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LAMINAR FLOW OF DILUTE FIBRE SUSPENSIONS

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BY

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SUMMARY

Flow of dilute fibre suspensions such as paper stock is considered on the basis of observed characteristics. It is assumed that there is a separation into two regions, one moving as a solid body and containing all of the fibre, and the other a thin film of liquid. Flow in pipes is considered in detail, particularly the consequences of this type of motion on the friction characteristics. Flow between rotating concentric cylinders is also considered, and it is shown how this type may be used to predict the friction properties of pipes.

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LAMINAR FLOW OF DILUTE FIBRE SUSPENSIONS

1.0 INTRODUCTION

The low speed flow of a fibre suspension is unique and unusual compared to flows of air, water, and other common fluids. It is, furthermore, not like the behaviour of any of the classical non-Newtonian fluids. A separation occurs with the bulk of the flow field filled by a mass which contains all of the fibres. This is called the plug and moves as a solid body. It is far from solid, however, being of the order of 95 percent liquid in observations to date. Near solid boundaries a very thin liquid film is observed. Several investigators have referred to the very large velocity gradient between the plug and the wall as a form of wall "slip". Some have actually postulated that the velocity is not zero at the wall. For any particular suspension the range of the velocities at which this type of flow is observed is limited. At very low speeds the plug exhibits internal shearing displacements. At higher speeds the flow is in turbulent motion. In between these critical limits laminar flow is definitely present in the thin film.

A common fibre suspension is pulp stock, the basis of the manufacture of paper. It is a mixture of flexible wood fibres and water. The specific gravity of the fibres is only slightly greater than one and thus settlement is very small. The concentration of fibre is of the order of 5 percent or less by weight and hence the mixture is referred to as a dilute suspension. The size of the individual fibres ranges from 1 to 5 mm. in length and of the order of 25 microns in diameter. In suspension the fibres do not act independently but flocculate in masses many times their size. The entanglement of the flocs into larger masses results eventually in the plug. It is postulated that no fibres are found in the liquid annulus because any that were in this space have become entangled in the plug and the surface shear is insufficient to remove them. In all cases observed, the film thickness is of the order of the fibre length or smaller, which gives weight to this explanation.

This report is devoted exclusively to this type of laminar flow, primarily in pipes. The consequences of the observed properties in the light of viscous flow theory are discussed in detail. Particular attention is paid to the friction characteristics. Brief mention is also made of flow between rotating concentric cylinders.

2.0 THEORETICAL DEVELOPMENT FOR PIPE FLOW

Steady flow in a circular pipe is probably the most common and most readily observed type of laminar flow for a fibre suspension. It is thus the one of greatest practical interest. Because of symmetry it is readily amenable to mathematical solution. For these reasons there is much experimental data, and many theoretical formulations are available.

The general solution of the Navier-Stokes equation for pipe flow is(2):

$$v = -\frac{1}{4\mu}\frac{dp}{dx} r^2 + A \log r + B \qquad (1)$$

where r = radial distance from centre line of pipe

v = velocity at a point

p = pressure across a section perpendicular to the flow

x = distance parallel to centre line in direction of flow

μ = dynamic viscosity of fluid

A, B = constants determined by boundary conditions.

At the pipe walls $r = D/2$ the velocity must be zero and at the edge of the plug, $r = \frac{CD}{2}$, the velocity is assumed to be V_o, the constant velocity over the central portion of the pipe, (note Fig. 1). Applying these two boundary conditions to equation (1) results in the velocity distribution.

$$v = \frac{1}{4\mu}\frac{dp}{dx}\left[\left(\frac{D}{2}\right)^2 - r^2\right] + \frac{\log r/D/2}{\log C}\left[V_o - \frac{1}{4\mu}\frac{dp}{dx}\left(\frac{D}{2}\right)^2 (1 - C^2)\right] \qquad (2)$$

Equation (2) does not supply an explicit solution to the flow between the plug and the pipe wall so that another relationship must be obtained.

A section of the plug itself, Δx in length, is considered. Because it is in steady motion the forces acting thereon must be in equilibrium. There are only two such forces under the assumptions made above. The first is the result of the pressure gradient. This is a force in the direction of flow of the magnitude $\frac{dp}{dx}\Delta x \frac{\pi}{4}(CD)^2$. In opposition is the shear stress between the plug and the fluid film. The magnitude of the stress is specified as θ and defined as the surface stress. The equation of motion of the plug is hence:

$$\frac{dp}{dx}\Delta x \frac{\pi}{4}(CD)^2 = \theta \Delta x \pi CD$$

which reduces to

$$C = \frac{4\theta}{D\, dp/dx} \qquad (3)$$

The shear stress at the edge of the plug must be continuous between the plug and the fluid. Hence the shear obtained from equation (2) for $r = CD/2$ must be identical to θ. This requirement when imposed on equation (2) yields:

$$V_o = \frac{1}{4\mu}\frac{dp}{dx}\left(\frac{D}{2}\right)^2 (1 - C^2) \qquad (4)$$

Equation (4) upon recombination with equation (2) results in the following velocity distribution:

$$v = \frac{1}{4\mu}\frac{dp}{dx}\left(\frac{D}{2}\right)^2 \left[1 - \left(\frac{r}{D/2}\right)^2\right] \qquad \frac{r}{D/2} \geqq C \qquad (5a)$$

$$v = \frac{1}{4\mu}\frac{dp}{dx}\left(\frac{D}{2}\right)^2 (1 - C^2) \qquad \frac{r}{D/2} \leqq C \qquad (5b)$$

In the annulus this distribution is identical with the parabolic distribution for the flow of a Newtonian fluid. Thus the effect of the plug has been to truncate the parabolic distribution leaving a sharp corner on the velocity curve at the edge of the plug.

There are two useful relationships which can be derived from equations (5a) and (5b) which may be verified experimentally. In each the average velocity V is the total stock discharge divided by the cross-section area. First the friction equation is derived by integration across the section giving:

$$64 = (1 - C^4)\, fR \qquad (6)$$

where $f = \dfrac{dp}{dx}\dfrac{D}{\rho V^2/2}$ = friction factor

$R = \dfrac{VD\rho}{\mu}$ = Reynolds number

Several points should be noted about this equation. The friction factor can also be expressed in terms of the wall shear τ_o as

$$f = 4\frac{\tau_o}{\rho V^2/2}$$

This relationship when combined with equation (3) leads to several other forms.

For example, it is readily seen that

$$C = \frac{\theta}{\tau_o}$$

and by defining a new parameter similar to f namely,

$$g = \frac{\theta}{\rho V^2/2} \tag{7}$$

the following relationships result

$$C = \frac{4g}{f}$$

$$64 = (f - \frac{g^4}{f^3})\, R$$

It is important to note that the fluid properties used in defining the Reynolds Number refer to the fluid in the annulus and thus are not related to the properties of the material comprising the plug. The variation of viscosity of stock as pulp properties change has been a barrier in several previous theoretical formulations and experimental verifications.

The second relationship is the ratio of plug velocity to mean velocity which is obtained by combining equations (4) and (6)

$$\frac{V_o}{V} = \frac{2}{1 + C^2} \tag{8}$$

For a very small plug V_o/V approaches 2, the value for a full parabolic velocity distribution. For a very large plug V_o/V approaches 1, which merely indicates that the annulus makes a very small contribution to the total discharge.

It has been observed that long fibre stock invariably forms a plug which virtually fills the pipe. In terms of the above parameters, $C \geq 0.95$ and equations (6) and (8) can be simplified somewhat. In place of C, the plug size, the thickness of water film relative to the pipe diameter, $t = \frac{1 - C}{2}$, adjacent to the walls, is used in the equations. The usual approximations are introduced for small quantities and the friction equation becomes

$$8 = t f R \tag{9}$$

and the plug velocity ratio becomes

$$\frac{V_o}{V} = 1 + 2\ t \tag{10}$$

These expressions are identical with those obtained assuming that the velocity distribution is linear across the film. This is reasonable considering that only a very small part of the parabolic velocity distribution is present. However, the linear velocity distribution also requires that the surface stress be equal to the wall shear τ_o and hence $g = f$. For this identity to be true in the case of the flow under consideration requires $C = 1.00$ (see equation (3)). Thus the linear approximation should not be carried too far.

The friction equations, (6) and (9), are not the most convenient form for consideration of flow in an individual pipe. In the usual experiment stock is pumped through a pipe of constant diameter at various speeds. The result is a plot of head loss vs. velocity, or in dimensionless terms f vs. R. In the friction equations C (or t) are also functions of f and R and hence cannot be solved directly for the type of curve desired. Another parameter must be considered which does not include these quantities but contains the surface stress and other quantities which remain constant throughout the experiment. Using dimensional analysis it is possible to show that the only possible grouping of these quantities is $U = \rho\,\theta\,D^2/\mu^2$. If the surface stress is constant then U is constant, as the discharge through the pipe is varied. It will be seen later that θ is not a simple property of the stock but does vary slightly with flow parameters. The friction equations suitable for direct comparison with experimental results are thus, for the general case,

$$64 = \left(1 - \left(\frac{2\,U}{f\,R^2}\right)^4\right) f\,R \tag{6a}$$

and for the linear approximation

$$2\,U = f\,R^2 \tag{9a}$$

Equation (9a) is the most interesting in that it indicates that if θ were constant, the friction factor would vary with R^{-2} for a given pipe. In other words, the head loss over a particular length would be constant for all velocities. Intuitively, one would expect that this is unreasonable, inasmuch as head loss is a measure of energy dissipation in the flow.

3.0 EXPERIMENTAL EVIDENCE

A large number of investigators have measured the friction loss of pulp stock for ranges of velocity, pipe diameter, and concentration. The most favoured stock has been bleached sulphite because of its stability and ease of preparation. Robertson and Mason[3] have assembled the most significant results, but were not able to correlate them in a simple manner. By following the above analysis it is possible to correlate available results so that the significance and properties of the surface stress become evident. Figure 2 is a plot of equation (6) together with a typical experimental series. For a constant plug size the friction factor is inversely proportional to the Reynolds number and for constant U and large plug size f is inversely proportional to R^2. All experimental points lie on curves between these two extremes. For sulphite the author has found that $f \propto R^{-1.6}$, and other investigators have found similar relations. For all of the points plotted on Figure 2 the flow was observed to be a continuous plug with a clear water annulus. Only a rough measurement of the thickness of water film could be made because the pipe diameter was 2 inches, and optical difficulties made measurements of less than 1/100 inch impossible. In every case the measured film thickness agreed with the theoretical value, and more significantly, the film became thicker by an observable amount as the velocity increased.

In the light of Figure 2 the surface stress must increase as the velocity increases. Hence it must be a function of a flow parameter in addition to being a function of the stock. It is obvious that θ will be a function of both the type of stock and the concentration. However, the studies of the flow of stock made to date have not given an indication of this parameter. It may be linked to the internal structure of the plug which of course cannot be examined at the scale of these experiments. In some studies[3] the f-R plots have not yielded straight lines as shown on Figure 2, but have been S-shaped curves. The peaks have been correlated with measured variations in flocculation. This gives added weight to the contention regarding structure of the plug.

A flow parameter of some significance is the Reynolds number of the water film. This is defined as

$$R_t = \frac{V_o t D \rho}{\mu} \tag{11}$$

For thin films this can be combined with equation (9) to give:

$$R_t = Rt = \frac{8}{f} \tag{12}$$

The friction factor is inversely proportional to the film Reynolds number; thus higher velocities will mean larger R_t. It would be expected, following the theory of hydrodynamic stability[4], that there would be a minimum initial R_t for turbulent flow in the film. Above this value it would be expected that the flow would tend to amplify certain disturbance frequencies and thus become turbulent. It is assumed that a full range of disturbance frequencies is always present, which is reasonable considering the rough surface of the plug.

In all of the author's measurements with "sulphite" pulp stock, there is a definite break in the f vs. V curve at the particular value $f = 0.022$. This seems to occur for all batches and all concentrations. At the same conditions the water film appears to be disrupting the plug surface and individual fibres appear in the film. This is taken to be evidence of turbulence. Inserting $f = 0.022$ in equation (12) yields a critical $R_t = 364$, which is a reasonable value in the general theory. It is thus concluded that the stability of the plug depends upon the water film's being laminar and that this flow regime will not exist for a turbulent film.

3.1 Comparison of Different Pipe Diameters

As noted above, flow of stock is invariably in the regime where the plug size is very large, and thus the linear approximation provides a good description. It is significant that with this approximation the pipe diameter does not directly enter the friction factor relationship. This is best illustrated by rewriting equation (9a) in the following form:

$$f = \frac{\theta}{\rho V^2/2} \tag{9b}$$

If θ were a constant for a given stock it would be expected that plots of f vs. V would yield straight lines on log-log paper of -2 slope. This would be true for all pipe diameters. That θ is a function also of a flow parameter has been noted previously, but this parameter does not include the pipe diameter. Figure 3 illustrates this clearly. The f - V plots for two pipe diameters produce the same curve including even the same peaks. Similar results are readily obtained from the data presented by Brecht and Heller[5] by converting the plots to the f - V basis.

3.2 Comparison of Different Consistencies

The author's studies[1] concentrated on the range of consistency from 0.25 to 1 percent in a 2-inch pipe. From these it was shown that the surface stress increases with consistency and that changes of shape in the curves of f vs. V occur at constant values of f over this range. These facts are even more clear upon examination of the results of Brecht and Heller[5]. On Figure 4 one set of their results is plotted directly in terms of θ, the surface stress (derived by equation (9b)). For a given concentration, θ increases with V up to a critical value, decreases to a second critical value and increases again. A significant fact becomes evident when lines of constant R_t (also constant f) are drawn on this plot. The critical values lie on lines of constant film Reynolds number. Thus the changes in plug structure, which it is assumed are reflected by these points, depend on the film flow characteristics. It is thus concluded that the surface stress reflects both the density of the plug (as shown by effects of increased concentration) and fine structure.

Comparison of results taken by various investigators on the same type of pulp shows large variations in the measured surface stress, although variations within a set of studies are small. There are variations in the magnitude of θ and the location and existence of the critical points. As would be expected, different types of fibre produce different results. It is, nevertheless, instructive to compare various studies in a consistent manner. This is attempted in the plot shown in Figure 5, where θ and consistency are plotted for $R_t = 50$. This type of plot was first introduced by Robertson and Mason[3], but in a slightly different form. Most of the points represent measurements on sulphite, but there are a few for other types of pulp. The large amount of scatter is not surprising, although the relatively small difference between types of pulp is not expected. General reasoning leads to the expectation that θ would be greater for the long fibre pulp and this is roughly verified. More detailed and careful measurements will have to be made before conclusions can be drawn on this aspect.

The points on Figure 5 appear to scatter about a straight line which has the equation

$$\theta = 6300 \ c^{2.5} \tag{13}$$

where θ is surface stress in lb./ft.^{2}, c is concentration. This equation is very similar to the yield stress relationship proposed by Head[6]

$$S = 10{,}000 \ c^{2.4} \tag{14}$$

A close examination of the experiments to which Head fitted this relationship leads to the conclusion that he may have measured a surface stress instead of a yield stress, or it may be in fact that the two stresses are distinct properties of stock which are closely related. Further research on the small scale properties will be required before these conjectures can be verified.

3.3 Slope of Pipe

In a previous paper[1] the author concluded that the settling of the fibre flocs is important in the flow in a horizontal pipe. It was observed that the plug settled to the bottom of the pipe, leaving the water gap at the top up to ten times the thickness at the bottom. In more recent measurements the author has measured the friction factor for a loop consisting of vertical and horizontal elements in series. To the accuracy of measurement (about 1 percent), f was identical in each case. Furthermore, the thickness of water annulus in a vertical pipe was the same as the thickness at the bottom of the horizontal pipe.

In the horizontal pipe it was possible to measure the actual plug velocity V_o by timing the passage of a dye cloud trapped therein. The mean velocity was obtained by weighing the stock collected at the end of the pipe in a given time. A result was obtained which could be compared to that predicted from the friction measurements together with equations (9) and (10). The agreement was poor in every case. In a typical case the measured value of V_o/V was 1.07 and the predicted value 1.02. However, if the measured value of t at the bottom of the pipe was used in equation (10), the result was $V_o/V = 1.02$. In addition, if the average of t measured at the top and bottom was used, the result was $V_o/V = 1.07$.

The conclusions drawn from these observations are:

(i) The controlling film thickness in a pipe at any slope is the thinnest value.

(ii) In a horizontal pipe the surface stress is a maximum at the bottom of the plug and here it has the uniform value found in a vertical pipe.

4.0 FLOW BETWEEN CONCENTRIC CYLINDERS

A common form of viscometer consists of two smooth concentric cylinders with the fluid contained in the annular space between. One of the cylinders is rotated and the torque measured on the other. If the annular thickness is small compared to either radii, then the fluid viscosity can be readily computed.

This device can also be used to measure the surface stress of a stock, but the annulus must be large compared to the mean size of the flocs. Figure 1 shows the resulting flow pattern for the inner cylinder rotated and the outer one fixed. It has been observed that the bulk of the stock is motionless and there is a thin water film next to the inner cylinder. All of the fluid shear is concentrated in this layer, which is operating as a small-gap viscometer. Because the layer is thin the velocity distribution across it can be assumed to be linear, and hence the shear stress on the inner cylinder is given by the fundamental equation

$$\tau = \mu \frac{V_o}{tB} \tag{15}$$

where μ = liquid viscosity, i.e. water

tB = film thickness

B = gap thickness

As in the linear approximation for pipe flow, the shear stress on the rotor must be numerically equal to the surface stress on the stock. If now the friction factor and Reynolds number are defined in a similar manner to pipe flow, i.e.

$$f = \frac{4\tau_o}{\rho V_o^2/2} \qquad R = \frac{V_o B\rho}{\mu}$$

then equation (15) becomes identical with the friction factor relationship for pipe flow, i.e.

$$8 = \text{tfR} \tag{9}$$

It remains to be shown that the surface stress is the same in the two flow situations. If this is the case, then the friction properties of a stock may be determined in a wide-gap viscometer and the pipe friction curves drawn directly by use of equation (9).

The author attempted a measurement of the surface stress in a wide-gap viscometer which was immediately available. Unfortunately, the design of the viscometer was such that the outer cylinder rotated and the inner was fixed. Furthermore, the radii were small (of the order of 1 inch). Both of these factors restricted the surface velocity which could be used. At high rotational speeds the fibre was forced to the outer cylinder because of the differential centrifugal force. This enlarged the water film thickness next to the stationary cylinder, and the resulting surface stress was reduced. This difficulty could be reduced by constant stirring. A further complication arose with the paraboloidal free surface of the rotating stock; the higher the rotation speed the lower was the depth of water film next to the stationary cylinder. This difficulty could not be overcome. One accurate measurement was made on sulphite of 1 percent concentration and the result is shown on Figure 2. The experimental points for pipe flow appear to line up well with this point. However, there is too large a gap between them for the result to be conclusive.

Further experimental work should be performed with this type of flow in order to verify this contention. A specially designed viscometer should be constructed in which the stock might be agitated but otherwise kept at rest. With this apparatus further fundamental studies could be made on the characteristics of the surface stress of many types of pulp. These could be performed much more rapidly and cheaply than in a pipe.

5.0 CONCLUSIONS

Laminar flow of fibre suspensions has been observed to occur with a separation into two components. Near a solid boundary a thin film of water is found. In the mass of the flow a motion of fibres with water, called a plug, moves without internal motion. It has been observed that for pulp stock the water film is so thin that the velocity distribution can be considered linear.

The equations of motion for pipe flow upon imposition of these conditions indicate that the stress on surface of the plug is an important flow parameter. The following conclusions are drawn from consideration of experimental results.

(i) The surface stress is a function of the type and concentration of fibre primarily, but is also a function of flow properties, decreasing with increasing velocity.

(ii) The surface stress is the same for all pipe sizes, all other factors being unchanged.

(iii) The surface stress increases with increasing fibre concentration and appears to be roughly proportional to the 2.5 power of the concentration.

Flow between concentric rotating cylinders exhibits the same properties as pipe flow and from this fact it is postulated that:

(i) the same surface stress exists on the plug surface as in the pipe;

(ii) pipe friction factors can be easily deduced from measurements of torque on the cylinder.

Limited experimental proof indicates these to be valid.

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FIG. 1
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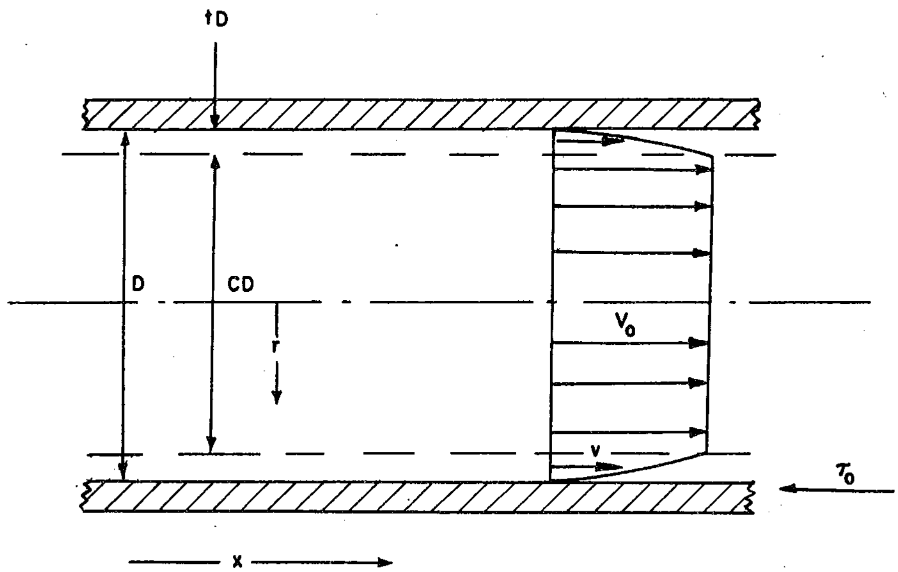
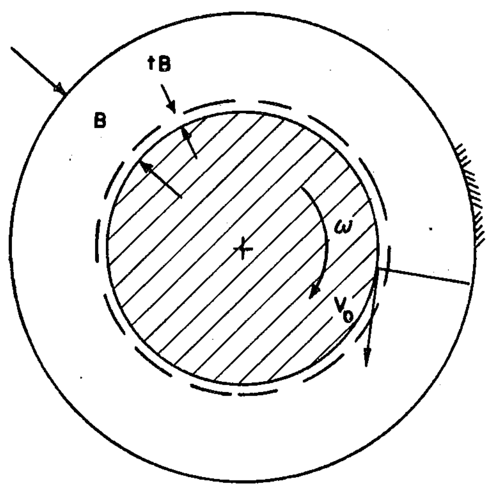


FLOW IN CIRCULAR PIPE



FLOW BETWEEN CONCENTRIC CYLINDERS

DEFINITION SKETCHES OF TWO SIMPLE TYPES OF STOCK FLOW

FIG. 2
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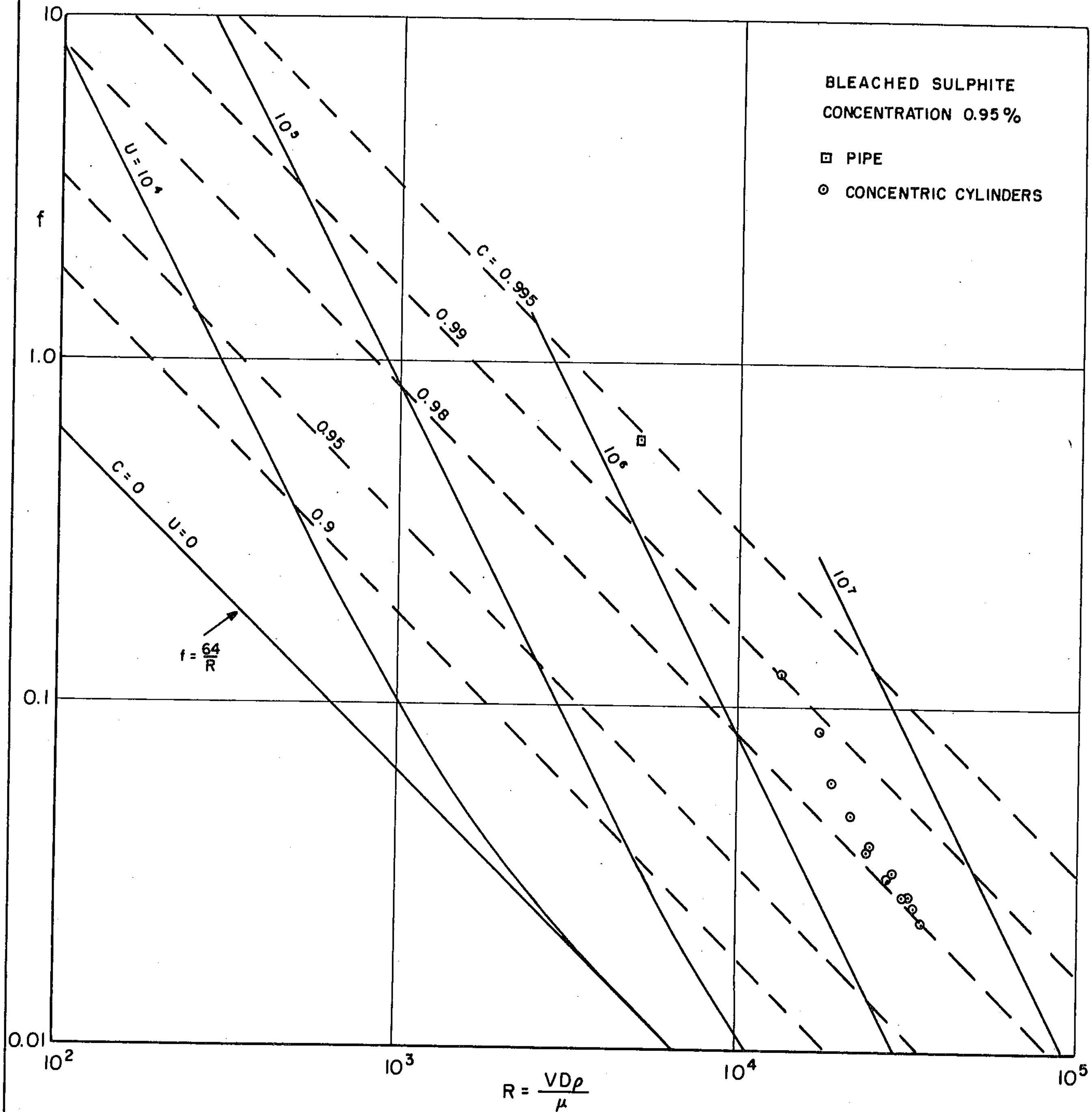

RESISTANCE DIAGRAM FOR LIQUID-FILM THEORY

FIG. 3
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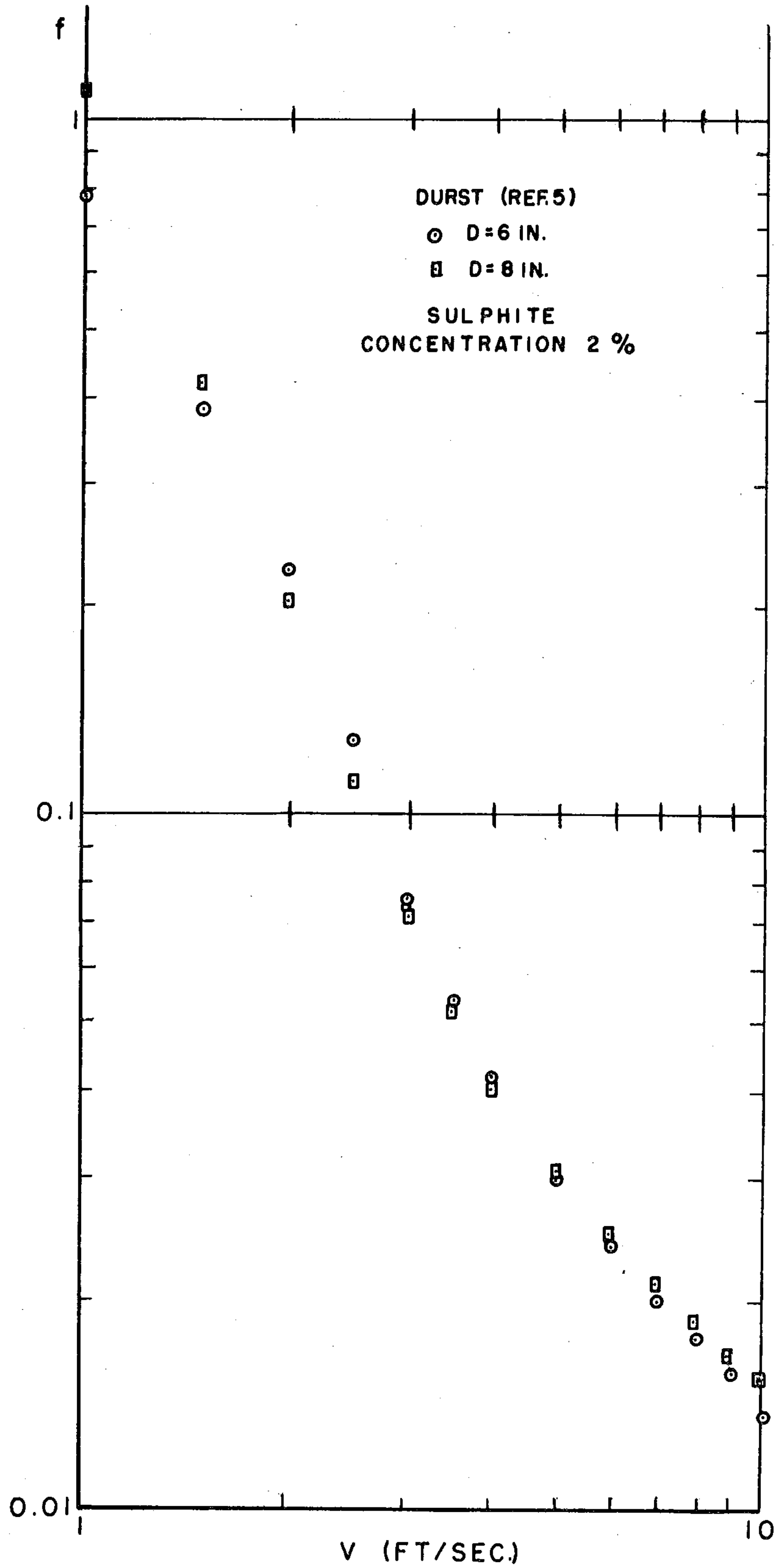
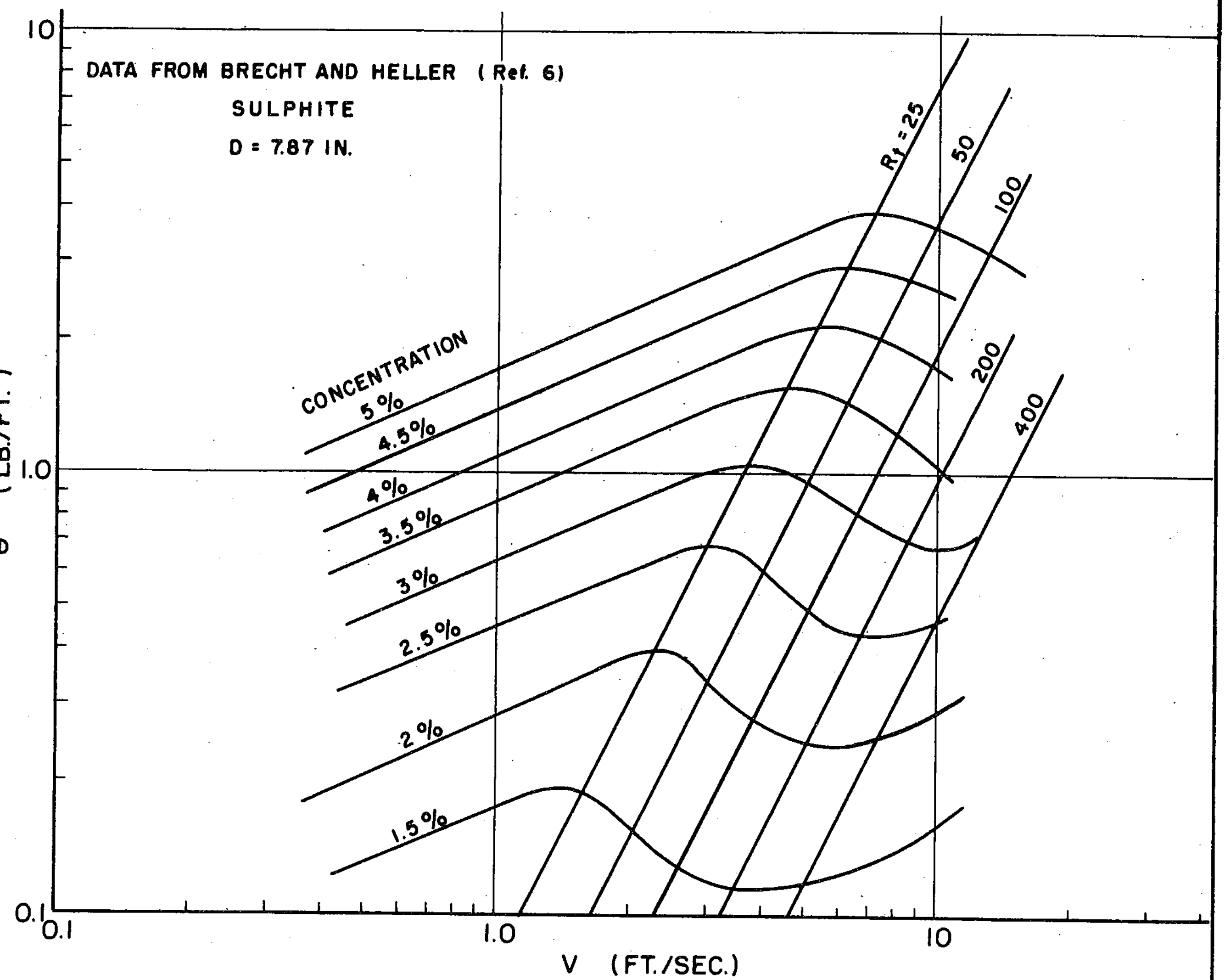


FRICTION MEASUREMENTS
ILLUSTRATING INDEPENDENCE OF PIPE DIAMETER



MEASUREMENTS OF SURFACE STRESS FOR VARIOUS CONCENTRATIONS

FIG. 5
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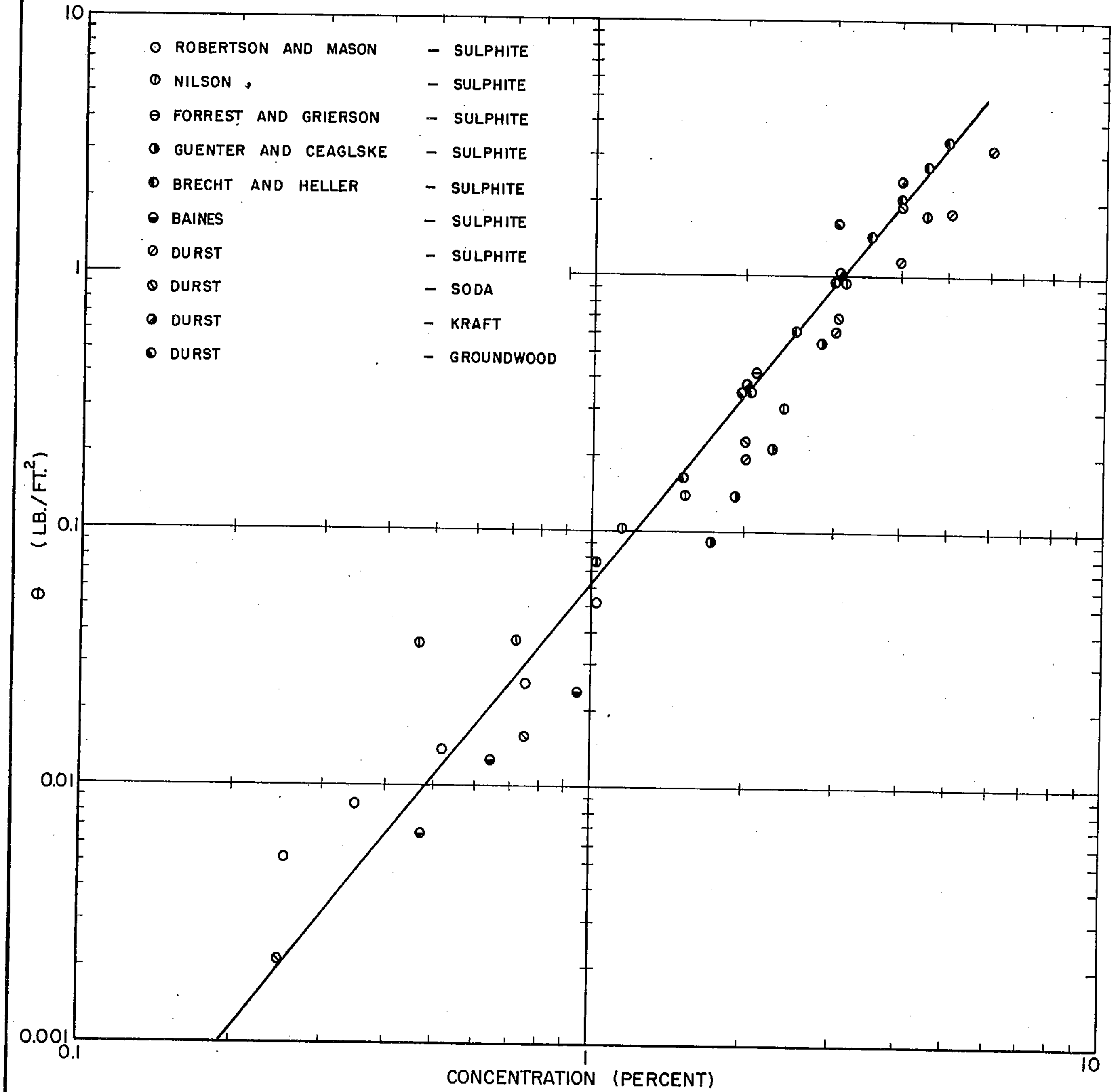

MEASUREMENTS OF SURFACE STRESS FOR VARIOUS PULPS

$R_t = 50$