation.

Is validation conducted?	Quantity	Studies
Yes	6	Browne et al. (2022), Guinness et al. (2014), Jeong et al. (2018), Kotovirta et al. (2009), Lee et al. (2021), Lehtola et al. (2019), and Mishra et al. (2021).
No	26	Others

suggested route from the route optimization with credible sources, such as the expert opinion of seafarers and historical AIS data. Six out of 32 studies validated the route selection of the system with real-world scenarios, such as Browne et al. (2022), Guinness et al. (2014), Jeong et al. (2018), Kotovirta et al. (2009), Lee et al. (2021), Lehtola et al. (2019), and Mishra et al. (2021).

4. Discussion

4.1. Operational objectives

All route optimization for vessels in ice considered either travelled distance, or voyage time, or fuel consumption, or a combination of them as the operational objectives. This choice is understandable because all three indicators are related to the cost of operation. The optimization is straightforward when the navigation uses only a single metric among the three: the best route is the shortest route in length or time or the route in which the vessel consumes the least amount of fuel. However, when two or more operational objectives are selected, a method to determine optimality is required.

Three main approaches to multiple criteria decision-making were used in the articles. Firstly, the scalarization method was applied by Choi et al. (2015) and Browne et al. (2022). These works assigned weights, from 0 to 1, for each operational objective. They compared actions by their costs. The action with the least cost was more optimal than the others. A cost function was determined by the weighted summation of all objectives. The determination of weights was at the discretion of the system's designers. While there was no specific justification for the choice of Choi et al. (2019), Browne et al. (2022) simulated different sets of weights where each set represented a decision style and resulted in different optimal routes. Weight setting leads to bias issues. The dominance of the fuel consumption or voyage time in the cost function must be calibrated by users in specific contexts. Similarly, other authors applied the scalarization approach from a monetized viewpoint. They converted all optimized objectives to the economic cost (Nam et al., 2013; Topaj et al., 2019; Wang et al., 2018). In these models, the operational cost of the voyage was calculated. The route whose cost was the lowest achieves optimality. Secondly, other research applied lexicographic optimization (Nam et al., 2013). Nam et al. selected distance and time as the two operational objectives where distance was prioritized over time. Initially, the optimal route was the shortest in terms of length. Then the attained speeds along the shortest route were determined to ensure the selected route arrived at the destination in the fastest time. Finally, another approach to multi-criteria route optimization in this literature was the Pareto front used by Frederking (2003). The selected route did not have any objective that was inferior to the same indicator of other routes. In summary, many multi-criteria decision-making approaches were used in route optimization for vessels in ice, in which scalarization was more commonly applied than other approaches because it is simple to use. The optimality was dependent on the system designer's discretion. The system designers have to justify how to set weights to provide transparency and to avoid bias issues.

4.2. Ship performance model

The purpose of route planning is to select the best route based on defined criteria, such as travelled distance, voyage time, and fuel consumption. While the determination of distance is trivial, the estimation of voyage time and fuel consumption requires a means to estimate ship performance in ice and open water. A ship performance model is used to estimate the resistance of the vessel, as a function of speed, and the associated required power and fuel consumption. The speed of a vessel along a route depends on the actual situation and is typically not constant across varying ice conditions and open water. Operating at different speeds leads to variations in estimated voyage time and consumed fuel. The ship performance model can support the determination of optimal speed for a given operating scenario.

Approaches to modelling ship performance can be classified into three main categories: semi-empirical or empirical methods (Keinonen et al., 1996; Riska, 1997; Tillig et al., 2017), data-driven methods using log data, and hybrid approaches with the combination of the first two methods. All models involve a lot of technical details of ice properties as well as ship particulars. The semi-empirical approach uses numerical analysis and experiments to formulate the ice resistance. This process required an analysis based on physics to determine the parameters of the model with their coefficients. The values of coefficients are estimated by experiments, such as from ice tank tests. Montewka et al. (2015) claimed that these models omitted the joint effect of sea ice on the vessel's speed. The data-driven approach regresses the resistance based on the historical data of specific vessels given an ice condition. This method has a problem when an essential variable of the learning process is hidden, because it does not model the physics of the ice breaking process (Montewka, 2019). A hybrid model can tackle the problem mentioned above. However, a critique of the ship performance models is out of the scope of this research. Route optimization in ice should consider an accurate model to have a better plan.

4.3. Operational constraints

Safety is of utmost importance for maritime operations, and operational constraints are often imposed to promote safe operations. Safe operations are modelled within the research literature using different

operational constraints. All the optimizations must be executed after the operational constraints have been met.

In general, ships must operate in adherence to a network of maritime regulations. Specific to Polar regions, regulatory guidelines impose operational constraints to promote safe operations in ice. One regulatory guideline is POLARIS, introduced through the IMO Polar Code (IMO, 2016). Another similar regulatory guideline for safe navigation in ice is the Canadian AIRSS. As mentioned in Section 3, a lot of work showed it was feasible to adhere to these two regulations in the route optimization framework. For instance, Browne et al. (2022) evaluated the implications of POLARIS and AIRSS constraints on Arctic ship operations. However, the two regulatory constraints are insufficient to ensure safety in ice navigation (Browne et al., 2022). Other constraints in the literature were proposed, such as ship speed reduction in ice (Nam et al., 2013; Choi et al., 2015; Browne et al., 2022), sea ice risk index (Hsieh et al., 2021), preference rules (Lehtola et al., 2019), minimum ice thickness (Zvyagin and Voitkunskaia, 2016), and iceberg avoidance (Li et al., 2019; Voitkunskaia et al., 2019).

4.4. Route optimization techniques

The optimization algorithm is the core element of the pathfinding system. It processes all input data to search for the best suitable routes according to the predefined constraints and optimized objectives. Multiple techniques were introduced in the literature for the route optimization problem. Each method has advantages and disadvantages based on the focus area of the studies. Among these algorithms, the graph-based method is dominant.

The graph-based algorithm is the most popular technique. It is straightforward to convert a map into a discretized grid world. Each cell plays a node role, and this cell's connectivity to each neighbour creates an edge relation. When the pathfinding problem is formulated as a graph, multiple algorithms can be used to solve for the optimal path, such as A* and Dijkstra's algorithm (e.g. Browne et al., 2022; Choi et al., 2015). The principal idea is to search for the path whose cost is the lowest among the possibilities based on the defined objectives and associated cost function. The cost of the path is the sum of the costs to traverse all nodes along the path. The advantage of graph-based algorithms is that they can solve the problem in linear time proportional to the total number of edges and nodes in the graph. However, a limitation is that the method requires the graph to be known in advance and remain unchanged during the process.

Other optimization methods are used. Frederking (2003) manually applied a ship performance model to compare two candidate routes for a voyage. Waypoints along two routes were predetermined, and the routes were compared considering voyage time, distance, and fuel consumption. It is similar to a divide-and-conquer strategy to find the total cost values of each objective. Despite the manual method, it provides a good strategy and foundation for dealing with a complicated ice regime decomposition.

Wave-based methods are applied in May et al. (2018), May et al. (2020), Topaj et al. (2019), and Zvyagina and Zvyagin (2022). This approach does not need to discretize the map with ice information into grid cells like the graph-based method. It searches the optimal route directly using the vector format of the ice charts, in which propagation of equal level curves is generated from the start point to the endpoint in the geographical space. The suggested route of the wave-based method is smoother than the results of the graph-based approach. However, the number of reference points in subsequent wavefronts increases significantly, which impacts the computational cost (May et al., 2020; Topaj et al., 2019). Wave-based methods are often not used directly, rather they are modified to reduce complexity, such as done by May et al. (2020) and Topaj et al. (2019).

Powell's method is another approach for route optimization for vessels in ice (Kotovirta et al., 2008). They created the cost function and minimized it using Powell's conjugate direction method. This approach does not require the cost function to be differentiable because the derivatives of this function are not needed. However, the method only results in a locally optimal solution.

Piehl et al. (2017) formulated route optimization in the form of potential field problems. A mesh is generated by the coastal and ice data for an area with some boundary conditions. The authors applied the finite element method to solve the Poisson equation. Then, the gradient vector field is derived from the computed potential field to determine the best route.

Genetic-based algorithms were selected by Choi et al. (2013), Lee et al. (2019), and Lee et al. (2021). This approach can address route selection on a continuous map despite long runtime. Zhang et al. (2019) used the ant colony algorithm, which is a metaheuristic approach similar to a genetic algorithm. This algorithm's advantage is the capability to adapt to a changing environment. Stochastic dynamic programming (Schütz, 2014) and rapid-exploring random tree (Hsieh et al., 2021) are helpful in the stochastic environment and searching for the optimal route in the continuous domain, respectively. A data-driven technique using multiple linear regression and the Least Absolute Shrinkage and Selection Operator (LASSO) regression to get an optimal route from AIS data was applied by Koyama et al. (2021).

4.5. Ice data sources

Modelling sea ice conditions is one of the requirements for a route optimization system for vessels in ice. Modelled ice conditions may include ice thickness, ice floe size, ice age, and ice concentration. As mentioned in section 3, sources of ice data are categorized into three groups. Firstly, many national ice services provide sea ice charts in specific regions, including the Canadian Ice Service for Canadian waters, the Finnish Meteorological Institute and the Swedish Meteorological and Hydrological Institute for the Baltic sea, the National Marine Environmental Forecasting Center in China, and the UK Meteorological Office for the Arctic region. They issue ice charts on a regular basis (e.g. daily or weekly) developed using satellite imagery and marine or aerial observations. Secondly, ice conditions can be estimated by forecast models, for example, Ice–Princeton Ocean Model (De Silva et al., 2015), National Snow and Ice Data Center (National Snow and Ice Data Center, n.d), the Towards an Operational Prediction system for the North Atlantic European coastal Zones version 4 (TOPAZ4) (Sakov et al., 2012), and Arctic and Antarctic Research Institute (AARI) (Arctic and Antarctic Research Institute, n.d). These models keep updating sea ice estimation frequently. The first two sources (published ice charts and sea ice forecast models) are generally considered reliable and widely used in ice navigation. Note that this study only mentioned ice sources and ice forecast models that have been used in the identified literature. There are other available sources of ice information in the world, such as Danish Meteorological Institute, Norwegian Ice Service, and US National Ice Center. Thirdly, some studies have developed customized sea ice models to generate the ice information for pathfinding and optimization. Models in the third approach should be validated before applying the ice navigation system to a real-world scenario. It should be acknowledged that each model also has its own format as well as spatial and temporal resolution.

4.6. Dynamic ice environment

Sea ice is a dynamic environment being driven by wind, tide, and current forces. Sea ice drift and deformation processes result in ice conditions that are spatially and temporally variable. Sea ice conditions can be forecasted using models as described in section 3. Navigating in such a dynamic environment is a challenging problem. Almost all of the research literature assumes that the ice estimation for a specific day is unchanged. A few others incorporate sea ice dynamics in their pathfinding and optimization, including May et al. (2020), Schütz (2014), Voitkunskaia et al. (2019), and Zvyagina and Zvyagin (2022). As can be

seen from Table 7, this important factor was first addressed in 2014 by Schütz, and it has been considered again recently since 2019.

Schutz (2014) used a scenario tree to evaluate uncertainty in ice prediction. The author showed the effect of uncertainty on route planning decision-making. The work used the non-anticipativity principle, which means the decision is valid at a time, and it does not consider future events.

Voitkunskaia et al. (2019) and Zvyagina and Zvyagin (2022) assumed a kinematics model of ice where the translational and rotational movements are considered. The drift ice information is known in this model. They also integrated an avoidance maneuver to drive the vessel away from unexpected ice floes.

May et al. (2020) also addressed the temporal changing ice from the ice data input. They used a prognostic model for changing sea ice. This model included the oceanic and atmospheric factors, such as wind and current.

4.7. Route validation

The routing system should be validated carefully before being adopted for real-world applications. Validation can have several levels to test effectiveness and accuracy, from simulation to field test, from open water to harsh sea ice environments. In the literature, the validation task is divided into three categories: validation using historical AIS data, validation by a field test, and face validation with experts.

The first approach is the field test. Each ice navigating tool helps determine a suitable route based on assumptions. The assumption could be the known environment as aforementioned. When the tool is applied in reality, the assumption might be invalid, so the route is no longer optimal. For this reason, a field trial for route validation is appropriate to check the reliability. However, conducting a field test validation may not be feasible. It can be costly to have multiple vessels run on multiple routes simultaneously for one scenario to compare performance. Another issue is that when a ship faces an unexpected navigational challenge, the captain might change the route to maintain safety. Only Jeong et al. (2018) conducted a real voyage for validation. They had five trial groups in five different ice conditions from 75°N to 78°N. The validation results showed that the mismatching between estimated speed and actual speed varied from 0 to 50%.

Due to the difficulties of field testing, some researchers chose to validate optimized routes with historical AIS data for similar voyages (Guinness, 2014; Kotovirta et al., 2009; Lehtola et al., 2019; Mishra et al., 2021). The advantage of this approach is that the route optimization can be tested against known ice conditions. It provides a reference for the comparison of a voyage for a specific vessel using historical AIS data. However, when comparing against AIS data, an assumption is that the navigators on that ship perform perfectly. In other words, the AIS routes are assumed to be optimal routes. If the optimized route matches the AIS data, the pathfinding performance is considered to be as good as a human decision-maker. If the optimized route is different from the AIS data, the pathfinding performance is considered to be poor. However, it could be that the AIS data reflect a non-optimal route. When using AIS data, it is crucial to consider the reference vessels and ensure the AIS data reflects a similar ice class and operation in similar ice conditions as the testing vessel.

The third validation method is face validation by consulting the ice navigation experts (Browne et al., 2022; Lehtola et al., 2019). Lehtola et al. (2020) described details of how a validation process for a route optimization tool is conducted with sea captains. All essential information is given to the experienced captains and seafarers and they are asked how they plan routes. The other form of this validation method is showing the expert the proposed routes of the tools and asking for feedback. This category is advantageous because the routes are validated directly by end-users. If the expert agrees with the route selection, the tool gains more trust for implementation. The approach is also helpful when validation is conducted in a wide range of scenarios where no or few voyages are taken. The downside is that the test might miss some uncertain factors.

In summary, different approaches for validation are available in the literature. Each method has its own advantages and disadvantages. Future research might combine them to boost the effectiveness and reliability of the routing tool.

5. Future research suggestions

The application of route selection for vessels in ice has potential. Some work needs to be done to improve the performance. Firstly, the consideration of more constraints is essential because the safety of operation in ice requires more than regulatory constraints such as AIRSS, POLARIS, and speed reductions. Further guidance to reduce the likelihood of structural damage to the vessel may be considered. Carbon emission reduction is a new constraint that shipping operations will have to adapt to in the near future. Secondly, the calculation of fuel consumption requires an accurate ship performance model to estimate the power needed for vessels sailing in both open waters and sea ice. Ice route finding research should employ a ship performance model that can be used for a wide range of vessels in different ice conditions. Thirdly, future work may further investigate the integration of sea ice dynamics for pathfinding and optimization. Sea ice is drifting and deforming over time. Applying a sea ice dynamics model for navigation optimization may help improve route optimization and navigation decision-making. Fourthly, there are many machine learning approaches to pathfinding problems, such as reinforcement learning. This method helps solve the route planning when the environment is not deterministic, such as the increasing uncertainties associated with visibility at night, and differences between actual ice conditions and the conditions reported in ice charts. Fifthly, multi-criteria decision-making must be informed by transparent and more comprehensive weight schemes that reflect the relative importance of factors as viewed by a range of stakeholders. The weighting scheme will certainly influence optimality, and the weights themselves are likely to be different according to different types of stakeholders. Finally, route optimization involves assumptions and uncertainty. Continued and further validation is necessary.

6. Conclusion

A literature review on pathfinding and optimization for vessels in ice was performed in this study. A total of 32 articles was analyzed to address seven research questions. The dataset was sourced from Scopus, Science Direct, Web of Science, plus articles found in the review section of Lehtola et al. (2019). A limitation of the current study is that it does not consider literature from other sources. The research questions are related to the objectives, ship performance model, operational constraints, optimization methods, sources of ice data, temporal changes in the environment, and route validation. The findings of these questions are as follows. The main objectives for the route optimization problem are voyage distance, voyage time, and fuel consumption. Individual studies consider either one, two, or three of these operational objectives. The ship performance models and ice input data are various and depend on the application. Regarding the route optimization method, the graph-based algorithm is widely used in the research literature. The temporal changes in the environment and route validation are important, but just a few studies considered them. In conclusion, this review provides some insights into the route optimization for vessels in ice-covered waters. The suggested directions for subsequent research are to implement more operational constraints and to treat the ice navigating problem under uncertainties.

CRediT authorship contribution statement

Trung Tien Tran: Data curation, Formal analysis, Investigation, Methodology. **Thomas Browne:** Conceptualization, Formal analysis,

Writing – review & editing. **Mashrura Musharraf:** Methodology, Supervision, Writing – review & editing. **Brian Veitch:** Conceptualization, Funding acquisition, Project administration, Supervision, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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