Properties and Behaviour of Freezing Soils

Williams, P. J.

Publisher's version / Version de l'éditeur:

Research Paper (National Research Council of Canada. Division of Building Research); no. DBR-RP-359, 1967

NORGES GEOTEKNISKE INSTITUTT

NORWEGIAN GEOTECHNICAL INSTITUTE

PUBLIKASJON NR. 72
PUBLICATION

P. J. WILLIAMS:

Properties and Behaviour of Freezing Soils.

Avec résumés françaises.

OSLO 1967

NORGES GEOTEKNISKE INSTITUTT

NORWEGIAN GEOTECHNICAL INSTITUTE

PUBLIKASJON NR. 72

PUBLICATION

P. J. WILLIAMS:

Properties and Behaviour of Freezing Soils.

Avec résumés françaises.

OSLO 1967

The work described in this volume was carried out at the Norwegian Geotechnical Institute and the Division of Building Research, National Research Council of Canada as an interrelated program. This publication is also available in North America from the National Research Council, Ottawa; order No. NRC 9854.

PROPERTIES AND BEHAVIOUR OF FREEZING SOILS

BY

P. J. Williams

ERRATA

Pages 23 and 26:
After Williams *read* 1963 *for* 1962 (b)
After Williams *read* 1966 *for* 1964 (b)
After Miller *read* 1966 *for* 1964
Pages 51, 52, 58, 63, 64:
After Williams *read* 1967 *for* 1966 (c)
Page 64:
Equation should read: $\ln \frac{T}{T_0} = -\frac{(p_i - u_i)V_l}{L}$

The papers on pages 1–10, 11–26, and 51–72 are reprinted from Geotechnique: Vol. XIV, 2, 1964; Vol. XIV, 3, 1964; and Vol. XVI, 3, 1966 respectively.

The paper on pages 27–35 appeared in Proceedings, Permafrost International Conference, Nat. Acad. Sci. – N. R. C. Publ. 1287, 1966.

The paper on pages 73–90 first appeared in «The Engineer», Vol. 223, 5796, pp. 293–298, 1967.

The cover and the papers on pages 27–48 and 73–119 are printed in Norway.

FOREWORD

The characteristic behaviour and properties of frozen soils arise because ice and water coexist under the conditions imposed by the porous structure of the soil. The results of an investigation into the relative amounts of ice and water in frozen soils led to a series of related studies, described in the present volume, each of which was suggested by the conclusions arising from the preceding one.

The first paper in this volume describes experimental measurements of the apparent specific heat of frozen soils. The second describes the determination, from similar measurements, of the proportions of ice and water present in frozen soils. The temperature at which a certain quantity of unfrozen water occurs in the frozen soil was found to be related to the pressure at which an equivalent quantity of water occurs in a conventional suction-moisture content test. This implies a dependence of pore water pressures on temperature in frozen soils. This is confirmed by experimental investigations described in the third paper, of the mechanical effects of the pressures of the unfrozen water in frozen soil at various temperatures.

The fourth paper analyzes theoretically the relationships of temperature to the pressures of the ice and water phases, in terms of interfacial energy and the restricted sizes of ice-water interfaces within the soil. It is shown that where interfaces are sufficiently large not to be affected significantly by surface adsorption forces of the soil particles, a relatively simple physical picture based on capillary theory gives relationships in good agreement with the experimental observations. In practice the limits, for all soils, within which this picture is realistic correspond to temperatures between 0° C and about −2° C. This range is, of course, of prime importance in many engineering and geological considerations.

Striking advances recently made in thermodynamic analysis from the point of view of adsorption phenomena promise interpretation of similar experimental observations for lower temperatures, where much of the remaining pore water is under the influence of surface forces. The ideas and concepts involved will probably have considerable practical significance but as yet are unfamiliar to many. They lie outside the scope of the present work, although appropriate literature references are included.

The fifth paper extends the concepts of the preceeding paper to an experimental investigation of conditions at the frost line (the boundary between frozen and unfrozen soil). It appears possible to predict whether frost heaving will occur and under which field conditions from simple measurements of the pressure at which air will intrude into the pores of a saturated soil sample. The sixth paper describes experimental investigations of the manner in which air may replace water in the pores of the soil. It clarifies the extent to which the behaviour of air and water in soil, and ice and water in soil, are analogous.

The seventh and final paper differs in nature from the others. It attempts a simple, composite presentation of the soil freezing processes based on the work described in the previous papers. The main equations relating pressures in the soil, ice and water, of frozen and adjacent unfrozen ground, to temperature are reviewed. The implications of the relationships are considered in respect to the frost-heaving process, and various mechanical and thermal properties of frozen soil. It is intended that this paper can be read without detailed reference to the other papers. Both engineering and geological applications are considered. The extrapolation to field conditions of knowledge gained from laboratory experiments, in the absence of a correspondingly comprehensive programme of field research, involves some degree of speculation. It remains to be seen whether factors associated with field conditions and as yet unexplored will prove on occasion to be of dominant significance.

In general each paper was prepared and in some cases published shortly after completion of the research it describes. The investigations have been carried out over six years. Occasionally discussion of minor points occurs which, in the light of the subsequent work, appears irrelevant. In some cases, papers also included in the present volume are referred to only by the journal, volume and date of their original publication. It is hoped that the papers are consistent in development of the picture of the soil freezing process, in the use of symbols and in terminology.

The investigations were carried out at the Division of Building Research, National Research Council, Canada, and at the Norwegian Geotechnical Institute during tenure of a Fellowship 1963–65, awarded by the Royal Norwegian Council for Scientific and Industrial Research. The Institute and its Director, Dr. Laurits Bjerrum, provided the stimulus and facilities which led to the publication of the present volume. Mr. Rolf Kirkedam has kindly given considerable editorial assistance. More specific acknowledgements are made at the end of each article, but note must be made here of the author's gratitude to colleagues too numerous to name who have discussed, often at length, aspects of the work.

CONTENTS

SYMBOLS

(Symbols appearing only once are defined in the text where they occur.)

A	area
L	latent heat of freezing of water
L_{fus}	molar latent heat of fusion
L_{subl}	molar latent heat of sublimation
H	negative pressure (suction) expressed as height of a column of water
p_a	pressure of air
pF	logarithm of suction expressed as height (cm) of column of water (see also footnote p. 16, and 'Definitions')
p_g	pressure of gas phase
p_i	pressure of ice
p_l	pressure of liquid phase
p_s	pressure of solid phase
p_w	pressure of water
r	radius
r_{aw}	radius or equivalent radius of air-water interface
r_c	radius equivalent to size of largest continuous openings through soil pore system
r_{ia}	radius or equivalent radius of ice-air interface
r_{iw}	radius or equivalent radius of ice-water interface
r_{lg}	radius or equivalent radius of liquid-gas interface
r_{sg}	radius or equivalent radius of solid-gas interface
r_{sl}	radius or equivalent radius of solid-liquid interface
S_g	molar entropy of gas
S_l	molar entropy of liquid
S_s	molar entropy of solid
T	temperature
T'	temperature °C, at which soil water has a given suction in the absence of effects due to salts in solution
ΔT_s	freezing point depression due to dissolved salts
ΔT_{sx}	freezing point depression due to dissolved salts at temperature x
u	pore water pressure
u_a	pore water pressure immediately adjacent to air penetrating capillary, or soil pores
u_i	pore water pressure immediately adjacent to ice at penetrating frost line
u_{ix}	pore water pressure immediately adjacent to ice at penetrating frost line at depth x
u_x	pore water pressure at depth x under conditions of hydrostatic equilibrium
V	volume
V_g	molar volume of gas
V_l	molar volume of liquid, *also* specific volume of water
V_s	molar volume of solid
X	depth from ground surface
x	depth or thickness
x_m	maximum depth below ground surface at which frost heave can occur
z	depth of water table from ground surface
γ	bulk density of soil
Δ	finite increment
δ	infinitesimal increment of
θ	contact angle
μ	micron ($= 1 \cdot 10^{-4}$ cm) *also* molar chemical potential
ϱ_w	density of water
σ	total stress, *also* interfacial energy
σ'	effective stress
σ_{aw}	interfacial energy (surface tension) air-water
σ_{ia}	interfacial energy (surface tension) ice-air
σ_{iw}	interfacial energy (surface tension) ice-water
σ_{lg}	interfacial energy liquid-gas
σ_{sg}	interfacial energy solid-gas
σ_{sl}	interfacial energy solid-liquid
ω	shape factor

DEFINITIONS

Frost line: The boundary between frozen and unfrozen soil.

Negative pore pressure: Pore water pressure less than atmospheric.

Negative temperature: Temperatures below 0° measured on the Centigrade scale. 'Increasing negative temperature' refers to a state of cooling.

Pore water pressure; pore pressure: Pressure of water in pores of soil, or pressure of water external to soil pores but continuous with it and at the same pressure.

Saturated soil: Soil, the pores of which are filled with water and/or ice. Pores at the surface of the soil are also considered completely filled, such that there is no curvature at the air-water or air-ice interfaces. Scattered entrapped air bubbles within the soil pores are ignored.

Suction: (1) The difference between the pore water pressure and the air pressure external to the soil, as a result of curvature of the air-water interfaces. If the external air pressure is atmospheric the suction is numerically equal to the negative pore pressure. Some authors, referred to in the text, use the term in a more general sense to describe the potential or free energy of the soil moisture.

(2) The difference in pressure between the ice and water in a frozen soil, due to the curvature of the ice-water interfaces, when the pressure of the ice is equal to the external atmospheric air pressure.

Surface tension: The tension which apparently exists in an interface between *int. al.* a liquid and a gas, a liquid and a solid, or a solid and a gas. Ascribed to the effects of attractive forces between the molecules at the interface, it is often referred to as interfacial tension, or interfacial energy. The latter term is also used synonymously with surface tension in the present volume. See also descriptive illustration, p. 91.

EXPERIMENTAL DETERMINATION OF APPARENT SPECIFIC HEATS OF FROZEN SOILS

by P. J. Williams*

SYNOPSIS

A property of water in porous materials is that it freezes at temperatures below 0°C. There is no single freezing temperature for water in soils. As ice is formed the freezing point of the decreasing quantity of unfrozen water falls further below 0°C. Latent heat of fusion is thus involved in temperature changes over a range extending to several degrees below 0°C. The latent heat and specific heat together constitute an apparent specific heat.

Apparent specific heats for various silt, clay and organic soils have been measured in a calorimeter. The apparent specific heats generally rise as temperatures approach 0°C, and in a clay soil may be ten times as great at −1°C as at −5°C. The apparent specific heats for a given temperature depend on whether the soil is freezing or thawing, and on various other factors. The precise determination of heat quantities involved in temperature changes in soil in situ is difficult, mainly because of the several factors influencing the freezing of the soil moisture.

Une propriété de l'eau dans les corps poreux est qu'elle gèle à des températures inférieures à 0°C. Il n'y a pas de température unique de congélation de l'eau dans le sol. Au fur et à mesure que la glace se forme, le degré de congélation descend au dessous de 0°C. Une chaleur de fusion latente est donc en cause lors des changements de température couvrant plusieurs degrés au dessous de 0°C. La chaleur latente et la chaleur spécifique constituent ensemble une chaleur apparente spécifique.

Les chaleurs spécifiques apparentes pour des limons, des argiles et des sols organiques divers ont été mesurées dans un calorimètre. D'une manière générale, la chaleur spécifique apparente s'élève lorsque la température s'approche de 0°C, et dans le sol argileux elle peut être dix fois plus grande à −1°C qu'à −5°C. La chaleur spécifique apparente pour une température donnée dépend soit du gel, soit du dégel du sol, et d'autres facteurs divers. La détermination précise des quantités de chaleur en cause lors des changements de température dans le sol, sur place, est difficile, surtout à cause de plusieurs facteurs qui influencent la congélation de l'humidité se trouvant dans le sol.

INTRODUCTION

When water in finely porous materials freezes, much of it does so at temperatures lower than 0°C. This phenomenon is important in the freezing and thawing of soils. Half of the water in clay soils may be unfrozen at −2°C and generally, smaller quantities in coarser-grained soils. The equilibrium freezing point temperature is different in different parts of the soil water, so that the process of freezing or thawing of soils generally takes place over a wide range of temperature. Consequently, the quantity of heat required to raise or lower the temperature of soil within this range is made large by the latent heat of freezing (Williams, 1962). In this present Paper, the term "apparent specific heat" refers to such heat quantities to distinguish them from "true" specific heats which are not associated with change of phase.

This Paper describes determinations made with a calorimeter of the specific heats and apparent specific heats of various soils, for temperatures above 0°C and down to −10°C. The use of such values in estimating the apparent specific heats of soils met with in field problems is discussed briefly.

CALORIMETRIC INVESTIGATIONS

A calorimeter has been constructed (Fig. 1) that permits measurement of the amount of heat added to or removed from a specimen to raise or lower its temperature by a certain amount. During warming of a sample, the only source of heat is that supplied at a measured rate by a heating coil attached to the specimen holder. During cooling, heat is lost from the sample holder at a practically constant rate determined by the temperature of the outer container which, in this case, is maintained lower than that of the sample holder by a certain amount.

* Snow and Ice Section, Division of Building Research, National Research Council, Ottawa, Canada—at present Research Fellow, Norwegian Geotechnical Institute.

Temperatures are measured by thermocouples at three points within the sample. Readings from each thermocouple are recorded by a Speedomax recorder with preamplifier, at intervals of 5 minutes or less. Also recorded at 3-minute intervals on the same apparatus are temperatures from four thermocouples placed on the inner side of the outer container (Fig. 1). Under normal conditions these latter readings serve merely to check the operation of the calorimetric equipment.

The calorimeter is immersed in a tank of ethylene glycol solution, cooled by a compressor. During warming tests, when the only source of heat is to be the measured quantity supplied through the heating coil, heat exchange between the sample holder and its surroundings is avoided by maintaining the outer container at substantially the same temperature as the surface of the sample holder. This is achieved by regulation of the temperature of the ethylene glycol in which the calorimeter is immersed. The temperature of the ethylene glycol

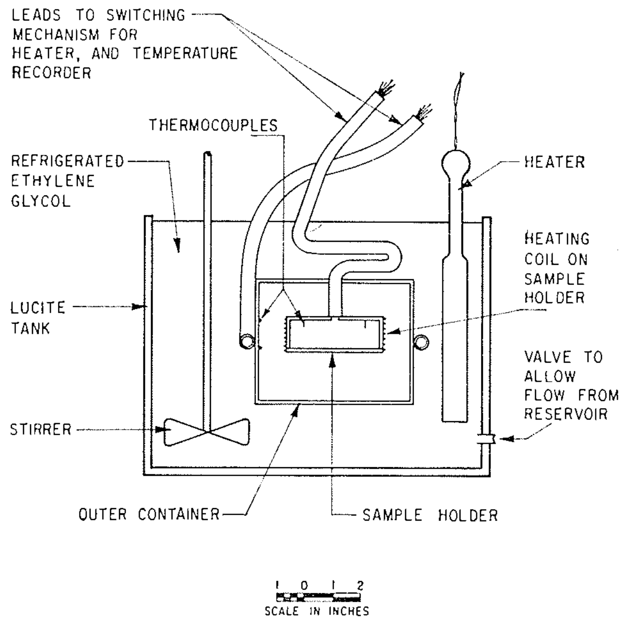


Fig. 1. The calorimeter

normally tends to fall slowly. When its temperature and hence that of the outer container falls about 0·1 °C below that of the sample holder, a blade heater in the glycol is automatically switched on until the temperature of the outer container is about 0·1 °C warmer. The switching mechanism is operated by the amplified signal received from two thermocouples in series, on the sample holder and outer container.

During cooling (freezing), when heat is extracted at a nearly constant rate, the temperature of the outer container is maintained consistently lower than that of the sample holder. This is achieved by adjustment of the zero control of the amplifier, such that its output is sufficient to actuate the relay mechanism and heater only when the ethylene glycol becomes cooler than the sample holder by more than the predetermined amount.

The rates of heat extraction during cooling for various temperature differences were measured in calibration tests. In most soil tests, however, observations were made over a cycle involving cooling followed by rewarming to the initial temperature. The rate of heat loss is then calculated by dividing the total heat input in the "thawing" part of the freeze/thaw cycle, by the time taken to cool the sample in the "freezing" part of the cycle. Both procedures involve minor errors, but the latter gives more consistent results. 3–5 days were normally required for each freeze/thaw cycle.

Calculation of results

Although the method used in calculating the specific and apparent specific heats was occasionally varied in minor respects, the following was the general procedure.

The procedure is similar in most respects for both thawing (warming) tests, and for freezing (cooling) tests. On the time-temperature record on the Speedomax chart, starting from the point corresponding to 0 mV (i.e. 0°C), points are marked for every 0·25 mV, (equal to 0·13°C) change in the temperature, for one of the thermocouples in the soil sample. The heat supplied to or removed from the specimen and sample holder in the time taken for each of these intervals of temperature change (ΔT), is then calculated. For warming tests the relationship used is:

$$0{\cdot}24\ VIt = Q \text{ calories}$$

where

$0{\cdot}24$ = conversion factor, cal/J
V = voltage across heating coil on sample holder, volts
I = current through heating coil on sample holder, amps
t = time (seconds) for sample temperature to change through ΔT

During cooling the relationship used is:

$$Q = \frac{\text{Total heat input for warming part of cycle}}{\text{Time (sec) to complete cooling part of cycle}} \times t_f$$

where t_f = time (sec) for sample temperature to change through ΔT.
The specific heat, or apparent specific heat, is then calculated and expressed in calories per gram of soil per °C:

$$\lambda = \frac{Q - \Delta T\, C_c}{W_s\, \Delta T}$$

where:

W_s = total weight of soil sample, g
C_c = calorimeter "constant", cal/°C, i.e. the heat required to change the temperature of the empty sample holder by 1°C.

Values obtained for various soils are shown in Figs 2 and 3 where they are plotted as a function of temperature.

For many soils, the large heat quantities (largely composed of latent heat) involved in temperature changes in the range 0°C to −0·5°C cannot be shown satisfactorily on this type of graph. These heat quantities are shown as a separate value beside each graph. In calculating these values, the same expressions were used, except that ΔT has a value of 0·5°C.

Experimental accuracy

The temperatures of the soil are observed with an accuracy of about ±0·05°C for individual points, but a line drawn through several recorded points generally gives the thermocouple temperature with somewhat greater accuracy. Temperature differences within the sample are normally less than 0·1°C. When there is a very small quantity of ice present locally in the sample, the temperature of the part of the soil furthest from the ice may change more rapidly than that adjacent to it, because of the absorption of latent heat by the melting ice. Temperatures in different parts of the soil may then differ by 0·2°C, and sometimes more. This is probably the main reason for an observation frequently made during thawing: the time/temperature curves indicated that the effect of liberated latent heat is still present at

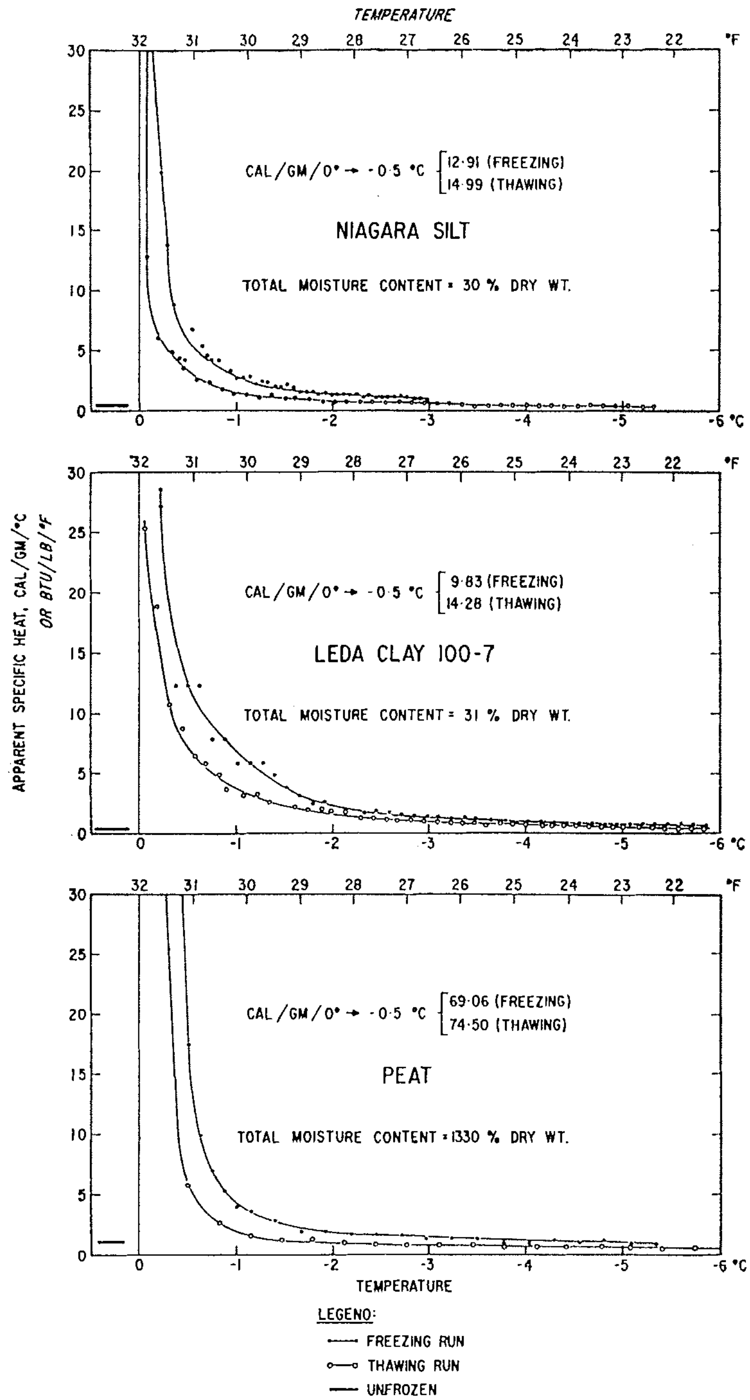


Fig. 2. Specific heats and apparent specific heats of various soils. Also shown are the heat quantities exchanged in a temperature change between 0°C and −0·5°C

recorded temperatures of +0·4°C or even higher.* The last remaining ice is probably remote from the thermocouples and thaws at or below 0°C, so that the latent heat of the last thawing ice has been calculated as part of the heat quantity involved in temperature change between 0°C and −0·5°C. The specific heats above 0°C are therefore shown as a single value on the graphs.

In addition to the quality of the equipment that measures temperature and heat input, the accuracy of the determined specific heats depends substantially on the control of the temperature of the outer container of the calorimeter. If the temperature of the outer container deviates on the average from that of the sample holder, there will be a corresponding unmeasured quantity of heat added to or removed from the specimen. The amount of heat lost will be proportional to the time taken for the test. For soil samples of given weight, the percentage error in apparent specific heat due to this heat loss, will, for practical purposes, be inversely proportional to the measured rate of heat input or extraction. Conversely, for a

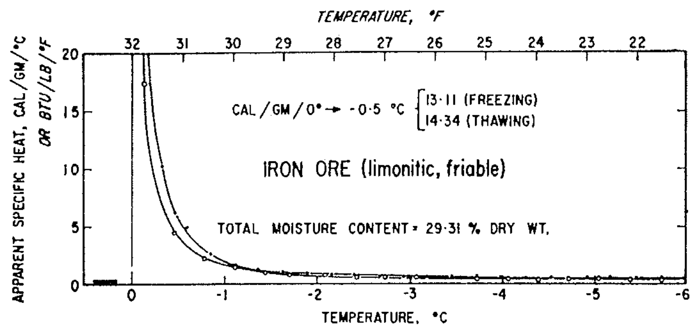


Fig. 3. Specific heats and apparent specific heats of iron ore. Also shown is the heat quantity exchanged in a temperature change between 0°C and −0·5°C

given rate of heat transfer, the error will be proportional to the weight of the sample. During tests of long duration (as in the case of most soils) errors of this type could be reduced by occasional slight readjustments of the temperature controlling mechanism.

The amount of heat involved in warming or cooling the sample holder was determined in calibration tests. The value used for this calorimeter "constant" was 87 cal/°C. It may be in error by 1 to 2 calories in some cases, mainly because of minor changes that became necessary in the mounting of the thermocouples. Since the weight of the sample is normally about 200 g it follows that an error of perhaps 1–2% may occur in the observed specific heats, when these are very low (e.g., 0·2 cal/g/°C). For higher (apparent) specific heats the error due to inaccuracy of the calorimeter constant becomes negligible.

The effects of the limitations of instrumental accuracy and of the various sources of error mentioned here, together with other minor ones, are not predictable quantitatively for every test. To some extent, in each test these effects cancel out. In a series of five tests carried out with distilled water, the observed values of the specific heat showed a standard deviation of 0·04 cal/g/°C, or 4·0%, about the true value. The average of the deviations of the observed values from the true values was 0·02 cal/g/°C (Table 1).

Although this gives a good indication of the accuracy that can be obtained with the calorimeter, investigations of soils involve additional complications, especially those associated with the rapid change of apparent specific heats with temperature. These are illustrated by tests on two different-sized samples from the same soil. For each soil sample the

* This may be compared with temperatures which are recorded by a thermocouple placed in an ice bath to which heat is being slowly supplied. Temperatures of several tenths of a degree above 0°C, are observed even while ice is still present, but there is no marked increase in rate of temperature rise until all the ice has melted.

Table 1

Control tests carried out on distilled water

Temp. °C	Specific heat cal/g/°C		Deviation $= x$	
	observed	Hdbk of physics and chem., 1962	cal/g/°C	As %
12	0·978	1·001	−0·023	2·3
3·3	0·958	1·005	−0·047	4·7
7·6	0·996	1·002	−0·006	0·6
17·3	1·032	0·999	0·033	3·3
6·5	1·017	1·003	0·014	1·4

$$\text{Standard deviation} = \sqrt{\frac{\Sigma x^2}{5}}$$

$= 0{\cdot}04$ cal/g/°C
$= 4\%$
Average of deviations $= 0{\cdot}02$ cal/g/°C

apparent specific heat was obtained for fifty-two temperatures at increments of 0·13°C, from −0·5°C to −7·3°C. With the two values thus obtained for each temperature (one from each test), a mean value was calculated. The deviations of the observed values from their respective means were expressed as a percentage of these means. The standard deviation of these percentages for the whole range was 9·4%. The smoothed curves drawn through the two sets of observed points are closely similar, however (Figs 6(a) and (b)). The freezing process (and consequently the apparent specific heats) are unlikely ever to be exactly similar in any two tests on the same soil, because natural soils are not entirely homogeneous. Small differences in the nature and distribution of voids, as well as different stresses that may arise at each freezing, result in slightly different amounts of moisture freezing or thawing at a given temperature.

These considerations, together with the fact that the temperature of soils in situ is rarely definable to within 0·1°C, show that the reproducibility of the results is quite satisfactory for practical purposes.

Experimental results and their interpretation

Figs 2 and 3 illustrate the dependence of apparent specific heat on type of soil. At least two freeze/thaw tests were made on each of five soil types and in some cases six or more. These repeated tests showed that the apparent specific heats are independent of the rate of temperature change, at least for those rates likely to occur under field conditions. The grain-size composition of the soils is shown in Fig. 4. The finer-grained soils have, in general, higher apparent specific heats. This is a result of the larger quantity of water that freezes, or thaws, at temperatures below 0·5°C in these soils. For each increment of temperature change in such soils, there is a correspondingly larger component of latent heat involved. The freezing of water in soils at these temperatures is due mainly to the state of stress (pF) produced in the soil water by capillary and other effects (see Schofield, 1935; Edlefsen and Anderson, 1943). In general the stresses developed in the water in finer-grained soils are such that larger quantities of water will freeze at temperatures below −0·5°C than in coarser-grained soils. The conspicuous peak shown in the freezing curve of the Leda clay (Fig. 5(a)) is probably due to an unusual amount of water in the soil under a state of stress resulting in freezing at about −1·1°C. This may be compared with the sharp drop in moisture content for a small change of suction, that occurs in the suction–moisture content curves (see e.g. Coleman and Croney, 1961) for some clay soils.

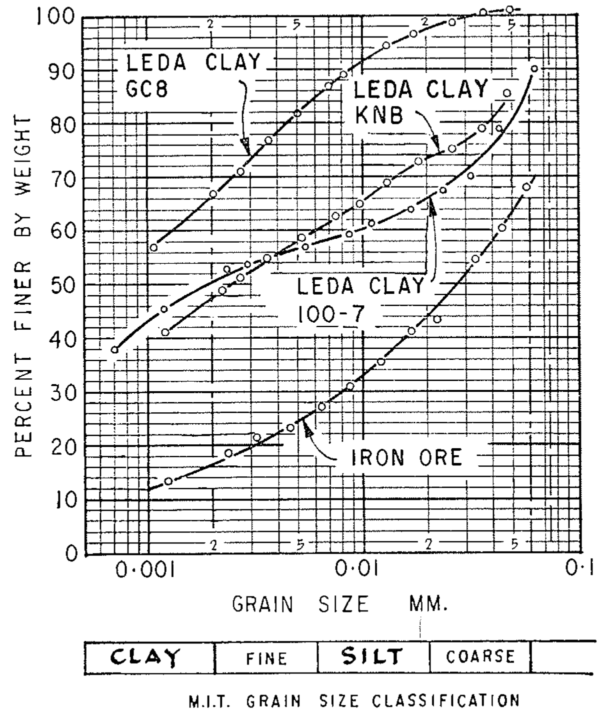


Fig. 4. Grain-size composition curves for soils investigated calorimetrically. The curve for Niagara silt was closely similar to that for iron ore, only differing by being 0 to 10% higher in the range 0·02 to 0·06 mm

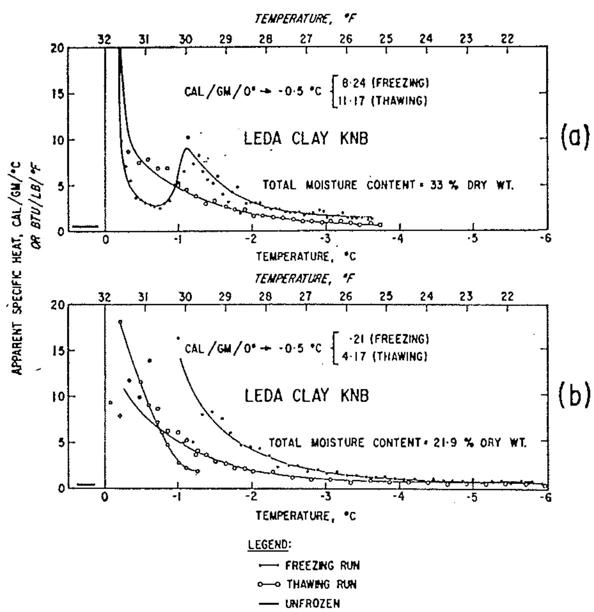


Fig. 5. Specific heats and apparent specific heats of Leda clay at natural moisture contents (a) 33% dry weight and (b) partly dried (21·9% dry weight). In the latter case, freezing began at a temperature of $-0{\cdot}99$°C; two thawing curves are shown, corresponding to minimum temperatures during freezing of $-8{\cdot}3$°C and $-1{\cdot}4$°C

Different values of apparent specific heat are obtained depending on whether the soil is freezing or thawing and in the latter case on the lowest temperature reached during freezing (Fig. 5(b)). This may be compared with the different moisture contents under similar stresses which exist in porous materials undergoing drying in the one case and wetting in the other.

In the case of compressible soils, a freeze/thaw cycle on previously unfrozen soils gives results different from those obtained in subsequent tests. This is illustrated by the examples in Figs 6(b) and 6(c). It will be seen that in the latter example the apparent specific heat for temperatures of $-0{\cdot}5°C$ to $-3{\cdot}0°C$ are smaller, whereas the heat quantities involved between $0°$ and $-0{\cdot}5°C$ are larger in the second freezing than in the first. In this respect, freezing may be regarded as similar to drying, which is well known as causing consolidation (see e.g. Warkentin and Bozozuk, 1961) and a corresponding change in the stress/moisture content relationships of compressible soils. After the second freezing, subsequent freeze/thaw cycles do not increase the effect significantly unless carried out to even lower temperatures. The effect of subsequent freezing to temperatures lower than about $-2{\cdot}5°C$ is small, in any case. It is to be expected that consolidation due to applied loads will similarly result in somewhat different apparent specific heats. It might be noted that the different apparent specific heats observed on freezing and on thawing are not solely the result of consolidation during the freezing (or "drying") process, since they occur even in the case of non-compressible soils.

Particularly important is the effect shown, for example, by comparison of Figs 5(a) and 5(b) where apparent specific heats were determined on a saturated sample and on a similar sample from which about one third of the water was removed by drying. The apparent specific heats for temperatures below $-0{\cdot}5°C$ were rather similar in both cases. This illustrates the point that there is a considerable amount of water held in soils, which freezes in the range $0°C$ to perhaps $-0{\cdot}2°C$, and which can be varied or even totally removed with little alteration of the apparent specific heats for lower temperatures.* Slightly lower apparent specific heats are to be expected when the total moisture content is very high. This is because, per unit weight, there will be more ice and less soil material. As the apparent specific heats are largely composed of latent heat of freezing of water under the influence of the porous structure of the soil, the latent heat quantity will be reduced if there is less soil material.

VALUES OF APPARENT SPECIFIC HEATS FOR APPLICATION IN FIELD PROBLEMS

Considerations involved in comparison of experimental and field values

Because of the wide variety of soil type and moisture content, and the complex relationships involved in the freezing of soil water, it is rarely possible to estimate precisely the heat quantities involved in temperature changes below $0°C$, on the basis of grain-size composition. Even for soils tested calorimetrically, field conditions may be such as to give rise to values substantially different from those determined experimentally. The important factors will be summarized here.

1. In the finer-grained, so-called frost susceptible soils, substantial quantities of water migrate to the freezing soil, the amount depending, among other factors, on water availability, rate of freezing, and soil type (see e.g. Penner, 1958). The addition of water in this way cannot, of course, occur to the calorimeter sample (although internal migration will occur). Large quantities of heat are involved in the freezing of this additional water. It is not possible to predict the amount accurately but it can be allowed for by measurement of the total moisture content of the frozen soil.

2. Theoretical considerations, based on the work of Schofield, 1935; Edlefsen and Anderson, 1943; *et al.*, suggest that over-burden pressures or high pore-air pressures, will change

* This and the ensuing remarks are more easily understood when it is remembered that most of the ice in freezing soils is in discrete (and often quite large) masses larger than pore size. Variation in size of the ice masses will not alter the pore structure and neither therefore the freezing temperatures of the remaining unfrozen water within the pores.

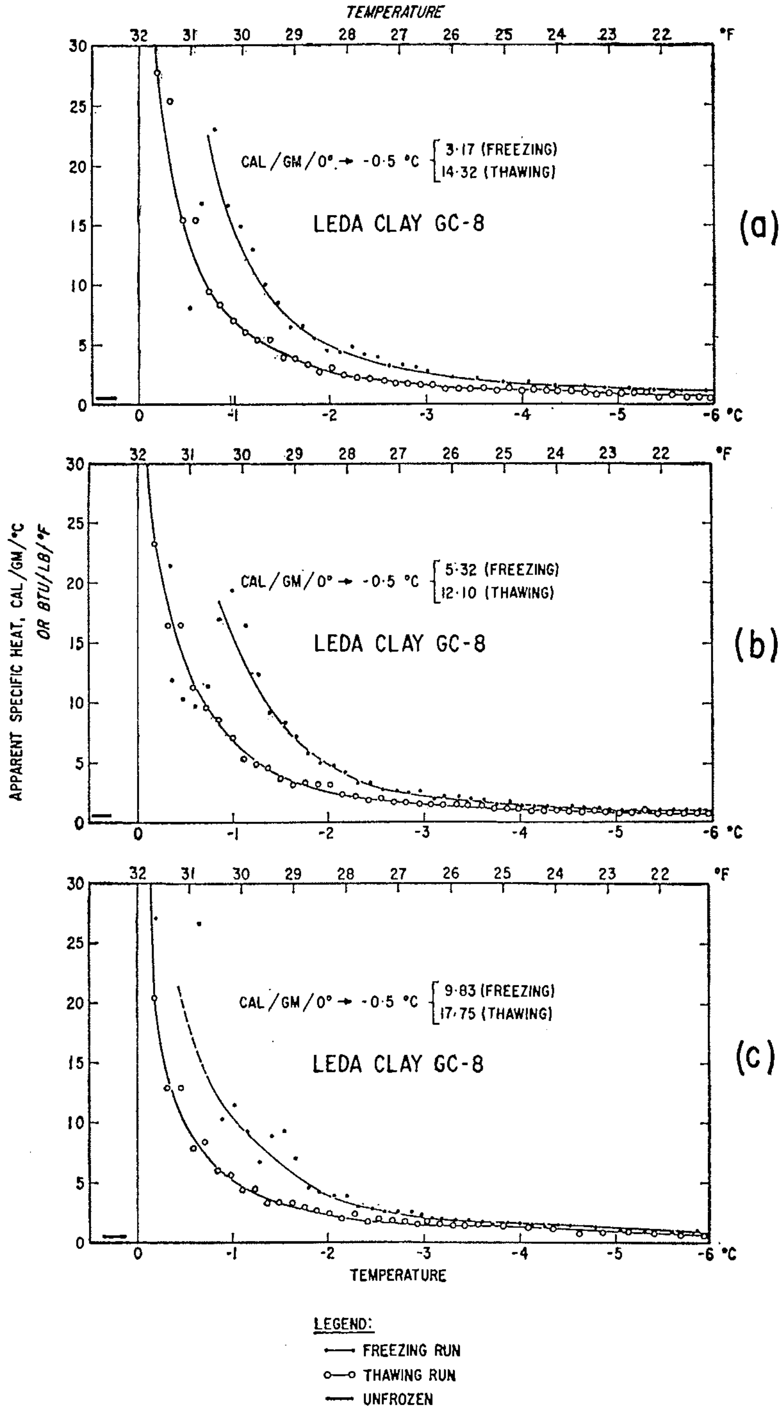


Fig. 6. Specific heats and apparent specific heats of Leda clay GC–8. (a) and (b) show the results of identical tests on identical samples, and illustrate the reproducibility of the calorimetric observations. (b) and (c) illustrate the different results obtained on first freezing (b) and second freezing (c) of a sample

the amount of water that freezes over a given temperature range. There will be corresponding changes in apparent specific heats. At present these effects are not fully understood quantitatively but they are probably of little significance at one to two metres depth in the ground, without other over-burden (compare Penner, 1959).

3. Although the freezing of water in soils at temperatures below 0°C is broadly related to the pore sizes and void ratio and thus to grain-size composition of the soil, other effects, especially those due to salts and to surface forces of the clay particles, may be significant. Thus, Leonards and Andersland (1960) give information on the freezing of a clay, which, on the basis of grain-size composition alone, does not seem compatible with the findings presented in this paper. Except possibly for obviously saline soils, the occurrence of these effects is not easily predictable at present without calorimetric tests.

Work recently completed indicates that the suction-moisture content relationship of a soil can be used to obtain a good estimation in many cases, of the apparent specific heat. This will be considered in a subsequent publication.

CONCLUSIONS

1. Determinations of the apparent specific heats of various soils at temperatures below about 0·5°C have been made and are presented graphically. The heat quantities involved in warming and cooling through 0° to —0·5°C are also given.

2. Apparent specific heats are dependent on soil type, being generally larger for finer-grained soils.

3. For a given soil apparent specific heats are dependent on:

(*a*) The temperature, being in general larger at temperatures nearer 0°C.

(*b*) Whether the soil is freezing or thawing and, in the latter case, the lowest temperature reached during freezing.

(*c*) In compressible soils, whether the soil has been previously frozen and thawed, or dried and rewetted, and if so, to what extent.

Differences in total moisture content, however, within the range that might occur naturally have little effect on the apparent specific heats for temperatures below about —0·5°C.

ACKNOWLEDGEMENTS

This Paper is a contribution from the Division of Building Research, National Research Council, Canada, and is published with the approval of the Director of the Division. Technical assistance was provided by R. Armour; discussions with L. W. Gold and other members of the Division are also gratefully acknowledged.

REFERENCES

EDLEFSEN, N. E. *and* A. B. C. ANDERSON, 1943. "Thermodynamics of soil moisture." *Hilgardia,* 15:2: 31–298.

CRONEY, D. *and* J. D. COLEMAN, 1961. "Pore pressure and suction in soil." *In: Pore pressure and suction in soils. Butterworths, London.* pp. 31–38.

LEONARDS, G. A. *and* O. B. ANDERSLAND, 1960. "The clay water system and the shearing resistance of clays." *Research Conf. Shear Strength Soils,* June 1960. *Amer. Soc. civ. Engrs,* pp. 793–818.

PENNER, E., 1956. "Soil moisture movements during ice segregation." *Highw. Res. Brd, Bull.* 135, pp. 109–118.

PENNER, E., 1959. "Pressures developed in a porous granular system as a result of ice segregation." *Highw. Res. Brd, Special Report* 40.

SCHOFIELD, R. K., 1935 "The pF of the water in soil." *Third Congr. Soil Sci.,* 2:37–48.

WARKENTIN, B. P. *and* M. BOZOZUK, 1961. "Shrinking and swelling properties of two Canadian clays." *Fifth Int. Conf. Soil Mech.,* 1:851–855.

WILLIAMS, P. J., 1963. "Specific heat and unfrozen water content of frozen soils." *In: Proc. First Can. Conf. on Permafrost.* (National Research Council, Associate Committee on Soil and Snow Mechanics Technical Memorandum 76.)

PRINTED IN ENGLAND BY WILLIAM CLOWES AND SONS, LIMITED, LONDON AND BECCLES

UNFROZEN WATER CONTENT OF FROZEN SOILS AND SOIL MOISTURE SUCTION

by

P. J. WILLIAMS*

SYNOPSIS

The unfrozen water content of various soils has been measured by calorimeter. The suction-moisture content characteristics of the same soils were also determined at room temperature, with conventional pressure plate and pressure membrane apparatus. Using the values of unfrozen water content measured during freezing, and the suction characteristics obtained during drying, a relation was found between the two sets of results. The relationship approximates that of Schofield, between suction and initial freezing point, of soils at various moisture contents. Knowledge of the relationship permits prediction of the amount of water remaining unfrozen in a soil at negative temperatures down to $-1{\cdot}0$°C and often somewhat lower. Such prediction requires only determination of suction-moisture content characteristics by conventional methods together with a simple determination of the freezing point of an extract of the soil solution. In many cases, the accuracy of the prediction is apparently as great as can be obtained using even complex calorimetric methods. Using these relatively easily determined values of unfrozen water content at various temperatures, realistic estimates can be made of the apparent specific heats of frozen soils. The observed relationship provides a basis for studies of the effects of both load and temperature, on the proportions of ice and water in frozen soils and on the state of stress within the ice and unfrozen water.

La teneur en eau non gelée de divers sols, a été mesurée par calorimètre. Les caractéristiques des teneurs eau-succion des mêmes sols furent aussi déterminées à la température ambiante, avec plaque conventionnelle de pression et appareil à membrane de pression. En utilisant les valeurs des teneurs en eau non gelée mesurées pendant la congélation, et les caractéristiques de succion obtenues pendant le séchage, un rapport fut trouvé entre les deux séries de résultats. Le rapport se rapproche de celui de Schofield, entre la succion et le point initial de congélation de sols de diverses teneurs en eau. La connaissance du rapport permet de prédire la quantité d'eau restant non gelée dans le sol à une température descendant jusqu'à $-1{\cdot}0$°C et souvent quelque peu plus basse. Une telle prédiction nécessite seulement la détermination des caractéristiques de la teneur eau-succion par les méthodes conventionnelles, ainsi qu'une détermination simple du point de congélation d'un prélèvement de solution du sol. Dans beaucoup de cas, l'exactitude de la prédiction est apparemment aussi grande que celle qui peut être obtenue même en utilisant des méthodes calorimétriques complexes. En utilisant ces valeurs déterminées d'une manière relativement facile de teneur en eau non gelée à diverses températures, on peut faire des évaluations réalistiques des chaleurs spécifiques apparentes des sols gelés. Les rapports observés fournissent une base pour les études des effets à la fois de charge et de température, sur les proportions de glace et d'eau dans les sols gelés et sur l'état des contraintes dans la glace et l'eau non gelée.

INTRODUCTION

Early studies of the freezing of soil moisture were mainly concerned with the determination of a freezing "point", that is, the temperature at which freezing commences in a sample (Bouyoucos and McCool, 1916). More recently it has been realized that the freezing of water in finely porous materials takes place over a range of temperatures (Lovell, 1957; Institut Merzlot., 1953–57); and that the equilibrium freezing temperature varies in different parts of the soil water. The greatest depression of the freezing point below 0°C presumably occurs in water that is most firmly held and is closest to the soil particles.

Schofield (1935) showed that as the moisture content of soil samples is reduced the freezing point is depressed further below 0°C. Schofield carried out investigations on the suction with which water is held in soil and showed clearly that as the water content of the sample is reduced the suction to be applied to a column of water in contact with the soil (as in the suction plate apparatus) must be increased if water is not to enter it. He argued that this

* Research Officer, Snow and Ice Section, Division of Building Research, National Research Council, Ottawa, Canada. Research Fellow, Norwegian Geotechnical Institute, Oslo, 1963, 1964.

"pressure deficiency" or suction of the soil water is mainly responsible for the freezing point depressions observed; and proposed and verified experimentally that a relationship exists between the pressure deficiency at a given moisture content and the temperature at which the water in the soil (in the absence of supercooling) begins to freeze.

Comprehensive calorimetric investigations of the freezing of a variety of soils have recently been carried out; some are described in this Paper. They allow calculation of the amounts of unfrozen water present in the soils at various negative temperatures. These amounts may be said to have freezing points (i.e. to start to freeze) at those temperatures. Accordingly there might be expected (as pointed out by Croney, Coleman, and Black 1958), a relationship similar to that proposed by Schofield, between the temperature associated with a certain content of unfrozen water and the suction associated with the same moisture content in the soil (measured at room temperature). The establishment of such a relationship would have theoretical and practical interest and would allow, without the necessity for elaborate calorimetric tests, prediction of the unfrozen water contents of a soil at temperatures below 0°C (on the basis of determination of the moisture-content-suction characteristics of the soil). This Paper gives observations, determined calorimetrically, on the unfrozen water content of soils and on the suction-moisture content characteristics of the same soils. A relationship between these two quantities is shown to exist and is discussed.

EXPERIMENTAL METHODS

The determination of unfrozen water content in freezing soils

A calorimeter was constructed that permits the determination of the heat quantities required to raise or lower the temperature of soils by a measured amount. When expressed as calories/g/°C this value is the specific heat. If freezing temperatures are involved, it is usually markedly temperature-dependent, and in this Paper is called "apparent specific heat" because it includes some latent heat of fusion. A full description of the apparatus, together with observations on the apparent specific heat of soils, has been given recently (Williams, 1964(a)).

The soil sample (about 120 cc volume) is contained in a sample holder inside an outer container. During warming tests heat is supplied to the sample at a measured rate through a heating coil wound onto the sample holder. The outer container is maintained at the same temperature as the sample holder so that there is no loss of heat externally. During cooling tests the outer container is maintained at a temperature lower by a constant amount than that of the sample holder. There is then a steady and known passage of heat from the sample holder to the outer container and the exterior.

Such measurements permit the calculation of the amount of unfrozen water present in a frozen soil at various negative temperatures. The method has been outlined (Williams, 1963). The unfrozen water content at a given temperature is dependent on whether the temperature is reached by cooling or warming. The present Paper is concerned only with the former case.

In calculating the unfrozen water content of a frozen soil the time/temperature record (obtained during warming or cooling of a specimen) is divided into increments, ΔT_c, of 0·13 C° (other values occasionally being used). The quantity of heat associated with the change of temperature of the sample holder, soil, ice and water is subtracted from the quantity of heat removed from the sample in the time taken for this temperature change. The remainder is the latent heat involved in freezing or thawing of water. By adding the quantities of ice formed in each increment of temperature change (starting with the temperature of initial freezing), and by making appropriate corrections for the changing proportions of ice and water, the amount of ice present at any temperature can be calculated. This quantity is deducted from the total moisture content to give the unfrozen water content. These calculations can be expressed as:

Unfrozen water content of sample (g):

$$\text{total mc (g)} - \sum_{T_{\text{in}}}^{T_x} \frac{\Delta H - (K_s + \text{cal const})\Delta T_c + (K_w - K_i) I \Delta T_c}{79{\cdot}68} \quad . \quad . \quad . \quad (1)$$

where

ΔH = quantity of heat removed to lower temperature of sample by ΔT_c

ΔT_c = temperature change of sample

K_s = specific heat of wet soil (from determinations in unfrozen state)

cal const = heat required to change temperature of sample holder, cal/°C

K_w = 1 cal/g/°C (i.e. specific heat of water)

K_i = 0·508 cal/g/°C (average value of specific heat of ice)

T_{in} = temperature at which freezing commences

T_x = temperature at which unfrozen moisture content is required

I = amount of ice present at temperature of warm end of interval ΔT_c (change in the amount of ice present as the temperature changes through ΔT_c was ignored; the resulting error in the expression $(K_w - K_i)\, I\Delta T_c$ is not significant)

79·68 = latent heat of freezing of water, cal/g.

The results of observations on several soils are given in Figs 2 to 5, where the unfrozen moisture content expressed as a percentage of the dry weight is shown as a function of temperature during cooling (freezing). The grain size composition and a brief description of each soil are given in Fig. 1.

In compressible soils at a given temperature the unfrozen water content of a sample that has been frozen previously differs from that observed during the first freezing. The observations, therefore, include both cases. In determinations carried out on soils previously frozen and thawed (Figs 2(b), 3(b), and 4(b)) the lowest temperatures reached during prior freezing varied somewhat (although always lower than those in the determinations themselves). The unfrozen water content subsequently determined did not appear to be affected by these variations.

Accuracy of the calorimetric investigations

In calibration tests with water a standard deviation of 4% from the true value for specific heat was found in five tests; the average of the five results was within 1%. In tests with frozen soils further considerations are necessary. The greater part of the 4% error is due to an unmeasured loss or gain of heat in the sample during tests (Williams, 1964(*a*)). During prolonged tests it was often possible to rectify this so that a value better than $\pm 4\%$ could be expected. The calculated quantities of ice formed or melted should also be accurate to well within these limits. When the ice quantity, and subsequently the unfrozen water, is expressed as a percentage of dry weight, the latter may be in error by a maximum of 0·5 to 2% dry weight (depending on the total moisture content) at, for example, -5°C. The error does not accumulate as lower temperatures are reached, except insofar as I in the expression $(K_w - K_i)I\Delta T_c$ may be in error, and this expression has only a small effect on the whole calculation.

This accuracy is also indicated by a series of repetitive, though not entirely similar, tests on Leda clay (Fig. 4(b)). More serious are the errors associated with measurement of the sample temperature, especially within the temperature range 0° to − 1°C, on account of the rapid change of unfrozen water content with temperature. The sample temperatures were usually measured to an accuracy of ± 0·05°C, but occasionally to an accuracy of − 0·075°C. The curves can be displaced by this amount, and this will result in possible errors in unfrozen water content of about 2 to 3% dry weight and occasionally more. Most of the curves given in this Paper are more accurate than these figures indicate because they are based (as noted in each figure) on more than one test.

Repeated tests on the same sample (Fig. 4(b)) showed no difference, within the limits of experimental error, in unfrozen water content for a wide range of freezing rates. The temperatures, therefore, can be regarded as true equilibrium freezing temperatures. Temperature differences within the sample were very small and always less than 0·05°C.

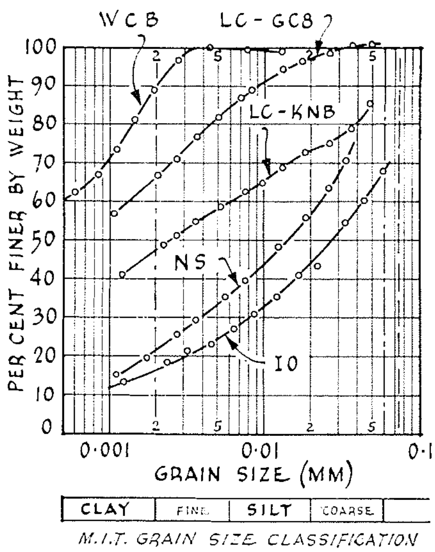


Fig. 1. Grain size composition and other characteristics of soils investigated:

NS	**— Niagara silt**
WCB	**— Undisturbed, mottled, montmorillonite clay; Winnipeg**
LC–GC8	**— Undisturbed Leda (marine) clay, mainly illite (for analysis, see Lambe and Martin, 1956); Green's Creek, Ottawa**
LC–KNB	**— Remoulded Leda (marine) clay, mainly illite**
IO	**— Haematitic iron ore, friable; Schefferville, P.Q.**

The determination of the suction–moisture-content characteristics

In the simplest form of apparatus for determining the suction–moisture-content characteristics of a soil, the sample rests on a porous plate to the underside of which is attached a column of water (Croney, Coleman, and Bridge, 1952). Various suctions are applied to this water, usually by means of a mercury manometer. At each suction the soil is allowed to establish its equilibrium moisture content, which is then determined. Because of the limitation on the suction that can be applied to a column of water, this method can be used only over a very limited range of soil moisture content.

In the present investigations, therefore, pressure plate and pressure membrane apparatus were used (Richards, 1947). Instead of placing the water source under reduced pressure, it is maintained at atmospheric pressure and the air pressure around the sample is raised. It is generally assumed that the air pressures are numerically equivalent to the suction* that would

* The word "suction" is now generally used in soil studies, to describe the property of the soil water system being measured here (ISSMFE, 1961). It does not imply, however, that the "pressure deficiency" of the water in the soil is limited to between 0 and 1 atmosphere. Furthermore, there is controversy (discussed later) as to the nature of the stresses giving rise to the observed suctions.

have to be applied to a column of water to produce the same moisture content in similar samples in the suction plate apparatus. As in the suction plate apparatus, water passes out of the sample through the plate or membrane until equilibrium is reached. Air does not escape from the chamber in significant quantity because of the small size of the pores in the pressure plate, which can be used at pressures up to about 1·02 kg/sq. cm, or of the pores of the membrane in the pressure-membrane apparatus, which can be used at pressures up to 14 kg/sq. cm. In the case of the pressure membrane apparatus a rubber diaphragm is pressed down by an additional air pressure of about 0·3 kg/sq. cm on to the soil sample, which is thus maintained in close contact with the membrane.

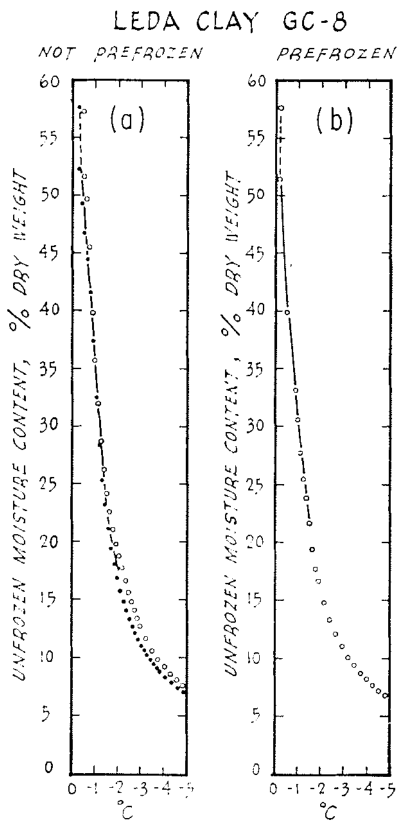


Fig. 2. Unfrozen water content during freezing of Leda clay GC–8: (*a*) during first freezing (based on observations during two tests); (*b*) following previous freeze and thaw

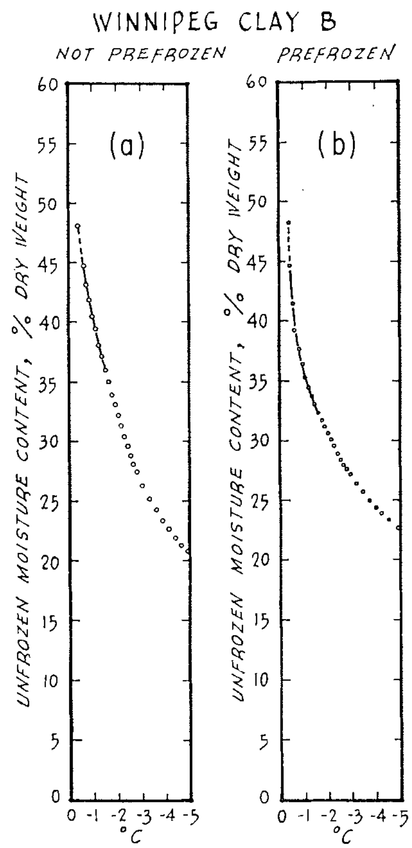


Fig. 3. Unfrozen water content during freezing of Winnipeg Clay B: (*a*) during first freezing; (*b*) following previous freeze and thaw

The thickness of the specimens was normally about 3 mm, and tests indicated that equilibrium was reached in 4 or 5 days. Moisture contents were determined by drying at 105°C. In the observations of Coleman and Marsh (1961) equilibrium was reached in 5 to 6 days, about 1% dry weight moisture content loss occurring during the last 1 or 2 days; but these specimens were about 5 mm thick. The results of the experiments are shown in Figs 6 and 7, where the pF of the soil is given as well (Schofield, 1935).* The samples for the calorimeter and suction tests should be as similar as possible. In the case of undisturbed samples, they should be adjacent parts of a single soil specimen. The same sample cannot be used for both tests, as the test procedure changes the character of the soil.

The suction–moisture-content characteristics of a given soil are known to vary with degree of consolidation, previous wetting or drying (Croney and Coleman, 1954). It was found in this investigation that a freeze/thaw cycle also resulted in similar changes. The figures give as well, therefore, determinations of suction–moisture-content characteristics of the soils following a freeze/thaw cycle.

Interpretation and accuracy of observations of suction–moisture-content characteristics

Suction–moisture-content characteristics were determined at room temperature (approximately 20°C). Gardner (1955); Wilkinson and Klute (1962); and Coleman, Croney, and Bridge (1952) have pointed out a dependence of suction on temperature. The question arises, therefore, of whether the suction values determined at room temperature should be corrected to a temperature near 0°C before comparison with the unfrozen moisture content/temperature observations. Gardner's experiments, which involved suctions up to 0·32 kg/sq. cm (pF 2·5), indicate a change of suction of about 0·14 kg/sq. cm between 20 and 0°C. Such a change corresponds to about 0·16 pF units at pF 2·5, but would correspond to only 0·06 units at pF 3, and to 0·01 pF units at pF 3·5. It will be shown, subsequently, that pF's lower than about 3 are of relatively little significance in the present investigation. It is apparent, therefore, that no serious error will result if the temperature correction is ignored in all cases. Tests by Penner (1958) lead to the same conclusion.

In previously frozen and thawed samples the lowest temperature to which they were subjected in prior freezing varied between − 5 and − 18°C. These variations appeared to have no significant effect on the subsequently observed suction-moisture content characteristics, and a single line was used to represent all points for a given soil.

Water from soils collected after passing through the membrane has a depressed freezing point (Table 1) and contains a quantity of dissolved salts. In the porous plate apparatus and certain of the pressure membrane apparatus fresh water is periodically circulated below the membrane to remove air bubbles; this procedure may reduce the salt concentration of the water in the soil. In the present experiments, however, moisture contents obtained with this technique did not differ significantly from those where the lower side of the membrane was always in contact with solution from the soil.

Effects of salts on freezing temperatures

Salts dissolved in the soil solution are partly responsible for depression of the temperature of freezing and for the associated unfrozen water content of freezing soils (Ayers and Campbell, 1951). To investigate this the soil solution was extracted and collected with the pressure membrane apparatus at various air pressures up to 4·2 kg/sq. cm. In this application the air pressures are applied entirely above the rubber diaphragm and are thus not exactly comparable with those used in the suction–moisture-content investigations. This procedure is necessary to avoid air leakage through the membrane (it is not possible to keep the membrane saturated by circulation of water below it—this would dilute the extracted solution). Using a mercury thermometer, simple freezing point determinations accurate to about ± 0·02°C

* pF is a widely accepted symbol to refer to the convenient value, the logarithm of the suction. It is used in this Paper solely in that sense and without reference to the "free energy" of the soil water.

Table 1

Freezing point determinations on extracted soil solutions

Soil	Air pressure used for extraction: kg/sq. cm	Initial moisture content: % dry wt	Freezing point: Δt_s, °C	Remarks
Winnipeg clay B	4·2	48	− 0·29	
Leda clay GC-8	4·2	57	− 0·22	
Leda clay KNB	4·2	31	− 0·10	Same sample subjected to two air pressures
Leda clay KNB	14·2		− 0·10	
Iron ore		30 approx.	taken as: −0·30 (see text and footnote *)	

(Table 1) were made on the solution obtained. Within this accuracy it was found that for each soil the freezing points were similar for water extracted at any pressure up to 14 kg/sq. cm. In the absence of ice, therefore, that part of the soil water of interest in the present investigation is assumed to have a uniform concentration of dissolved salts.

As the ice (which does not contain any significant amount of dissolved salts) is formed during freezing of the soil, the salts become increasingly concentrated in the unfrozen water. Because the increase in freezing point depression due to salts in solution is approximately proportional to the concentration, the effects of dissolved salts on the freezing temperature associated with each quantity of unfrozen water are easily calculated:

$$\Delta T_{s_x} = \frac{\Delta T_s \text{ total moisture content}}{\text{unfrozen water content at temperature } x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where

ΔT_s = the observed freezing point depression of the extracted solution °C

ΔT_{s_x} = the calculated freezing point depression °C of the unfrozen water content at temperature x (on account of dissolved salts)

The observed temperature corresponding to each quantity of unfrozen water must therefore be raised (giving a smaller negative temperature) by the amount, ΔT_{s_x}, before comparison with suction–moisture-content observations.

The freezing point depression of the extracted solutions is quite similar to the temperature at which ice first appears in the calorimeter samples (i.e. the highest temperature on the unfrozen water content-temperature graphs). They are not expected to be exactly the same because the calorimeter samples were usually not saturated. At their total moisture content, therefore, the pF value is not 0, and their freezing temperature is somewhat below that of the extracted solution.*

There is a possibility that the membranes used are semi-permeable with respect to some of the salts in the soil solution. The similarity in freezing point of the extracted solution and the initial freezing temperature of the calorimeter sample indicates, however, that this does not occur to any significant extent.

* For one soil, iron ore, the reverse was found to be the case, the calorimeter sample commencing to freeze at − 0·026°C; the extracted solution froze at − 0·13°C. Different samples were used for the calorimeter test and for extraction of the solution, however, and the difference in freezing temperature is almost certainly due to the variable nature of the material. In this case only, therefore, the calorimeter value, − 0·026°C., was taken as a better value for ΔT_s.

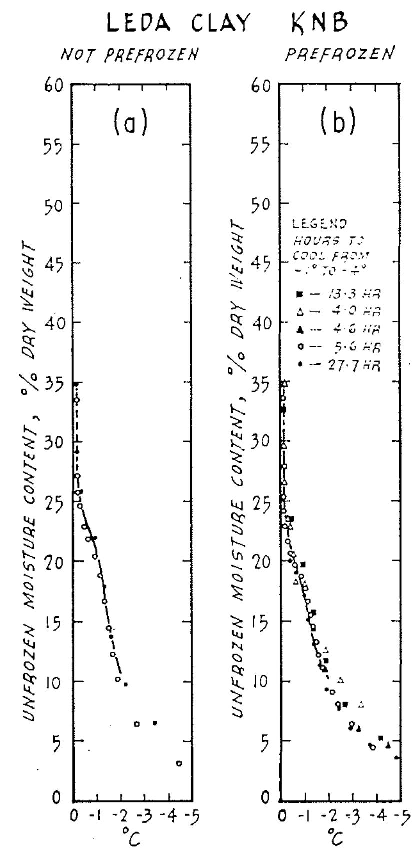


Fig. 4. Unfrozen water contents during freezing of Leda clay KNB: (*a*) during the first freezing (based on observations during two tests) (*b*) following previous freeze and thaw; the five tests were carried out at different rates of freezing as indicated; in one test the total moisture content was 19%, while in four it was approximately 35%; it may be seen that the different total moisture content has no significant effect on the unfrozen water content

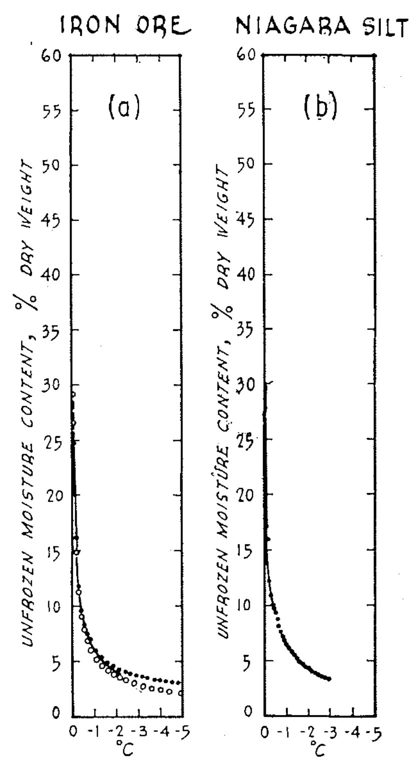


Fig. 5. The unfrozen water content during freezing of: (*a*) Iron ore (based on two tests); (*b*) Niagara silt

COMPARISON OF THE OBSERVATIONS OF UNFROZEN WATER CONTENT AND SUCTION–MOISTURE-CONTENT CHARACTERISTICS OF THE SOILS

As already noted, the moisture content at a given suction and the unfrozen water content at a given temperature are both affected by previous freezing and thawing. This must be borne in mind when comparing results.

Unfrozen water contents on first freezing, and suction–moisture-content characteristics of soil not previously frozen

For the first comparisons, freezing tests (Figs 2(a), 3(a), 4(a), and (5)) were completed on samples of several soils that had not been previously frozen (all samples were from deposits below the maximum annual frost penetration and from areas where permafrost has almost certainly never occurred). Further samples were taken from the same soils and the suction–moisture-content relationships determined (Figs 6(a), 6(c), 7(a), 7(c), 7(d)). To compare these

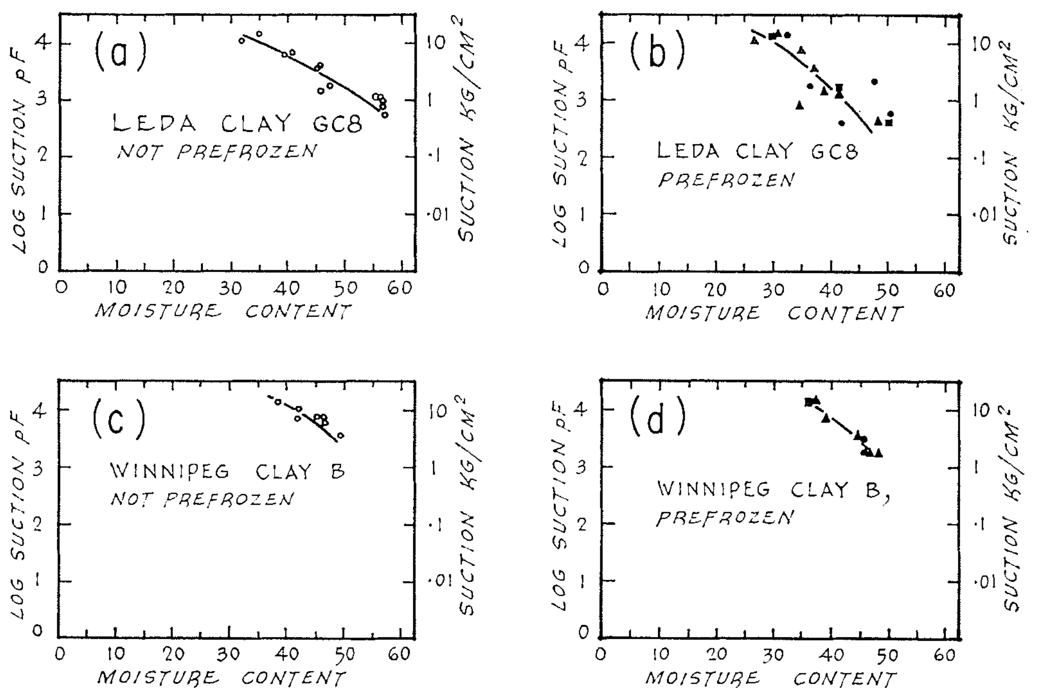


Fig. 6. Suction–moisture content curves for soils investigated calorimetrically: For the soils previously frozen, the following symbols are used in this Figure and Fig. 7 to indicate the lowest temperature for each sample:

▲	−5·0	●	−12·0
△	−6·5	□	−15·0
■	−9·5	×	−18·0

two sets of results a graph was drawn (Fig. 8) showing negative temperature and suction values. These were obtained by selecting a temperature and finding the unfrozen moisture content corresponding to it for a particular soil. The suction value corresponding to an equal moisture content in the unfrozen soil was then obtained from the appropriate graph in Fig. 6 or 7. To allow for the effects of dissolved salts a correction (as described in the previous section) was made to each freezing temperature before plotting. From the curves obtained (Fig. 8) it is quite clear that there is a strong relationship between the suction and the equilibrium freezing temperature.

Unfrozen moisture contents during second and subsequent freezing of a soil, and the suction–moisture-content characteristics of soils also previously frozen and thawed

For these next comparisons, freezing tests were carried out on soil samples previously frozen and thawed (Figs 2(b), 3(b), 4(b)). Suction–moisture content characteristics were also determined for samples of the same soil similarly pretreated (Figs 6(b), 6(d), 7(b)). These two sets of information were again used, with appropriate correction for the influence of salts, to give the curves shown in Fig. 9. It is clear that the relationship previously found also holds in this case. In view of the difference between observations made on the same soil, with and without prior freezing, it is equally clear that the relationship will only apply if the suction–moisture content and temperature–unfrozen water content measurements are made on samples of soil treated similarly with respect to previous freezing.

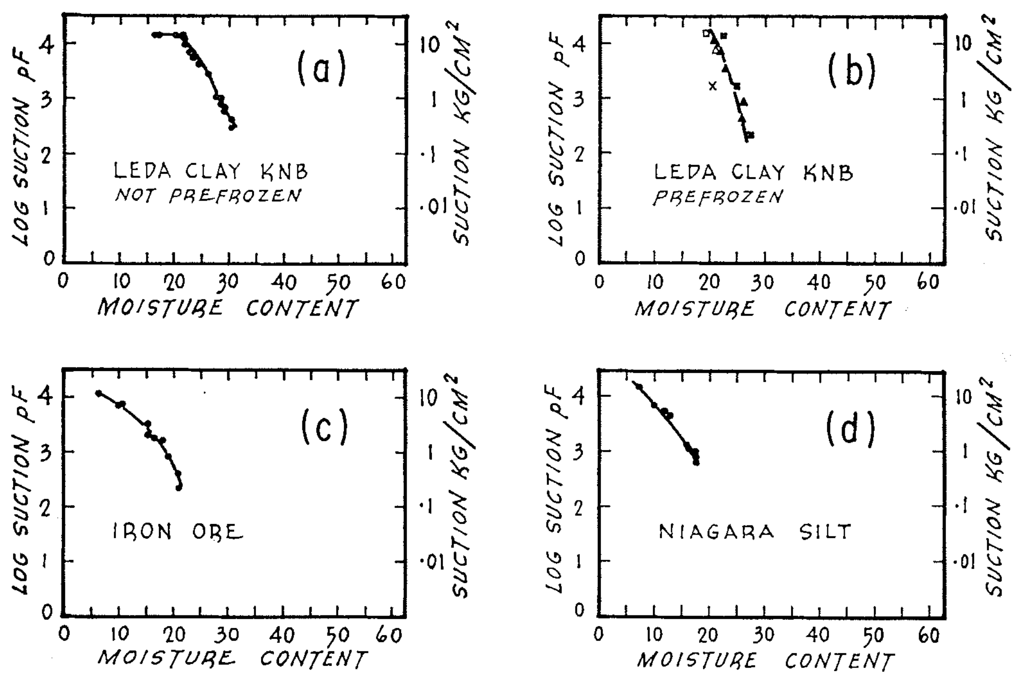


Fig. 7. Suction–moisture-content curves for soils investigated calorimetrically (symbols as in Fig. 6 except that ● is used for soils *not previously frozen*)

DISCUSSION

The results described above have important theoretical and practical implications. This Paper is not primarily concerned with the causes of the observed relationships, but a brief theoretical discussion is appropriate.

Theoretical

When a uniform pressure (above atmospheric) is applied to ice and water in contact, the freezing point is lowered below 0°C. Conversely, if the pressure is reduced below atmospheric, the freezing point is raised above 0°C. If the measured suctions in soils imply that the water is under pressure lower than atmospheric, it might be supposed that freezing would take place at temperatures above 0°C. Even when allowance is made for dissolved salts and the necessity for an ice nucleus, however, it has been frequently observed that this does not occur. Edlefsen and Anderson (1943) proposed that very little, if any, of the soil water is under reduced pressure. They believed that most of the water in a fine-grained soil is under the influence of adsorption forces acting normally to the particle surface and resulting in raised pressures in the adjacent water. Ice present would also be subjected to these pressures, and the result would be a depression of freezing point. They believed such adsorption forces could also be responsible for the "suction" of the soil.

Schofield, on the other hand, thought that much of the soil water was really under tension or reduced pressures, but that the ice first formed would be under atmospheric pressures. If such were the case the freezing point would again be depressed according to the relationship:

$$H = \frac{L}{Tg} . \Delta T \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

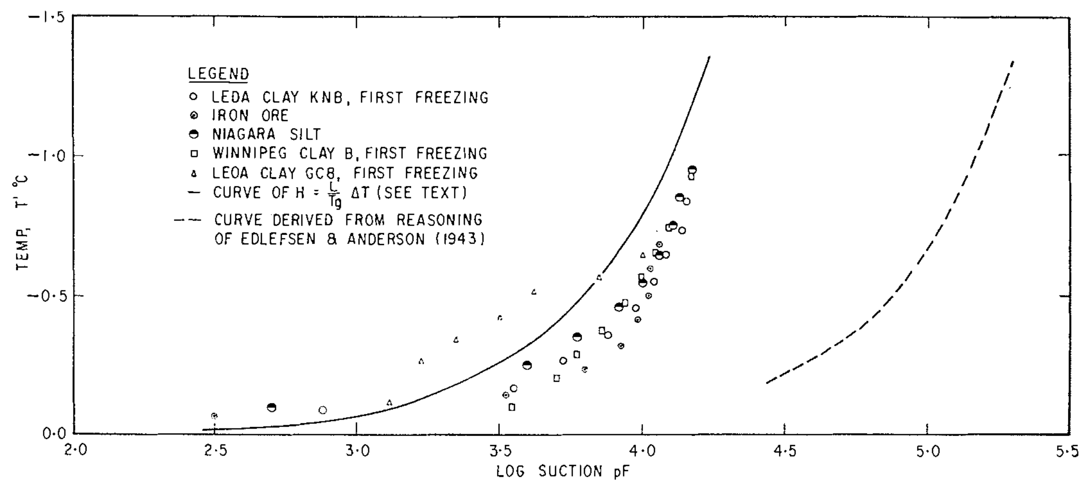


Fig. 8. Relationship for soils investigated between suction for a given moisture content, at room temperature, and temperature (after correction for dissolved salts) at which a similar amount of unfrozen water occurs in the freezing soil. (For descriptions of soils see Fig. 1)

where

H = suction expressed as height of a column of water, cm

L = latent heat of freezing of water $3{\cdot}336 \times 10^9$ ergs/g

T = temperature °K

g = gravitational acceleration, 981 cm/sec^2

ΔT = freezing point depression °C

In Schofield's (1935) paper, errors have apparently occurred in the printing of the equation, which is given here correctly. No derivation for the equation is given, but this may be followed in the work of Poynting (1881) and Hudson (1906).

Schofield and Botelho da Costa (1938) found that the initial freezing points for samples with various moisture contents (and hence various suction values) fitted this relationship well. The relationship has been plotted on Figs 8 and 9 ($\Delta T = T'$), and corresponds also to the experimentally observed relationship shown there.

Experimental

With respect to the present investigations the following should be noted:

(1) The various curves determined experimentally in Figs 8 and 9 coincide remarkably closely. The relationship cannot be fortuitous, because the soils differ substantially in both nature and experimental behaviour. The theoretical curve from equation (3) is also very similar, although it differs slightly from the curve that would most nearly approximate all the experimentally determined curves. Theoretical values calculated from an equation based on the work of Edlefsen and Anderson (1943), on the contrary, deviate significantly from the experimentally obtained curves.

(2) The agreement between the observations and the curve given by equation (3) implies that even when large quantities of ice are present for temperatures down to at least − 1°C the pressure on the ice is different from that on the water. Probably, as visualized by Schofield, the pressure on the ice approaches atmospheric and that on the water is appropriate

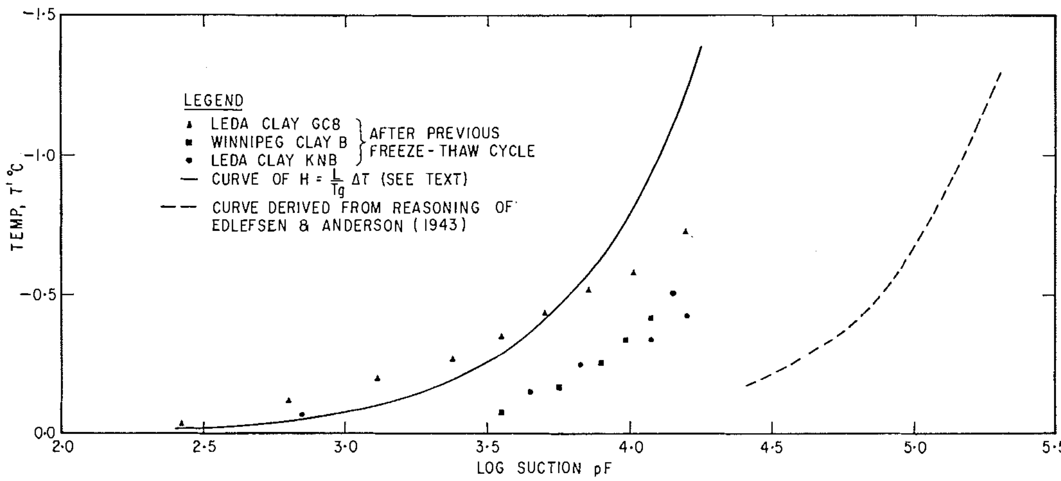


Fig. 9. Relationship for soils investigated between suction for a given moisture content, at room temperature, and temperature (after correction for dissolved salts) at which a similar amount of unfrozen water occurs in the freezing soil. (For descriptions of soils see Fig. 1)

to the remaining water content, as is shown by the suction–moisture-content characteristics of the soil in question. Alternatively, the pressure on the ice may rise somewhat. That on the water must then be assumed to rise by approximately the same amount (i.e. with respect to the suction–moisture-content curve, the pF values must decrease) to give the observed relationship.*

(3) The correspondence between the curve from equation (3) (which was experimentally verified by Schofield and Botelho da Costa (1938) as relating initial freezing points of water in

* According to the usual relationship: $\Delta P = \dfrac{L\Delta T}{(V_2 - V_1)T}$

where

ΔP = change of pressure, relative to atmospheric

V_1 = spec. vol. of ice

V_2 = spec. vol. of water

other symbols as in equation (3)

pressures applied uniformly to ice and water give a change in freezing point of 0·0073°C for each 1,000 g/sq. cm. If the pressure on ice and water in the soil changes by a uniform amount, the temperatures for each unfrozen water content will differ from those predicted by equation (3). Pressures of even 10 kg./sq. cm, however, would produce a temperature change so small as to be barely significant experimentally.

soil to suction at various total moisture contents) to the curves derived in the present investigation leads to an important conclusion: the equilibrium freezing temperature associated with a particular (unfrozen) water content is not greatly affected, if at all, by the amount of ice (formed at higher temperatures) already present.

The observed correspondence thus provides some theoretical justification for the experimental observations already made (Fig. 4(b), and Williams, 1962 (*b*)) that the unfrozen water content for a soil at a given negative temperature is independent of the total (water and ice) moisture content (so long, of course, as the total moisture content is not less than the appropriate unfrozen water content).

It should be remembered that this statement will not be strictly true if the different total moisture contents are associated with different amounts of soluble salts per unit weight of dry materials: in such cases ΔT_{sx} will vary somewhat, causing differences (whose magnitude will depend on that of ΔT_{sx}) in the temperature associated with a given unfrozen moisture content.

(4) Although the agreement between the curves (Figs 8 and 9) is generally good, it is useful to speculate on the causes of the deviations that do occur, because they have significance in regard to the discussion in the next section.

There are margins of error to be expected in the temperature/unfrozen water content and in the *pF* moisture content determinations discussed earlier. Especially where the *pF* increases rapidly with decrease in moisture content (as in Figs 7(a) and 7(b)), a quite small error in the curve will result in rather large errors in moisture content values for given *pF* values. The error in moisture content will also be proportionately greater where the soil moisture contents are always relatively low (as in Figs 7(c) and 7(d)).

Errors in determination of the freezing point of the extracted solutions (ΔT_s) are important, particularly where the moisture contents are always relatively low. They are especially important when the decrease in moisture content with increasing *pF* is rapid.

Another explanation for part of the deviations lies in the possible pressure changes resulting from some confinement of the freezing and expanding sample within the sample holder. The sample does not completely fill the holder, but various stresses might be set up in both the ice and pore-water that could cause some of the observed deviations.

There is no obvious explanation for the fact that four of the five soils illustrated in Fig. 8 lie below the theoretical curve in a consistent manner. It may be that with further observations on other soils, this pattern will cease to be apparent.*

CONSEQUENCES OF THE OBSERVED RELATIONSHIP

Prediction of unfrozen water contents

The most direct result of the relationship established is that it permits prediction of the unfrozen water present in frozen soils at temperatures from 0 to $-1{\cdot}0$°C, and often somewhat colder, without the necessity of elaborate calorimetric tests. Although the relationship has not been established for temperatures much colder than -1°C, soil temperatures in seasonally frozen layers and in islands of sporadic permafrost (situations of major practical significance) most frequently lie within the range 0 to -1°C. Prediction of unfrozen water content in this way requires careful determination by conventional methods (as described in this Paper) of the appropriate suction–moisture-content characteristics. The unfrozen moisture contents for the temperatures desired are then found by finding the corresponding suction values from the relationship as represented by the theoretical curve (of equation (3)) in Figs. 8 and 9.

* Very recent work, as yet unpublished, indicates that the difference between interfacial energy for ice-water, and air-water is partly responsible (see Miller, 1964; Williams, 1964(b)).

It is essential that the soil sample be typical for that for which the unfrozen water content is to be determined, particularly with respect to previous freezing and thawing or drying and rewetting. For soils with significant amounts of dissolved material, determination of the freezing point of the extracted solution is necessary.

To illustrate the procedure it will be assumed that the unfrozen moisture content of Leda clay KNB at a temperature of $-0{\cdot}7°C$ is required. The clay has not been previously frozen. The steps involved are:

(1) The total moisture content of a sample of (unfrozen) soil is found to be 33·4% dry weight.

(2) The suction–moisture-content characteristics are determined for suctions up to about pF 4 (as in Fig. 7(a)).

(3) A specimen of the soil solution is extracted in a pressure membrane apparatus using an air pressure of between 4 and 14 kg/sq. cm. The freezing point ΔT_s of this solution is found to be $-0{\cdot}10°C$ (as in Table 1).

(4) Using the theoretical curve representing equation (3) on the graph (Fig. 8) read off the suction appropriate to a temperature T'°C, which should be less than that temperature for which the unfrozen water content is required by at least the value ΔT_s. Taking T' as $-0{\cdot}5°C$, for example, the pF value is 3·8.

(5) From the suction–moisture-content curve (Fig. 7(a)) read off the moisture content corresponding to this suction (in this case 24·5% dry weight). This gives a rough value of the unfrozen water content required.

(6) The freezing point depression ΔT_{sx} caused by dissolved salts concentrated in the amount of unfrozen water found in § 5 is then calculated as $33{\cdot}4 \times 0{\cdot}1/24{\cdot}5 = -0{\cdot}14°C$.

(7) The quantity of unfrozen water, 24·5% dry weight, is thus that to be expected at temperature of $T' + \Delta T_{s_x} = -0{\cdot}64°C$.

Reference to the curve obtained calorimetrically for this soil shows that 22·7% dry weight unfrozen water content occurs at $-0{\cdot}64°C$, so that agreement is close.

If the curve of unfrozen water content versus temperature is required (for example, to obtain a value for $-0{\cdot}7°C$ exactly) a number of points can be obtained in a similar manner.

If the procedure used to obtain the points in Figs 8 and 9 is followed through, it will be seen that when salts are dissolved in a soil solution the curve for that soil will always terminate short of $-1{\cdot}2°C$ (corresponding to the highest measured pF's for a salt-free soil, according to equation (3)). The amount by which the curve falls short of $-1{\cdot}2°C$ depends on the value of ΔT_s, the suction moisture content relationship, and of course on any errors in the pF and unfrozen water content determinations.

The experimental data obtained do not at first sight verify the relationship for T' further than $-0{\cdot}5°C$ for some of the soils, but it is reasonable to assume that they do in fact apply also for those soils at temperatures of $-1{\cdot}0°C$ and even somewhat lower. If, for example, the graph (Fig. 8) is entered at $-1{\cdot}0°C$, a pF value of 4·09 is obtained (from the Schofield curve). Assuming that Winnipeg Clay B is considered, and proceeding from paragraph (5) above, an unfrozen moisture content of 36·75% dry weight is found to occur at $-1{\cdot}38°C$. As the calorimetrically determined value is 34·9% dry weight at $-1{\cdot}38°C$, the agreement is again close.

It appears that even for soils for which the pF temperature relationship (Fig. 8) deviates considerably from that of equation (3) a useful estimate can be obtained. For example, for Leda clay KNB that has been previously frozen, a value of 20·75% unfrozen moisture content would be predicted at $-1{\cdot}16°C$. According to the calorimetric tests, the value is

16·2% unfrozen moisture content at that temperature. An approximation accurate to this extent is likely to be of more use for the practical purposes outlined below than estimates based on grain size analysis or other simple appraisal. This appears especially true if the very large variations in unfrozen water contents between different soils are recalled.

Prediction of thermal properties

In an earlier paper, calorimetric measurements made during these and other tests were used to calculate values for actual and apparent specific heats of frozen soils (Williams, 1964 (*a*)). Most soils in a range of temperature to several degrees below 0°C have apparent specific heats largely composed of latent heat.

Estimation of these apparent specific heats (if calorimetric tests are not feasible), based on visual or grain size tests, is often not very satisfactory. The apparent specific heats can be calculated with reasonable accuracy if the corresponding unfrozen water contents are calculated from determinations of the suction–moisture-content characteristics. The difference in unfrozen water content between any two temperatures gives the latent heat involved in such a temperature change. Because the apparent specific heats change very rapidly with temperature, it is necessary that they be calculated for small increments of temperature change. For each such temperature change the latent heat component of the corresponding apparent specific heat can be calculated directly from the change in unfrozen moisture content.

To obtain apparent specific heat there must be added the heat involved in changing, by an equal amount, the temperature of soil solids, water, and ice present. These heats can be estimated with satisfactory accuracy using values for specific heat of dry soil from for example, Kersten (1948), water and ice, if the total moisture content is known. Determination of apparent specific heats in field problems may involve a number of variables (e.g. externally applied stresses, the effects of which are the subject of continuing investigations).

Strength properties of frozen soil

Relatively few studies of the strength of freezing or thawing ground and its consolidation properties have been carried out (Lovell, 1957; Vialov, 1955; Inst. Merzlot, 1953–57). Some of these are of an entirely empirical nature and the general applicability of the results is correspondingly limited. It is apparent that the change in proportions of frozen and unfrozen water, and especially the effects of loading, are of fundamental significance in this respect. The present investigation has shown the thermodynamic equation which relates the unfrozen water content of frozen soils to temperature and pressure. A basis is thus provided for studies of the effects of load and pore-water pressure on the deformation behaviour of frozen soils.

CONCLUSIONS

(1) The unfrozen water content of various soils during freezing has been determined calorimetrically. The suction–moisture-content characteristics of the soils at room temperature have also been determined. When allowance for experimental error is made and a correction applied for depression of the freezing point by dissolved salts, a unique relationship is found to exist between the negative temperature at which a given unfrozen moisture content occurs and the suction corresponding to a similar moisture content at room temperature.

(2) This relationship is similar to that given by Schofield (1935) and Schofield and Botelho da Costa (1938) as existing between the initial freezing temperatures of soils at various moisture contents and the suctions associated with those moisture contents.

(3) The correspondence of the observed relationship with that of Schofield provides theoretical justification of the earlier observed independence of unfrozen water content, at a given temperature, from the total moisture content.

(4) The observed relationship permits prediction, from the suction–moisture-content characteristics, of the unfrozen water content of a given soil at various negative temperatures down to $-1{\cdot}0$°C, and often somewhat lower, without the need for calorimetric studies.

(5) Such predictions permit calculation of the apparent specific heats of frozen soils, also without calorimetric tests.

(6) Establishment of the relationship between suction and unfrozen water content provides a basis for studies of the effects of pressure as well as temperature, and thus of applied loads, on the unfrozen water content of frozen soils.

ACKNOWLEDGEMENTS

Discussions with L. W. Gold and other members of the Division are gratefully acknowledged, as is the technical assistance of R. Armour. This Paper is a contribution from the Division of Building Research, and is published with the approval of the Director of the Division.

REFERENCES

AYERS, A. D. *and* R. B. CAMPBELL, 1951. "Freezing point of water in a soil as related to salt and moisture contents of soil." *Soil Sci.*, 72:201–206.

BOUYOUCOS, G. J. *and* M. M. MCCOOL, 1916. "Further studies on the freezing point lowering of the soils." *Mich. Agric. Coll. Exp. St., Tech. Bull.* No. 31, 51 pp.

COLEMAN, J. D. *and* A. D. MARSH, 1961. "An investigation of the pressure-membrane method for measuring the suction properties of soil." *Soil Sci.:* 12: 2:343–362.

CRONEY, D., J. D. COLEMAN *and* W. P. M. BLACK, 1958. "Movement and distribution of water in soil in relation to highway design and performance." *Highw. Res. Bd Spec. Rep.* 40:226–252.

CRONEY, D., J. D. COLEMAN *and* P. M. BRIDGE, 1952. "The suction of moisture held in soil and other porous materials." *Road Res. Tech. Pap. No.* 24 (DSIR, RRL, Harmondsworth, Middx.). 42 pp.

CRONEY, D. *and* J. D. COLEMAN, 1954. "Soil structure in relation to soil suction (*pF*)." *J. Soil Sci.*, 5:1:75–84.

EDLEFSEN, N. E. *and* A. B. C. ANDERSON, 1943. "Thermodynamics of soil moisture." *Hilgardia*, 15:2:31–298.

GARDNER, R., 1955. "Relation of temperature to moisture tension of soil." *Soil Sci.*, 79:257–265.

HUDSON, C. S., 1906. "The freezing of pure liquids and solutions under various kinds of positive and negative pressures and the similarity between osmotic and negative pressure." *Physical Rev.* 22:257–264.

INSTITUT MERZLOTOVEDENIA, *im.* V. A. OBRUCHEVA, 1953–57. "Materialy po laboratornym issledovaniiam merzlykh gruntov," (*Izd.*) *Akad, Nauk SSSR, Moskva, Sb.* 1,2,3.

INTERNATIONAL SOCIETY OF SOIL MECHANICS AND FOUNDATION ENGINEERING, 1961. "Pore pressure and suction in soils," *British Nat. Soc. of Int. Soc. Soil Mech. Butterworths, London*, 151 pp.

KERSTEN, M. S., 1948. "Specific heat tests on soils," *Proc. 2nd Int. Conf. Soil Mech.* Vol. III, Rotterdam, 5 pp.

LAMBE, T. W., *and* R. T. MARTIN, 1956. "Composition and engineering properties of soil." *Highw. Res. Bd, Proc.* 35th *Ann. Meeting*, pp. 661–677.

LOVELL, C. W. Jr, 1957. "Temperature effects on phase composition and strength of partially frozen soil." *Highw. Res. Bd. Bull.*, 168:74–95.

MILLER, R. D., 1964. Paper presented to First International Conference on Permafrost, 1963. *In Proc.* 1*st Int. Conf. Permafrost* 1964.

PENNER, E., 1958. "Soil moisture tension and ice segregation." *Highw. Res. Bd, Bull.*, 168:50–64.

POYNTING, J. H., 1881. "Change of state: solid-liquid." *Phil. Mag.*, 5:12:32–48.

RICHARDS, L. A., 1947. "Pressure membrane apparatus, construction and use." *Agric. Eng.*, 28:451, 454, 460.

SCHOFIELD, R. K., 1935. "The *pF* of the water in soil." 3*rd Int. Cong. Soil Sci.*, 2:37–48; 3:182–186.

SCHOFIELD, R. K., *and* J. V. BOTELHO DA COSTA, 1938. "The measurement of *pF* in soil by freezing point." *Agric. Science*, 28:645–653.

VIALOV, S. S., 1955. "Creep and long-term strength of frozen soils." *Dok. Akad. Nauk SSSR*, 104:6:850–853. (Translation, Defence Research Board, Canada, T 203 R, 1957, 4 pp.).

WILKINSON, G. E., *and* A. KLUTE, 1962. "The temperature effect on the equilibrium energy status of water held by porous media." *Soil Sci.*, 26:4:326–329.

WILLIAMS, P. J., 1963. "Specific heats and unfrozen water content of Frozen Soils." *In Proc.* 1*st Can. Conf. Permafrost. Nat. Res. Counc., Canada, Assoc. Cttee on Soil and Snow Mechs., Tech. Memo. No.* 76, pp. 109–126.

WILLIAMS, P. J., 1964(*a*). "Specific heat and apparent specific heat of frozen soils." *Géotechnique*, 14:2:133–142.

WILLIAMS, P. J. 1964(*b*). Author's closure to Paper: "Suction and its effects in unfrozen water of frozen soils." *In Proc.* 1*st Int. Conf. Permafrost*, 1964.

SUCTION AND ITS EFFECTS IN UNFROZEN WATER OF FROZEN SOILS

by P. J. Williams

SYNOPSIS

An experiment was devised to investigate effective stresses that should result from the suction (negative pore water pressure) supposed responsible for the presence of water remaining unfrozen in frozen soils. A soil sample is frozen while ice formation is prevented in part of the sample; dry density and water content of the part without ice are subsequently determined. Several soils were tested at various negative temperatures. Decrease of water content and consolidation of the ice-free part occurred. Since the samples remained saturated, the results could be compared with oedometer tests to indicate the magnitude of the effective stresses that were developed. The water content of the ice free part was equal for a given temperature, to that which remained unfrozen in normally frozen samples at the same temperature.

The existence and approximate magnitude of negative pore pressures, giving rise to effective stresses, in unconfined samples at negative temperatures is confirmed. Freezing of water in soils appears to take place according to an equation of the type proposed by Schofield, and this equation can be accepted as giving approximate values for the negative pore pressures and effective stresses arising, as a result of freezing, in unconfined samples.

On a établi un moyen de mesurer les pressions efficaces causées par la succion (pression interstitielle négative), supposées responsables pour la présence de l'eau non gelée en sols gelés. On gèle un échantillon de sol en évitant la formation de glace dans une certaine partie de l'échantillon. Subsequemment, on détermine la densité sèche et la teneur en eau de la partie non gelée. Plusieurs sols ont été examinés à des températures différentes. On a observé une diminution de la teneur en eau et une consolidation de la partie non gelée. Comme les échantillons demeurent saturés, des essais oedométriques pourraient indiquer l'amplitude des pressions efficaces developpées. La teneur en eau de la partie non gelée à une température donnée serait égale à celle qui demeurerait non gelée dans des sols gelés normalement à cette même température.

On a confirmé l'existance et l'amplitude approximative de la pression interstitielle négative dans la congélation des sols sans contraintes. La congélation de l'eau semble prendre place selon une équation du type proposé par Schofield. Donc, on peut adopter cette équation comme base du calcul approximatif de la succion et, par suite, des pressions efficaces résultant du gel, dans des échantillons sans contraintes.

Substantial quantities of water remain unfrozen in soils at temperatures of several degrees below 0° C (pp. 11–26) (Inst. Merzlot, 1953–57; Lovell, 1957; Williams, 1963). The proportion of unfrozen water decreases as the temperature is lowered, but as much as half of the water may exist unfrozen at –1° C. This unfrozen water has been attributed (p. 22) to the suctions or negative pore pressures that develop as a result of ice formation in the soil. A negative pore pressure in a saturated soil, in the absence of external loading, results in a positive effective stress (a stress acting across the grain-to-grain contacts) equal to the negative pore pressure. An increase in effective stress causes consolidation in compressible soils.

It is therefore of interest to investigate the consolidation that should occur in a frozen soil as a result of negative pore pressures in the unfrozen part of the moisture content. Such consolidation would also confirm the existence and, to some extent, the magnitude of the negative pore pressures and their relationship to temperature.

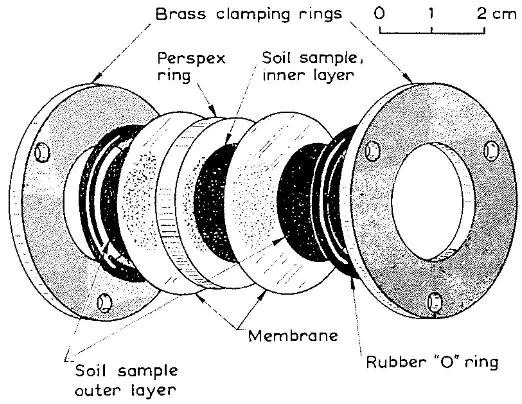


Fig. 1. Sample assembly for freezing experiments.

FREEZING EXPERIMENTS

Direct investigation of these effects is difficult because of the irregularly distributed ice lenses in a frozen soil which, on the one hand greatly reduce permeability, and on the other hand increase the overall soil volume. It is necessary to devise an experiment in which lens formation is prevented in a part of the soil large enough for subsequent study of degree of consolidation and water content.

Perspex rings 2,2 cm ID and 0,4 cm in height were made. A ring was filled with a soil sample prepared flush to the surface. Pieces of a membrane, previously soaked in water, were then placed across the faces of the ring and soil sample and pressed tightly against the perspex with a clamp arrangement (Fig. 1). A thin smear of vaseline was usually placed on the perspex ring where it came in contact with the membrane. Two further pieces of the same, or sometimes different, soil were then pressed against the exposed sides of the membranes.

This assembly was slowly cooled to a chosen negative temperature in the range 0° to –3° C. Freezing (with ice lens formation) occurs in the exposed soil, but not in that between the membranes because of the absence of an ice nucleus. Spontaneous nucleation does not occur – initially because the sample is small and subsequently for reasons which will become apparent – and ice growth cannot occur through the membrane. The membrane is of the type used in pressure membrane tests (Richards, 1947) and is permeable to water. The membrane pores are so small that ice growth only occurs within them at temperatures of at least several degrees below 0° C.

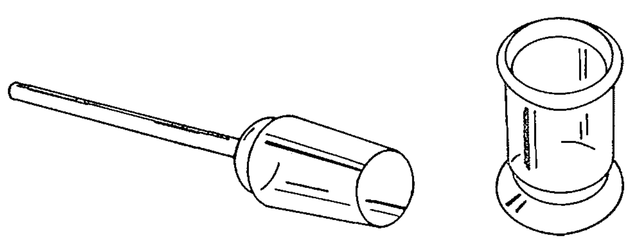


Fig. 2. Pycnometer used for dry density determinations.

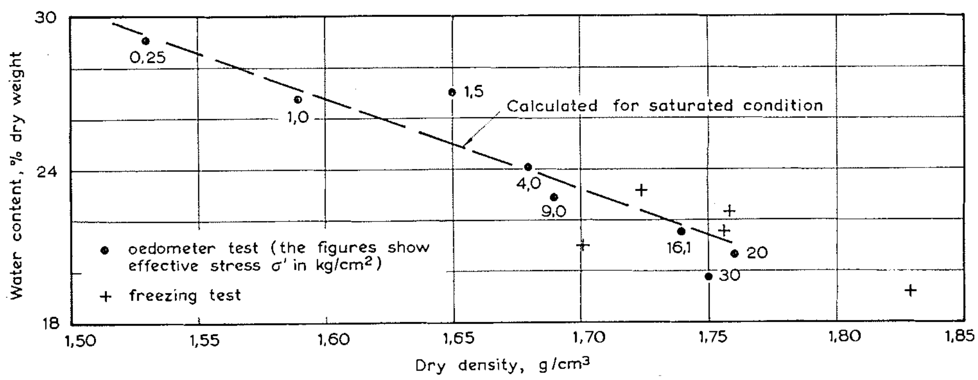


Fig. 3. Dry density as a function of moisture content, Leda clay KNB.

A modified domestic-type refrigerator was used. A mercury contact thermometer switch with a relay system maintained a fairly constant temperature. To avoid dessication and to ensure uniform temperature the sample assemblies were placed in closed jars, together with pieces of ice to start nucleation in the exposed soil. The specimens were generally maintained at the chosen temperature for three days. Tests with thermocouples indicated that the sample attained the chosen temperature from 5 to 10 hours after being placed in the refrigerator.

After removal from the refrigerator, the assembly was immediately dismantled and the water content and volume of the inner soil layer (located between the membranes) were determined. A specially constructed glass pycnometer (Fig. 2) containing paraffin was used to determine the volume. To obtain sufficient accuracy, considerable attention to temperature and other effects was necessary. Further tests were carried out involving freezing to various temperatures.

Many tests involving determination of water content only were also carried out. In all cases the water content of the inner layer was found to be substantially less than the initial content. Preliminary tests showed that the water content of the inner layers became practically constant within three days. It was lower for lower temperatures. In any one test similar soil samples placed in different parts of the refrigerator might give results differing by 1 to 2 % of dry weight. This was largely due to slight temperature differences. Samples of the same soil in one bottle consistently showed similar contents for the inner layer within 0,1 to 0,2 % of dry weight. When one sample was removed and the temperature then lowered for a further period, samples removed later showed lower water content. In all cases the moisture content (ice and water) of the outer soil layers (Fig. 1) was increased.

VOLUME MEASUREMENTS

The results of investigations on Leda clay KNB are shown in Fig. 3. From the volume determinations the dry density (weight dry material per unit volume of soil in moist condition, g/cm^3) was calculated and plotted (Fig. 3, crosses) as a function of water content. Consolidation of the samples from the inner layer of the membrane assembly occurred during freezing. This is clearly shown by comparison with results of oedometer tests on similar material (Fig. 3, circles).

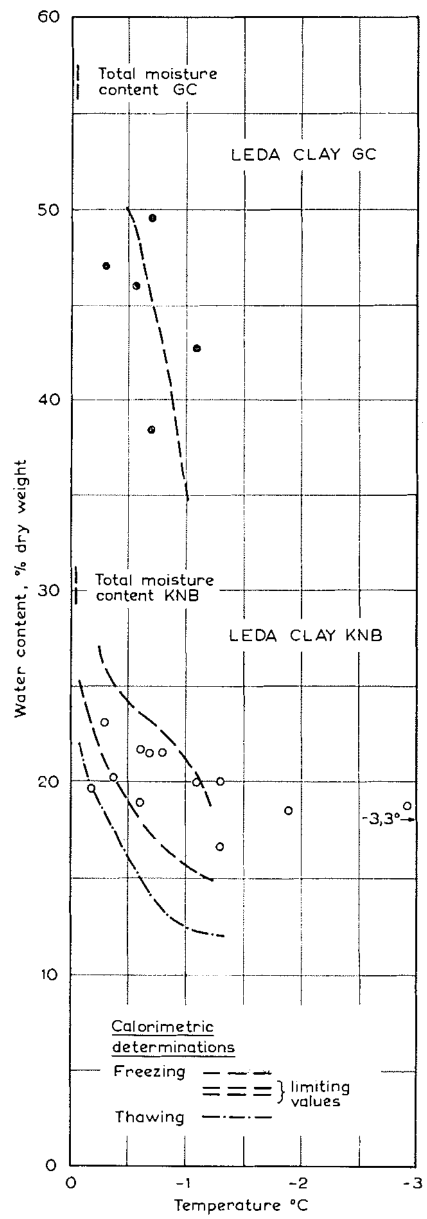


Fig. 4. Equilibrium water contents of inner layers (where ice formation did not occur). When two or three assemblies with the same soil were frozen simultaneously, the average water content is shown. Also shown are calorimetrically determined curves of (unfrozen) water content at various temperatures. In these experiments the soils were frozen normally and therefore also contained ice, making total moisture contents as shown.

If the material is saturated there is a unique relationship between dry density, water content, and effective stress, for the observations shown in Fig. 3. The specific weight of the soil particles was found to be 2,78 g/cm³, and from volumetric consideration the dry density and water content relationship for the saturated state was calculated (Fig. 3, dashed line). The relationship of the observed points to this line shows that the soil was saturated over the range shown.

According to the effective stress equation

$$\sigma' = \sigma - u \qquad \text{(1)}$$

where

- σ' effective stress
- σ total stress (equals applied load in oedometer)
- u pore water pressure,

the load applied in the oedometer is equal to the effective stress (which is responsible for consolidation) after equilibrium is reached, because the pore water pressure is then atmospheric *(i.e.* zero). The relationship between dry density (and moisture content) and effective stress is thus given by the oedometer tests. The figure for effective stress shown beside each point in Fig. 3 obtained from the oedometer tests, illustrates this relationship.

The magnitude of effective stresses to which samples from the membrane experiment have been subjected is indicated by those stresses that in the oedometer tests produced a similar moisture content and dry density. In the absence of external loading (as in the membrane freezing experiment) the effective stress is equal, but of opposite sign, to the pore water pressure. The positive effective stress giving rise to consolidation arises from, and is numerically equal to, the negative pore pressure developed by freezing.

RELATIONSHIP OF TEMPERATURE TO NEGATIVE PORE PRESSURE

Negative pore pressure as a function of temperature cannot be determined from the few observations in Fig. 3. However, if the water content of the inner layer from the membrane experiment was known as a function of temperature with sufficient accuracy, it would be possible to combine this with the information in Fig. 3 to obtain values of negative pore pressure as a function of temperature. The water content of the inner (ice-free) layer, determined in many tests is shown in Fig. 4 (points and circles) as a function of temperature. In general, the water content is as expected less for lower temperatures. In addition to the two soils illustrated, tests were also made on an illitic clay from Åsrum, Norway, and a bentonite from Winnipeg, Canada. These tests also showed decreased water content of the inner layer for lower temperature. From the information in Fig. 3, it should be possible to find the negative pore pressure (which is numerically equal to the effective stress causing consolidation) corresponding to the different water contents of the inner layer (Fig. 4) and thus to temperature. This direct procedure, while giving the approximate magnitude of the pore pressures, is not, however, entirely satisfactory. The reasons for this apparent from earlier experimental work (pp. 11–26) which can however be used to obtain a more accurate approximation of the temperature-negative pore pressure relationship.

Interpretation of observed water contents from calorimetric studies of frozen soils

In this earlier work, calorimetric methods were used to determine the quantity of water remaining unfrozen at various negative temperatures. Results of this type can be expressed as

$$\frac{\text{weight of water remaining unfrozen}}{\text{dry weight of soil}} \times 100$$

and then shown on Fig. 4. It is seen that at least to about $-1\,^\circ C$, the water content of the inner (ice-free) layer in the membrane experiments is, within the limits of experimental error, equal to the water content of 'normally' frozen soil (in which ice, of course, is also present). During the freezing process in the membrane experiment, water is transferred from the inner layer to the outer layers (where ice is formed) because of a pressure gradient in the water. When the water content of the inner layer becomes constant, the water in both inner and outer layers would be expected to have the same negative pressure. Thus the situation of the inner soil layer, isolated by membranes, does not differ fundamentally from parts of the normally frozen soil, in which ice may happen to be absent. The inner soil layer has the same (unfrozen) water content under the same negative pressure.

In the calorimetric investigations it was also found that the unfrozen water content as a function of temperature has, because of hysteresis, somewhat different values depending on whether the soil is in process of freezing or thawing (Fig. 4). In the membrane experiment, similar small differences in water content are to be expected depending on whether the measured temperature of the refrigerator is reached by cooling or slight warming. The temperature control of the refrigerator was such that fluctuations of $\pm 0{,}1\,^\circ C$ might occur and these are largely responsible for the scatter in the points shown. In addition, the water content was to some extent dependent on the degree of disturbance of the sample.

This relatively small scatter in the observed points from the membrane experiment is sufficient to prevent an accurate calculation of the relationship between temperature and effective stress. Fig. 3 shows that small changes in water content are associated with very large changes in effective stress, so that it is necessary to know the water content-temperature relationship (Fig. 4) for the freezing condition with a high degree of accuracy.

Determination of temperature – negative pore pressure relationship

Pressure effects are generally accepted as mainly responsible for the presence of unfrozen water in frozen soils. Earlier work (pp. 11–26) provided a quantitative evaluation of these effects. Suction-moisture content relationships (Croney, Coleman and Bridge, 1952) were determined at room temperature for the soils investigated calorimetrically. A hypothesis was proposed that as freezing occurred and water was transferred into ice lenses, the water remaining in the pores would be under an increasing suction (negative pore pressure). This suction could be predicted from the suction-moisture content tests and is probably responsible for the presence of the unfrozen water because of its effect on the freezing point of the latter. The suction predicted for each unfrozen water content was found to be related to the temperature by an equation of the type given by Schofield (1935).

$$H = \frac{L}{Tg}\Delta T \qquad \text{(2)}$$

where

- H suction expressed as height of a column of water, cm
- L latent heat of freezing of water $3{,}336 \times 10^9$ erg/g
- T temperature °K
- ΔT negative temperature °C
- g 981 cm/sec²

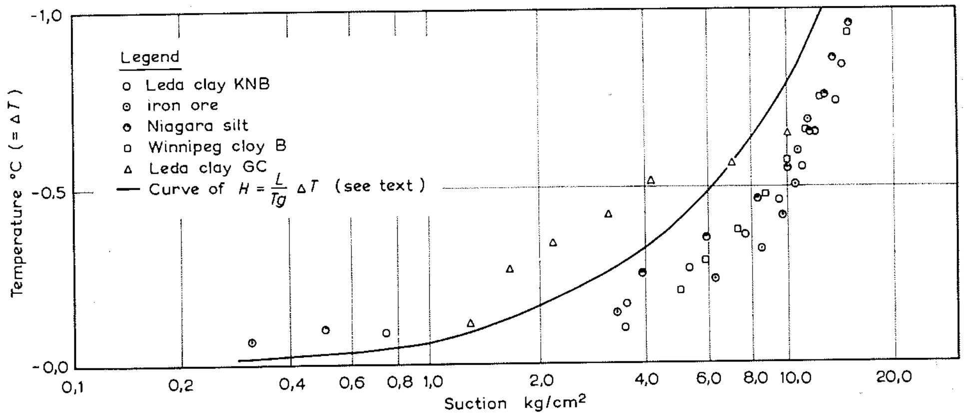


Fig. 5. Theoretical and experimental relationship between temperature and suction.

This relationship could only be determined following a large number of carefully controlled tests on various soils for reasons similar to those noted above. Results are summarized in Fig. 5 (modified from pp. 11–26). Except for a necessary minor correction to allow for the effect of dissolved salts on freezing point, the equation is apparently valid for all soils. The equation gives the freezing-point depression of water under negative pressure, which is in contact with ice under atmospheric pressure (Edlefsen and Anderson, 1943). Although it appears unlikely at first, this situation probably occurs in soils where the ice phase is mostly in discrete lenses or masses larger than pore size, while the water lies within small pores under the influence of capillary and other effects.

TEMPERATURE – NEGATIVE PORE PRESSURE: CONCLUSIONS

Although the earlier work did not show directly the existence of the negative pore pressures, the present experiments clearly demonstrate these pressures and also confirm that their magnitude is similar to that predicted from equation 2. A thermodynamic equation of the type of (2) does therefore correctly describe the temperature-pressure relationship for water freezing in soils. On the basis of the detailed observations made in the earlier work it can then be concluded that equation 2 gives the best available approximation of the relationship between negative temperature and negative pore pressure (and thus effective stress) in unconfined samples.

Applicability of equation 2

There is an important limitation to the present experiments in proving the range of application of Equation 2 which remains to be discussed. This is illustrated by Fig. 6, where the water content for samples in oedometer and suction tests is shown as a function of effective stress, (equal to the applied load, and applied suction respectively). So long as the soil is saturated these tests give, with minor qualification, similar results. At 21 % water content the two sets of results diverge. This is the shrinkage limit, and desaturation occurs in the suction tests. In Fig. 4 this is reached in freezing to about –1° C. On freezing to lower temperatures the membrane experiments always gave water contents substantially higher than those determined calorimetrically. This is probably because when desaturation occurs, hydraulic flow is substantially reduced. Transference in the vapor phase may occur, but even after three weeks at the low temperatures the water content remained higher in the membrane experiment.

The information in Fig. 3 can not be extrapolated to apply beyond the shrinkage limit (which in the case of Leda clay KNB is reached at about –1° C).

Pressures on the ice phase

Agreement of the experimental observations with Equation 2 implies that, as noted, the pressure on the ice (at least where it is in contact with the water) is always atmospheric. Most of the ice is in bodies considerably larger than pore size, and might be supposed to carry the effective stress, especially at points of 'contact' with soil grains (ice and grains may be separated by a bound film of water). The existence in the ice of a positive presure greater than atmospheric is in conflict with the evidence in Figs. 3–5, even when allowance is made for experimental errors.

Curvatures of the ice surface of small radii should cause increments or decrements of pressure locally within the ice (Everett, 1961; Gold, 1957). It is suggested that the ice surface will be concave over grains and convex into pores, with radii such that local stresses are relieved and the ice is uniformly at atmospheric pressure. Work now being carried out involves freezing tests on samples under various externally applied total-stress conditions and may give further information on this point.

CONCLUSIONS

Experiments involving freezing of soil samples, with restricted ice lens development show that water which remains unfrozen within a freezing soil has a negative pressure. This negative pore pressure gives rise to an effective stress which causes consolidation. The volume decrease will usually be obscured by ice lens growth.

Negative pore pressure is greater at lower temperatures. Unfrozen water in saturated soil moves freely along pore pressure gradients. The experiments confirm, for temperatures down to that at which the moisture content corresponds to shrinkage limit, that an equation of the type proposed by Schofield gives approximately the suction (negative pore pressure), in unconfined samples, as a function of temperature.

The effective stress developed, even at temperatures of –0,8° to –1,0° C, causes considerable consolidation in compressible soils. This, together with discontinuities left by ice lenses, results in a special structure in soils subjected to a freeze-thaw cycle.

ACKNOWLEDGMENTS

I am indebted to my colleagues at the Norwegian Geotechnical Institute, J. Moum and O. Ihlen Sopp, and to the Director, Dr. L. Bjerrum for assistance and stimulating discussion.

REFERENCES

Institut merzlotovedeniya (1953–57) *Materialny po laboratornym issledovaniiam merzlykh gruntov*. Vol. 1–3. Akademiya nauk, SSSR. Moscow.

Croney, D., Coleman, J. D. and Bridge, P. M. (1952) *The suction of moisture held in soil and other porous materials*. London. 41 p. (Road Research Laboratory, Harmondsworth. Road research technical paper, 24)

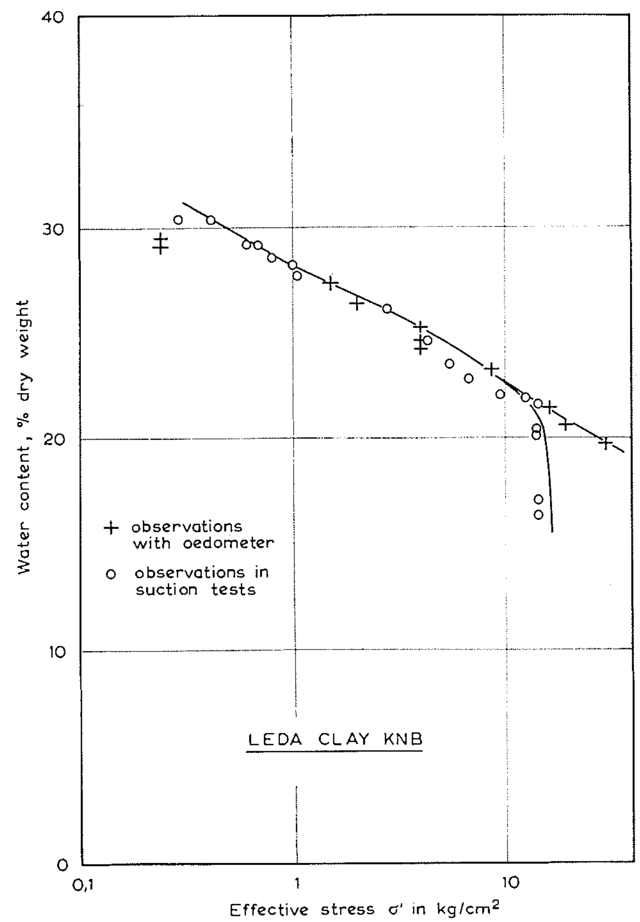


Fig. 6. Comparison of observations from suction-moisture content tests and from oedometer tests.

Edlefsen, N. E. and Anderson, A. B. C. (1943) *Thermodynamics of soil moisture.* Hilgardia. Vol. 15, No. 2, p. 31–298.

Everett, D. H. (1961) *The thermodynamics of frost damage to porous solids.* The Faraday Society. Transactions. Vol. 57, Pt. 9, p. 1541–1551.

Gold, L. W. (1957) *A possible force mechanism associated with the freezing of water in porous materials.* Highway Research Board, Washington D.C. Bulletin, 168, p. 65–73. (National Research Council, Canada. Division of Building Research. Research paper, 66)

Lovell, C. W. (1957) *Temperature effects on phase composition and strength of partially-frozen soil.* Highway Research Board, Washington D.C. Bulletin, 168, p. 74–95.

Richards, L. A. (1947) *Pressure-membrane apparatus; construction and use.* Agricultural Engineering, Vol. 28, No. 10, p. 451–454, 460.

Schofield, R. K. (1935) *The pF of the water in soil.* International Congress of Soil Science, 3. Oxford 1935. Transactions, Vol. 2, p. 37–48.

Williams, P. J. (1963) *Specific heat and unfrozen water content of frozen soils.* Ottawa. (National Research Council, Canada. Associate Committee on Soil and Snow Mechanics. Technical memorandum, 76, p. 109–126)

Williams, P. J. (1964) *Unfrozen water content of frozen soils and soil moisture suction.* Géotechnique, Vol. 14, No. 3, p. 231–246.

UNFROZEN WATER IN FROZEN SOILS:

PORE SIZE – FREEZING TEMPERATURE – PRESSURE RELATIONSHIPS

by P. J. WILLIAMS

SYNOPSIS

Experimental observations relevant to the freezing point of water in soils and the pressure state of liquid water in frozen soils are reviewed. By reference to vapour pressures, it is first shown as a general example, how with a curved interface between the air (and vapour) and water, the system ice-water-vapour may be in equilibrium at temperatures below 0° C while the pressure on the ice and air remains atmospheric.

The situation in a soil where the pore size restricts the size of ice crystals, and thus determines the curvature of the ice-water interfaces is then considered. An expression is derived relating freezing point depression to pore size and this relationship is shown to be in agreement with the results of earlier experimental observations.

Proceeding from this an equation is obtained relating freezing temperature and pressure in the unfrozezn water. It is shown how this relationship is also substantially confirmed by earlier experimental work.

Finally limitations to the applicability of the two relationships are briefly discussed.

On fait d'abord une revue des observations expérimentales se rapportant au point de congélation de l'eau en sols et les conditions de pression de l'eau liquide en sols gelés. En ce qui concerne les pressions de vapeur, on montre que le système glace-eau-vapeur, avec une surface courbée entre l'air (et vapeur) et l'eau, peut être en équilibre à des témperatures sous 0° C, tandis que la pression sur la glace et l'air reste atmosphérique.

Puis on examine la situation dans un sol òu la dimension des pores limite la dimension des cristaux de glace, et détermine ainsi la courbure de l'interface eau-glace. On arrive à une expression mettant en relation la dépression du point de congélation et la dimension des pores. On montre que ce rapport s'accorde avec des résultats d'observations expérimentales précédentes.

A partir de ce rapport, on a établi une équation entre la température de congélation et la pression dans l'eau non gelée. Il est montré que cette équation est aussi d'une façon pratique confirmée par des observations précédentes. L'article se termine par une courte discussion des limites de l'application de ces deux équations.

INTRODUCTION

Any theory accounting for the presence of unfrozen water in frozen soils must explain: (a) the freezing point depression of such water (b) the fall in pressure in such water associated with the freezing process. That some fall in pressure occurs during freezing at temperatures near 0° C is of course obvious from the migration of water to the frost line, occurring during frost heaving. That the pressure continues to fall with the decreasing freezing point of the remaining water is suggested by the relationship between unfrozen water content for various negative temperatures and the suction associated with a similar moisture content at above-freezing temperatures (Fig. 8, p. 21). It is substantially proved by observations of the consolidation, associated with increasing negative temperatures, produced in clay samples from part of which ice is excluded (pp. 27–35).

Earlier explanations of the fall in pore pressure have involved the assumption of an unstable supercooling of the soil water (Jackson and Chalmers 1958, Selmer-Olsen 1964), or have been based on effects associated with a stable freezing point depression resulting from the geometry of the soil particle and pore system (Miller *et al.*, 1960, Penner 1958, Gold 1957). In the experiments involving unfrozen water content (pp. 11–26) it was shown that within the limits of experimental error, the amount of unfrozen water present for a given temperature was independent of the rate of freezing. This, together with the widespread distribution of ice occurring in the frozen soil, indicates that supercooling does not occur to a marked extent. In the following it is shown that both qualitatively and quantitatively the presence of the unfrozen water, and the associated lowered pore water pressure with decreasing temperature, may be substantially explained on the basis of capillary theory and the assumption of thermodynamic equilibrium conditions.

Vapour-pressures, ice and water at negative temperatures

Under equilibrium conditions at a pressure of one atmosphere, ice, water and vapour are in equilibrium at 0° C so long as the interfaces gas-liquid, liquid-solid, solid-gas are essentially plane. If the temperature is reduced while a uniform one atmosphere pressure is maintained on all phases equilibrium is disturbed. The saturated vapour pressure falls but is no longer equal for ice and water. The vapour pressure for ice at temperatures below 0° C is lower than that for water and thus there is a transfer from the water to the ice phase. It is well known that when a water-air interface is curved as in a capillary, the vapour pressure above the meniscus is reduced in respect to that of free water. The reduction of vapour pressure (given by the Kelvin equation (Thomson, 1871, Defay and Prigogine, 1951, p. 176)) is ascribable to the fall in pressure produced in the water, as a result of the interfacial energy and radius of curvature r_{lg} of the interface. According to the well known capillary equation (see *e.g.* Taylor, 1948):

$$p_g - p_l = \frac{2\sigma_{lg}}{r_{lg}} \qquad \text{(1)}$$

where p_g = air pressure
p_l = water pressure
σ_{lg} = interfacial energy air-water
r_{lg} = radius of curvature of interface ($\approx$ radius of capillary)

The vapour pressure of water in a capillary may thus be equal to that of bulk ice (*i.e.*, ice with an essentially plane surface), if the capillary is small enough, the required radius of the capillary depending on the temperature of the ice. The water and ice are again in equilibrium; the water remains unfrozen and therefore shows a freezing point depression. As shown above the pressure of the water is also reduced relative to the ice (which has atmospheric pressure).

A similar situation exists in frozen soils, except that in this case the water phase is, in general, in direct contact with the ice, *i.e.* the curved interface water-ice must be considered.*

The internal pressure of small crystals

The internal pressure of a small crystal immersed in its own melt is given by (see *e.g.* Defay and Prigogine, 1951):

$$p_s - p_l = \frac{2\sigma_{sl}}{r_{sl}} \qquad \text{(2)}$$

* For an interface contained in a capillary, the right hand term in equations of the type of (1) should, strictly speaking, be multiplied by a factor, cos θ, where θ is the contact angle between the liquid and the walls of the capillary. The value of cos θ is generally considered to be close to 1 for water in soils.

where p_s = internal pressure of crystal
p_l = pressure of liquid
σ_{sl} = interfacial energy, solid-liquid
r_{sl} = radius of curvature of interface

(or more precisely, $p_s - p_l = \sigma_{sl}\, dA/dV$, where A = surface area of crystal, V = volume of crystal – but we assume here a spherical crystal, or an interface which is hemispherical).

In the freezing of a soil, the size of ice crystals depends on the size of the pore in which they lie. According to equation (2) there will be a difference in pressure between the ice in such crystals, and the water in the pore.

The internal pressure of a crystal in its gas is given by:

$$p_s - p_g = \frac{2\sigma_{sg}}{r_{sg}} \qquad \text{(3)}$$

where σ_{sg} = interfacial energy solid-gas
p_g = pressure of gas
r_{sg} = radius of curvature of interface

RELATIONSHIP OF FREEZING TEMPERATURE AND PORE SIZE

For a one component system of gas-solid-liquid to remain in equilibrium when pressure or temperature is changed, the Gibbs-Duhem relationship (see *e.g.* Guggenheim 1959) must be satisfied:

$$-S_g\, \delta T + V_g\, \delta p_g = \delta\mu \qquad \text{(4)}$$

$$-S_l\, \delta T + V_l\, \delta p_l = \delta\mu \qquad \text{(5)}$$

$$-S_s\, \delta T + V_s\, \delta p_s = \delta\mu \qquad \text{(6)}$$

where S_g, S_l, S_s = molar entropies of the three phases
T = absolute temperature
V_g, V_l, V_s = molar volumes of the three phases
p_g, p_l, p_s = pressures of the three phases
μ = molar chemical potential

The existence of curved interfaces will be associated with particular pressures and temperature of equilibrium, depending on their radii*. The Gibbs-Duhem relationships together with equations of the type of (1), (2) and (3) may be used to demonstrate how the equilibrium temperature is related to the radius of curvature of the solid-liquid interfaces, and therefore for a substance contained in an inert porous material to the size of the pores in which the interfaces lie.

Subtracting equation (5) from (6) and dividing by $V_l - V_s$

$$\frac{(S_l - S_s)}{V_l - V_s}\, \delta T + \frac{V_s \delta p_s}{V_l - V_s} - \frac{V_l \delta p_l}{V_l - V_s} = 0 \qquad \text{(7)}$$

* A detailed study of the influence of curvature on phase equilibria is given by Defay and Prigogine (1951).

and subtracting equation (4) from (6) and dividing by $V_g - V_s$

$$\frac{(S_g - S_s)}{V_g - V_s}\,\delta T + \frac{V_s \delta p_s}{V_g - V_s} - \frac{V_g \delta p_g}{V_g - V_s} = 0 \qquad (8)$$

adding $\dfrac{V_l \delta p_s}{V_l - V_s}$ to both sides of equation (7)

$$\frac{(S_l - S_s)}{V_l - V_s}\,\delta T + \frac{V_l \delta p_s}{V_l - V_s} - \frac{V_l \delta p_l}{V_l - V_s} = \frac{V_l \delta p_s}{V_l - V_s} - \frac{V_s \delta p_s}{V_l - V_s} \qquad (9)$$

adding $\dfrac{V_g \delta p_s}{V_g - V_s}$ to both sides of equation (8)

$$\frac{(S_g - S_s)}{V_g - V_s}\,\delta T + \frac{V_g \delta p_s}{V_g - V_s} - \frac{V_g \delta p_g}{V_g - V_s} = \frac{V_g \delta p_s}{V_g - V_s} - \frac{V_s \delta p_s}{V_g - V_s} \qquad (10)$$

since (from equations (2) and (3)): $\delta p_s - \delta p_l = \delta \dfrac{2\sigma_{sl}}{r_{sl}}$ and $\delta p_s - \delta p_g = \delta \dfrac{2\sigma_{sg}}{r_{sg}}$,

and subtracting (10) from (9)

$$\left(\frac{S_l - S_s}{V_l - V_s} - \frac{S_g - S_s}{V_g - V_s}\right)\delta T = -\frac{V_l}{V_l - V_s}\,\delta\,\frac{2\sigma_{sl}}{r_{sl}} + \frac{V_g}{V_g - V_s}\,\delta\,\frac{2\sigma_{sg}}{r_{sg}} \qquad (11)$$

ignoring V_s in relation to V_g, and multiplying by $(V_l - V_s)$:

$$\left\{S_l - S_s - \frac{S_g - S_s}{V_g}(V_l - V_s)\right\}\delta T = -V_l \delta\,\frac{2\sigma_{sl}}{r_{sl}} + (V_l - V_s)\delta\,\frac{2\sigma_{sg}}{r_{sg}}$$

since $(S_l - S_s)T = L_{fus}$

and $(S_g - S_s)T = L_{subl}$

where L_{fus} = latent heat of fusion

L_{subl} = latent heat of sublimation

and integrating (assuming V_l, V_s and L_{fus} to be constant) gives:

$$\left(L_{fus} - \frac{V_l - V_s}{V_g} L_{subl}\right) \ln \frac{T}{T_0} = -\frac{V_l 2\sigma_{sl}}{r_{sl}} + \frac{(V_l - V_s)2\sigma_{sg}}{r_{sg}} \qquad (12)$$

where T_0 is the normal equilibrium temperature.

If $\dfrac{V_l - V_s}{V_g} L_{subl}$ is small in relation to L_{fus}:

$$L_{fus} \ln \frac{T}{T_0} = -\frac{V_l 2\sigma_{sl}}{r_{sl}} + \frac{(V_l - V_s)2\sigma_{sg}}{r_{sg}} \qquad (13)$$

In the case of the soil samples investigated calorimetrically (see the first two papers in this volume), a constant air pressure of one atmosphere occurs. Since the normal freezing point of water (*i.e.* when there is no significant curvature of the interfaces between the phases) at one atmosphere pressure is 273,15° K (0° C), this temperature is to be taken for T_0, in performing the integration from $1/r_{sl} = 0$ to $1/r_{sl}$, the suffix $_{sl}$ referring to the ice-water interface. $1/r_{sl} = 0$ corresponds to the initial state when the

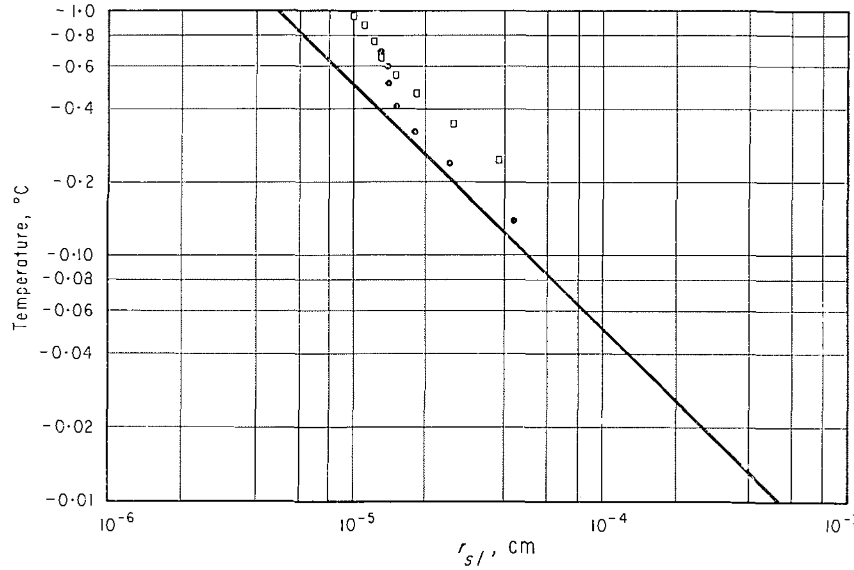


Fig. 1. Relationship between r_{sl} ($\approx$ pore radius) and the temperature at which ice can occur in the pore; values for Niagara silt (open squares) and iron ore (circular dots) determined experimentally, and those predicted by equation (14).

saturated soil commences to freeze and both ice and water are under atmospheric pressure, *i.e.* $r_{sl} = \infty$.*

In deriving equation (13) the relation

$$\delta p_s - \delta p_g = \delta \frac{2\sigma_{sg}}{r_{sg}}$$

was used. In the case of the soil the gas phase does not consist of water vapour alone, but also of air. The relationship is still valid, the interfacial energy σ_{sg} not being significantly affected by the total pressure, or the presence of air.

In the freezing of saturated soils the interface between ice and air is also essentially plane (lying at the surface of the sample) and $r_{sg} = \infty$. The last term in equation (13) is therefore equal to 0*, and the equation relating temperature T and the radius of the ice-water interfaces, and thus the radius of the smallest pores filled with ice, becomes:

$$\ln \frac{T}{T_0} = -\frac{V_l 2\sigma_{sl}}{r_{sl} L_{fus}} \quad \ldots\ldots (14)$$

taking the interfacial energy ice-water $\sigma_{sl} = 30{,}5$ ergs/cm² (after Hesstvedt, 1964) and $L_{fus} = 6{,}01 \times 10^{10}$ erg/mol, values of r_{sl} can be calculated, and are shown in Fig. 1 (continuous line). It may be noted that a rather similar equation, the Thomson equation, is often used in meteorology and other applications. An analogous equation, in terms of vapour pressures is given by Powers and Brownyard (1947), in their studies of hardened portland cement paste, while equation (14) was given by Kubelka (1932) on the basis of somewhat different assumptions.

* For this to be strictly true the interfaces must be outside the pores, and therefore it must be assumed that some ice can form outside the pores.

Experimental confirmation of freezing temperature – pore size relationship (equation 14)

Certain of the experimental observations summarized (fig. 8, p. 21) may be used to confirm equation (14). Each experimental point in that figure was obtained by first determining the amount of unfrozen water present in a particular frozen soil at a temperature T. As reasoned above, the ice-water interfaces will have a radius r_{sl} (according to equation 14), and should lie in pores of equivalent size. Considering now only those soils which are virtually incompressible, the value r_{sl} can be determined by reference to the suction (shown as pF in fig. 8, p. 21) measured in a suction-moisture content test, in the pore water in an unfrozen sample of the soil when containing exactly the same amount of water. In the latter situation, menisci should lie in the same pores, but would be those for air-water. It is generally agreed (*e.g.* Schofield 1938, Donat 1937) that the suction for a given moisture content in the suction moisture content test is a measure of the radius of the menisci and thus of the pores in which they lie according to equation (1).

Fig. 1 shows the results of converting the suction values of the experimental points for various temperatures (fig. 8, p. 21) for the two incompressible soils, Niagara silt and iron ore, to pore size by application of equation (1). There is seen to be close agreement between the theoretical relationship (of equation 14), and these experimentally determined values. The latter are slightly larger, and the probable reason for this is discussed in a later section (p. 46).

PRESSURE STATE OF THE UNFROZEN WATER

The pressure state of the unfrozen water at various freezing temperatures may be directly calculated by combining equations (14) and (2) to give:

$$p_s - p_l = -\frac{L_{fus}}{V_l} \ln \frac{T}{T_0} \qquad (15)$$

where $p_s - p_l$ = dyn/cm² ($\times 1{,}0198 \times 10^{-6}$ = kg/cm²)
L_{fus} = molar latent heat, ergs (= dyn cm)
V_l = molar volume of water cm³

This equation in fact corresponds to that given by Schofield in his paper (1935)* concerning initial freezing points of partly dried soils and reproduced in fig. 8, p. 21. A similar equation may be derived directly from the Clausius-Clapeyron equation (Edlefson and Anderson 1943) but the derivation given above demonstrates explicitly the effect of the porous structure of the soil in controlling the freezing process.

The same condition applies to equation (15) as applies in the derivation of equation (14): The sample is saturated so that the last term in equation (13) is equal to 0. Further, if the soil is subjected to one atmosphere air pressure $T_0 = 0°$ C. If both conditions apply, the pressure of the ice p_s is necessarily atmospheric, and the difference $p_s - p_l$ is equal to the negative pore water pressure (or suction). If during the freezing of a saturated sample the pressure on the system as a whole was greater than atmospheric

* It is however interesting to note that in addition to being misprinted, and expressed in unusual form, Schofields (1935) equation is apparently incorrect in so far as H, the suction in his equation, is considered as that of the water in an unsaturated sample prior to freezing of that sample. If such an unsaturated soil is frozen, the ice-air interfaces must be within the pores. In this case the condition in the derivation of equations (14) and (15), $r_{sg} = \infty$ is not met.

then T_0 would have to be reduced by 0,0073° C per kg/cm^2 of this pressure which also occurs in the ice. The difference $p_s - p_l$ would have the same value (*i.e.* with the menisci of the same radius r_{sl}) since the interfacial energy σ_{sl} is virtually constant for the temperatures and pressures of practical interest. Thus the pressure on the water would be increased by an equal amount (*i.e.* the negative pore pressure reduced).

For a soil which is not saturated, the solid-gas menisci lie within the pores and their radius r_{sg} determines the increased pressure of the ice relative to the air pressure. This increment of pressure is represented by the last term in equation (13). In the case of unsaturated soils therefore this term would also have to be carried into equations (14) and (15).*

One situation of particular significance should be mentioned in connection with the use of equation (2) in the derivation of equations (14) and (15). The value of the pore water pressure at the boundary between frozen soil in the ground, and the underlying unfrozen soil may be controlled by a variety of factors (*e.g.* depth to ground water table). This pore water pressure when inserted in equation (2) may be such as to give a value of r_{sl} which is greater than that of the pore radius of the soil in question, and so long as this persists ice cannot form within the pores. Instead freezing gives rise to ice lenses larger than pore size (p. 52).

Experimental confirmation of temperature-suction relationship

The experiments described on pp. 11–26 appeared to confirm that the values $p_s - p_l$, the difference in pressure between ice and water phases, are substantially those predicted by equation (15) (shown in its alternative form, p. 21). Although it was proposed there that $p_s - p_l$ was in fact equal to the negative pore water pressure developed (and that the ice accordingly was under atmospheric pressure) this was not proved by these experiments. Had some or all of $p_s - p_l$ found expression as an increased pressure in the ice, the temperatures (cf. fig. 8, p. 21) would have differed by an amount (as noted above) too small to be experimentally detectable. The occurrence of negative pore pressures approximately equal to $p_s - p_l$, in samples under one atmosphere pressure, is proved clearly however, by the consolidation effects observed in the further experiment (pp. 27–35).

When the derivation of equations (14) and (15) is considered in relation to certain well-known soil properties, a fuller theoretical interpretation of the experimental observations (summarised in Fig. 8, p. 21) is possible. The causes of certain deviations of the experimental points from the theoretical curve in that figure become apparent, and the experimental values can then be corrected to give a more precise verification of equation (15).

It is necessary to distinguish three categories of soil: incompressible, 'completely' compressible, and partly compressible (a similar distinction is considered by Miller, 1966).

Incompressible soils: These soils have a rigid "skeleton" of mineral particles, such that removal of water (in the suction-moisture content test) is necessarily associated with entry of air into the pores. There is no change in the bulk soil volume. Using similar reasoning as above (p. 42), for similar water (liquid water) contents in frozen soil, and in soil in the suction-moisture content test, the interfaces (ice-water, air-water respectively) lie in the same pores. Thus $r_{sl} = r_{lg}$ and by combining equations (1) and (2) it is

* See however p. 97.

apparent that the suction in the case of the frozen soil is smaller by a factor of σ_{sl}/σ_{lg} = 0,42. (taking σ_{lg} as 72,75 dyn/cm at 20° C). In Fig. 2 the suction values for the two incompressible soils, Niagara silt and iron ore, (fig. 8, p. 21), have been replotted after multiplying by this factor.

'Completely' compressible soils: By contrast, in many soils, the volume of water that is lost following the application of even a fairly large suction, is associated with an equal volume decrease of the bulk soil. The reason of course, is that the suction gives rise to stresses sufficient to compress the soil (see p. 27), while no air (or ice) enters the soil pores because all the pores are smaller than r_{lg} (or r_{sl}) as given by equation (1) or (2).

For such soils (*i.e.* those where pores are not emptied or filled with ice) there is a unique relationship between effective stress (or suction, cf. p. 30) and water content. Therefore equal water (liquid water) contents in the frozen soil and in the suction-moisture content test must also show equal suction. The suction values shown earlier (fig. 8, p. 21) should in this case equal those existing in the unfrozen water in the frozen soil, and are replotted directly in fig. 2.

From equations (1) and (2) it is apparent for these soils that the radii r_{lg} and r_{sl}, of the interfaces, cannot be equal (under the condition of equal water content) in contrast to the situation described for incompressible soils. The menisci do not lie in pores, but, in the suction-moisture content test remain on the surface of the soil, and in frozen soils lie on the surface of ice lenses (larger than pore size) on or in the soil.

However, when the suction becomes great enough (according to equation (1) or (2)) the radius of the interface becomes small enough to enter the pores. In the case of Leda clay KNB this is clearly shown to occur in the suction-moisture content test, at a suction of about 11 kg/cm² (fig. 6, p. 34). According to this, ice-water menisci would be expected to enter the pores (following the reasoning given in the previous sub-section) at about $0{,}42 \times 11 = 4{,}6$ kg/cm² suction, corresponding to a temperature of about −0,37° C. This is therefore the lowest temperature for which that soil can be regarded as 'completely' compressible. Corresponding information is not available for Leda Clay GC although approximately the same figure probably applies, while for the montmorillonitic Winni-

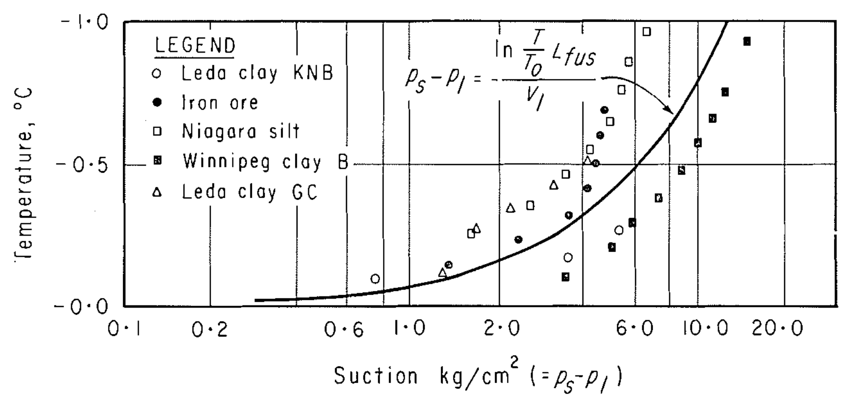


Fig. 2. Relationship between temperature and suction in the liquid water in freezing soils, determined experimentally, and as predicted by equation (15). In the preparation of this figure no account has been taken of the possibility of the emptying of 'isolated' pores in the suction-moisture content test (see p. 45). If this occurs and were allowed for, the majority of the experimental points would lie nearer the theoretical curve.

peg Clay B it is reasonable to suppose that the pores are smaller and that ice entry into the pores does not occur in the range 0 til –1 °C.

Partly compressible soils: Once ice-water interfaces occur within the pores it might be thought that the considerations valid for incompressible soils would apply. However, entry of ice or air does not necessarily prevent further compression of the soil. Thus we have the final group of soils in which the interfaces lie within the pores, but in which a reduction of moisture content occurs partly by advance of the interfaces into smaller pores, and partly by compression of the bulk soil.

Conversion of the suction in a suction-moisture content test, to the suction existing for the same liquid water content in the frozen soil, involves in this case a factor between 1 and 0,42, that is, between the two extreme cases considered in the previous sections. The value to be given to this factor, for a particular soil, could only be established by detailed observations of the compression behaviour of the soil over a range of suctions. However it may be noted that once ice or air has entered the pores in some quantity, it is unlikely that much further compression occurs (Haines, 1923). Thus in the majority of cases use of the factor 0,42 (with the assumption that the soils are virtually incompressible) would probably not involve too great an error. However this category of soils has not been included in Fig. 2.

It may be noted that most soils will be represented in at least two of the three categories depending on the suction occurring in them*. Thus at very low suctions, the two soils considered as incompressible, Niagara silt and iron ore, may lose water without interfaces entering the pores, and thus should be regarded as 'completely' or at least partly compressible for a very narrow range of suction.

Fig. 2 shows that there is a reasonable correlation between the theoretically and experimentally derived relationships of pore pressure (suction) and negative temperature. Such deviations as occur may be at least in part explained by the following consideration of the nature of the processes of ice-entry and air-entry into the pores.

Effects of air diffusion in the suction - moisture content test

In Fig. 2 it is seen that many of the experimental points indicate somewhat lower suctions than would be predicted. This could be due to a difference in the manner in which air replaces water in the pores of the soil compared with ice replacing the water.

A number of pores occur, which are connected to others only by much smaller openings. As ice formation takes place in the soil ice entry into such a pore will be delayed because of the small size of the openings. Only when the temperature has fallen sufficiently for stable ice crystals to grow in these openings (as given by equation (14)), will ice form in the pore as a result of nucleation by the crystals advancing through the openings**. Spontaneous nucleation of ice in 'isolated' pores of this type is very unlikely

* On p. 23 it was proposed that the unfrozen water content of soil at various negative temperatures could be predicted from the results of suction-moisture content tests and application of the theoretically determined relationship (*i.e.* of Schofield and equation (15), by assuming equal suction corresponded to equal moisture content. It is now apparent that while this is the case for 'completely' compressible soils, the suction values from the suction-moisture content test should be reduced by a factor of 0,42 for incompressible soils, and by a factor between 1 and 0,42 for partly compressible soils. However as will become apparent in the next section, for other reasons this factor may be ignored in most practical considerations.

** This manner of advance of ice through a porous material is also proposed for the case of hardened cement paste, by Helmuth, 1962.

and in any case would only occur after a considerable period of time. It may be noted that the water in such pores is thus slightly supercooled and represents an exception to the near-equilibrium conditions which are otherwise assumed to exist.

In the case of the suction-moisture content test the replacement of water by air in such pores is probably not hindered in any comparable manner. Air may pass into the pores by diffusion in solution, through the smaller (water-filled) openings, and can come out of solution in the pore when the water is saturated with dissolved air. Entrapped air bubbles can be enlarged in this way and a corresponding amount of drainage occurs (p. 76). Under certain circumstances such bubbles may arise from nucleating sites on the soil particles. Although the rate of diffusion might be thought to be small, the diffusion paths are short in samples of the size used in the suction-moisture content tests, and the time available relatively long.

It is not immediately possible to determine the size of the correction appropriate for the effects of such 'isolated' pores, although an estimate can be made of the amount of pore space that would have to be involved to account for deviations of the size shown in Figs. 1 and 2. In extreme cases the experimentally derived suctions (Fig. 2) are about one half, and correspondingly the experimentally derived pores sizes (Fig. 1) are about twice the theoretical values. Reference to figs. 6 and 7 pp. 19 and 20, shows that if the moisture contents had been greater by up to 10 % (of the volume of water) depending on the soil type and water content, the agreement between experiment and theory would have been exact. This volume can be assumed to correspond to that of 'isolated' pores, water-filled in the freezing tests, but air-filled in the otherwise analogous suction-moisture content determinations.

In Fig. 2 the experimental points for Winnipeg clay B do not lie to the left of the theoretical curve indicating that "isolated" pores which could become air-filled do not occur in that soil. This is quite likely since montmorillonitic clays are known to have very small pores. Thus over the relevant range of suction all the pores are probably too small (according to equation 1) for drainage to occur, whether directly or by diffusion of air.

Taking into account the limits of experimental accuracy of the procedures used to obtain the values shown in Figs. 1 and 2 and the limitations discussed in the next section, it can be concluded that they provide satisfactory proof of the theoretical relationships derived in this paper.

Limitations to the applicability of equations (14) and (15)

For the sake of simplicity certain factors bearing on equations (14) and (15) were overlooked in their derivation. In both equations (2) and (3) the interfaces between the phases were regarded as spherical or hemispherical. In fact, the crystalline form of the ice may require that $2\sigma_{sl}/r_{sl}$ be multiplied by a shape factor ω, the value of which is 1 for a sphere or hemisphere, and lies between 1,1 and 1,3 for different crystal forms (Hesstvedt 1964). Although the value to be given to ω is unknown, it may be noted that if it is other than 1, it would result in a displacement of the theoretical curve (Fig. 1) toward the experimental points.

The values σ_{sl} and L_{fus} are also not strictly constant, varying with the curvature of the interface (Hesstvedt 1964 and 1960). However over the temperature range of interest (as shown in Fig. 1) the combined effects of these variations would be to increase the theoretical values of r_{sl} by at most 1 to 2 percent *(i.e.* at the lower temperatures). σ_{sl} is also to be regarded as a mean value in so far as it varies for different crystal faces.

Throughout this paper, the radius of the interfaces has been assumed to be equivalent to the pore radius without this latter quantity being closer defined. The irregular nature of the voids in soils of course, cannot be defined by one or many 'pore radii'. By pore radius therefore, is to be understood an effective radius of interfaces between phases as determined by the internal geometry of the soil.

It is well known that the layer of water immediately adjacent to the mineral particles in a soil differs in several important respects from that of 'normal' or 'free' water, otherwise present in the pores. The 'thickness' of this layer (the boundary between it and the 'free' water being indistinct), is postulated for example as about $7{,}0 \times 10^{-7}$ cm by Anderson and Low (1958), and by Rosenqvist (1965). The terms V_l, σ_{sl} and L_{fus} may be expected to have substantially different values and to vary in a complex manner within these layers. Similarily equations of the type of (1), (2) and (3) will not be directly applicable.

It is obvious that for large values of r_{sl} and r_{lg} the presence of these layers may be ignored. According to equation (14) at $-1{,}0°$ C, r_{sl} has a value of $4{,}98 \times 10^{-6}$, and the presence of the bound water layers will then mean that the distance between the mineral surfaces will be twice this plus about 15 % corresponding to the thickness of the bound layer. The bound water may be assumed of course to freeze only at substantially lower temperatures.

At temperatures below about $-3{,}5°$ C, the calculated value r_{sl} refers to less than ⅔ of the actual radius of the pore, and the interpretation of r_{sl} in equation (14) as referring to pore size must be modified accordingly. Finally there is the situation in which the soil pores are so small that only 'bound' water occurs. The derivation for equation (14) given in this paper is then no longer relevant.

The experimental points do not extend below $-1°$ C, because at the time the experiments were carried out apparatus was not available for extending the suction - moisture content observations to suctions greater than that corresponding to that temperature. It is quite likely that the relationship shown between the experimentally determined points for various soils, and between these points and the theoretical curve, would in fact continue to substantially lower temperatures. This is because both the unfrozen water content-temperature relationship, and the suction moisture content-relationship are essentially measures of the same quantity, the free energy of the water. The simple physical picture used in this paper becomes, however, progressively less satisfactory for lower temperatures.

ACKNOWLEDGEMENTS

The author's thanks are due to Professor dr. philos. I. Th. Rosenqvist, Dosent dr. philos. Eigil Hesstvedt, cand. real. Johan Moum and siviling. S. E. Mortensen for helpful discussions.

REFERENCES

Anderson, D. M. and Low, P. F. (1958) *The density of water adsorbed by Lithium-, Sodium-, and Potassium-Bentonite.* Soil Science Society of America. Proceedings, Vol. 22, No. 2, p. 99–103.

Defay, R. and Prigogine, I. (1951) *Tension superficielle et adsorption.* Liège, Editions Desoer. 295 pp. Traité de thermodynamique; conformément aux méthodes de Gibbs et De Donder. Vol. 3.

Donat, J. (1938) *Das Gefüge des Bodens und dessen Kennzeichen.* International Society of Soil Science. Commission 6. Zürich 1937. Transactions, Vol. B, p. 423–439.

Edlefsen, N. E. and Anderson, A. B. C. (1943) *Thermodynamics of soil moisture.* Hilgardia, Vol. 15, No. 2, p. 31–298.

Gold, L. W. (1957) *A possible force mechanism associated with the freezing of water in porous materials.* Highway Research Board, Wash. D.C. Bulletin, No. 168, p. 65–73. (Also published as National Research Council, Canada. Division of Building Research. Research paper, No. 66.)

Guggenheim, E. A. (1959) *Thermodynamics.* Fourth Ed. Amsterdam, North-Holland Publishing Co. 476 pp.

Haines, W. B. (1923) *The volume-changes associated with variations of water content in soil.* The Journal of Agricultural Science, Vol. 13, Pt. 3, p. 296–310.

Helmuth, R. A. (1962) *Capillary size restrictions on ice formation in hardened portland cement pastes.* International Symposium on the Chemistry of Cement, 4. Wash. D.C. 1960. Proceedings, Vol. 2, p. 855–869.

Hesstvedt, E. (1960) *On the physics of mother of pearl clouds.* Oslo, Aschehoug. Norske videnskapsakademi. Geofysiske publikasjoner, Vol. 21, No. 9, 32 pp.

Hesstvedt, E. (1964) *The interfacial energy ice/water.* Norwegian Geotechnical Institute. Publication, No. 56, p. 7–10.

Jackson, K. A. and Chalmers, B. (1958) *Freezing of liquids in porous media with special reference to frost heave in soils.* Journal of Applied Physics, Vol. 29, No. 8, p. 1178–81.

Kubelka, P. (1932) *Über den Schmelzpunkt in sehr engen Kapillaren.* Zeitschrift für Elektrochemie und angewandte physikalische Chemie, Vol. 38, No. 8a, p. 611–614.

Miller, R. D. (1966) *Phase equilibria and soil freezing.* International Conference on Permafrost, 1. Lafayette, Indiana 1963. Proceedings, pp. 193–197.

Miller, R. D., Baker J. H. and Kolajan (1960) *Particle size, overburden pressure, pore water pressure and freezing temperature of ice lenses in soil.* Int. Congr. Soil Sci, 1960, p. 122–129.

Penner, E. (1958) *Pressures developed in a porous granular system as a result of ice segregation.* Highway Research Board, Wash. D.C. Special report, No. 40, p. 191–199. (Also published as National Research Council, Canada. Division of Building Research. Research paper, No. 81.)

Powers, T. C. and Brownyard, T. L. (1948) *Studies of the physical properties of hardened portland cement paste.* The American Concrete Institute. Journal, Vol. 18, No. 8, p. 933–992.

Rosenqvist, I. Th. (1965). Private communication.

Schofield, R. K. (1935) *The pF of the water in soil.* International Congress of Soil Science, 3. Oxford 1935. Transactions, Vol. II, p. 37–48.

Schofield, R. K. (1938) *Pore size distribution as revealed by the dependence of suction (pF) on moisture content.* International Society of Soil Science. Commission, 1. [Bangor 1939]. [Transactions], Vol. A, p. 38–45.

Selmer-Olsen, R. (1964) *Forelesningskompendium for grunnkurset i faget alminnelig geologi og ingeniørgeologi.* Trondheim, Tapir. 409 pp.

Taylor, D. W. (1949) *Fundamentals of soil mechanics.* New York, Wiley. 700 pp.

Thomson, W. (1871) *On the equilibrium of vapour at a curved surface of liquid.* The London, Edinburgh and Dublin Philosophical Magazine and Journal of Science, Vol. 42, No. 282, p. 448–452.

PORE PRESSURES AT A PENETRATING FROST LINE AND THEIR PREDICTION

by

P. J. WILLIAMS*

SYNOPSIS

An apparatus has been constructed for determining pore pressures developed immediately below a penetrating frost line. Tests have been carried out with natural soils, graded fractions, and ceramic discs under various confining pressures.

The 'air-intrusion value' of the same materials has also been measured. It is shown that a relationship exists between the air-intrusion value for a given soil, and the pore pressure developed at the penetrating frost line. This can be explained from the effects of pore size in determining the radii of ice–water and air–water interfaces and the relationship suggests a convenient test for determining the susceptibility of a soil to frost heave.

Un appareil a été fabriqué pour déterminer les pressions d'eau développées immédiatement sous la ligne de pénétration du gel dans l'échantillon. On a effectivé des essais avec des sols naturels, particules gradés et des disques de céramique sous diverses pressions.

La valeur d'intrusion de l'air (la pression de l'air nécessaire pour rompre les ménisques capillaires) a également été determinée. Il est démontré qu'il existe un rapport entre la valeur d'intrusion de l'air pour un sol particulier et la pression de l'eau développée sous la ligne de pénétration du gel. Cette relation peut etre expliquée par les effets des dimensions des pores dans la détermination des rayons d'interfaces (ménisques) glace–eau et air–eau, et la rapport propose un essai simple pour la détermination de la susceptibilité d'un sol au soulèvement dû au gel.

INTRODUCTION

Frost heaving in soils is the result of an increase in moisture content, as a result of migration of water to the freezing layer. This additional moisture appears for the most part in the form of discrete lenses or layers in the frozen soil. The migration of water occurs as a result of the hydraulic gradient set up by a fall in pore-water pressure at the lower boundary of the frozen layer. In this Paper, this boundary is referred to as the frost line and when advancing downwards as a penetrating frost line.

According to the well-known Thomson equation very small crystals in their own melt have a lowered freezing point and, at the same time, the internal pressure of the crystals differs from the pressure of the liquid phase. In the freezing of porous materials the pores themselves limit the size of the ice crystals and similar effects are thus observed in the freezing of soils. The pressure difference between the ice and water phases in general finds expression, at least in part, as a fall in the pore-water pressure, and it is this fall in pressure which is responsible for water migration to the freezing soil. The relationship between the pressures of the ice and water, freezing temperature and pore size have been discussed earlier (Williams, 1966(c)).

The pressure difference established on freezing, between the ice and adjacent water is given by

$$p_i - p_w = \frac{d(\sigma_{iw}A)}{dV} \qquad (1)$$

where

p_i = pressure of ice $\qquad p_w$ = pressure of water

A = area of ice–water interface (i.e. surface area of ice crystal(s))

V = volume of ice crystal(s) $\qquad \sigma_{iw}$ = surface tension ice–water

* Research Officer, Soil Mechanics Section, Division of Building Research, National Research Council, Ottawa, Canada. Research Fellow, Norwegian Geotechnical Institute, Oslo, 1963–1965.

If for simplicity, the pores of the soil are considered as cylindrical capillaries and the ice–water interface as hemispherical, then when the interface is within a pore (from Eqn (1)):

$$p_i - p_w = \frac{2\sigma_{iw}}{r_c} \qquad \qquad (2)$$

where r_c = radius of the curved interface ($\approx$ radius of the capillary)[1].

These equations have also been derived by Everett (1961) from theoretical considerations of the chemical potential of the ice and water phases. The stresses on the soil and the soil moisture conditions may be such that $(p_i - p_w)$ is less than that which follows from Eqn (2). This is the situation where an ice lens is forming. A similar equation then applies, but r_c is replaced by r, the radius of an interface which is too large to occur within the pores. For the frost line to penetrate downwards it is required that ice–water interfaces pass through the pores. The value of $p_i - p_w$ must then equal that given by Eqn (2) with r_c having a value corresponding to the pores of the soil in question.

The particular value to be understood by r_c requires consideration. The pores of a soil are not regular capillaries but consist largely of wedge shaped or conical spaces such that curved interfaces with a wide range of radii occur. It has been shown earlier that the interfaces become progressively more curved (lie in smaller openings) as the temperature is reduced (Williams, 1966(c)). Since the temperature at the frost line is the highest occurring in the frozen soil, it might be thought that the value of r_c to be used in Eqn (2) to describe the situation at the frost line should be that corresponding to the largest pores of the soil in question. This is apparently not correct because the largest pores are isolated from one another by smaller openings or 'necks'. It is probable that the penetration of ice further into the soil occurs solely by extension of already existing ice (Williams, 1966(c) and Helmuth, 1962). For ice to form in the largest pores, therefore, the temperature and pressure conditions must be such as to enable the ice to grow through the openings to these pores. Spontaneous nucleation of ice in the pores is very unlikely in view of their small size and the small supercooling involved. It is therefore the size of the 'necks' which largely determines the value r_c which is to be used in Eqn (2) to describe the pressure conditions at the penetrating frost line. More precisely, for a penetrating frost line, the radius r_c is that of the largest *continuous* openings (the soil is considered to be uniformly packed in this consideration, such that cracks or other discontinuities are not considered).

The formation of an ice lens at the frost line represents a pause in the penetration of the frost line, brought about because the temperature and/or pressure conditions are not at once appropriate for development of ice–water interfaces with a radius sufficiently small to pass through the underlying pore 'necks'. The continued growth of the ice lens occurs by migration of water from the underlying soil; when this takes place sufficiently rapidly or over a sufficient length of time, a fall in the pore pressure occurs which is great enough for the ice–water interfaces (according to Eqn (2)) to advance further into the soil.

According to the well-known capillary equation (see e.g. Taylor, 1948) air replaces water in a capillary when:

$$p_a - p_w \geqslant \frac{2\sigma_{aw}}{r_c} \qquad \qquad (3)$$

where

p_a = pressure of air

σ_{aw} = surface tension air–water

r_c = radius of capillary

[1] The radius of the interface is slightly less than that of the pore because of the presence of an adsorbed (unfrozen) water layer on the walls of the pore.

In soil mechanics terminology, the symbol u is normally used for the pore-water pressure. The symbols u_i and u_a will therefore be used henceforth in this Paper for the pressures p_w represented by Eqns (2) and (3) respectively, with r_c being defined for porous materials in the manner explained above. Combining Eqns (2) and (3):

$$\frac{p_i - u_i}{p_a - u_a} = \frac{\sigma_{iw}}{\sigma_{aw}} \qquad (4)$$

so that if σ_{aw} and σ_{iw} are known, measurements of $p_a - u_a$ (Eqn 3) should allow prediction of $p_i - u_i$ provided that the air–water and ice–water interfaces in both cases can be assumed to lie in the same openings (i.e. that r_c is equal in the two cases). Two well-known types of soil test, the suction-moisture content (or pressure membrane) test and those involving measurements of 'capillarity' (or 'height of capillary rise'), are essentially concerned with the relationship shown in Eqn (3). However for reasons which are discussed later, neither gives values of $p_a - u_a$ which are suitable for comparison with values of $p_i - u_i$ at the penetrating frost line. Instead an apparatus incorporating features from both these types of test has been devised for obtaining appropriate values of $p_a - u_a$.

FREEZING EXPERIMENTS

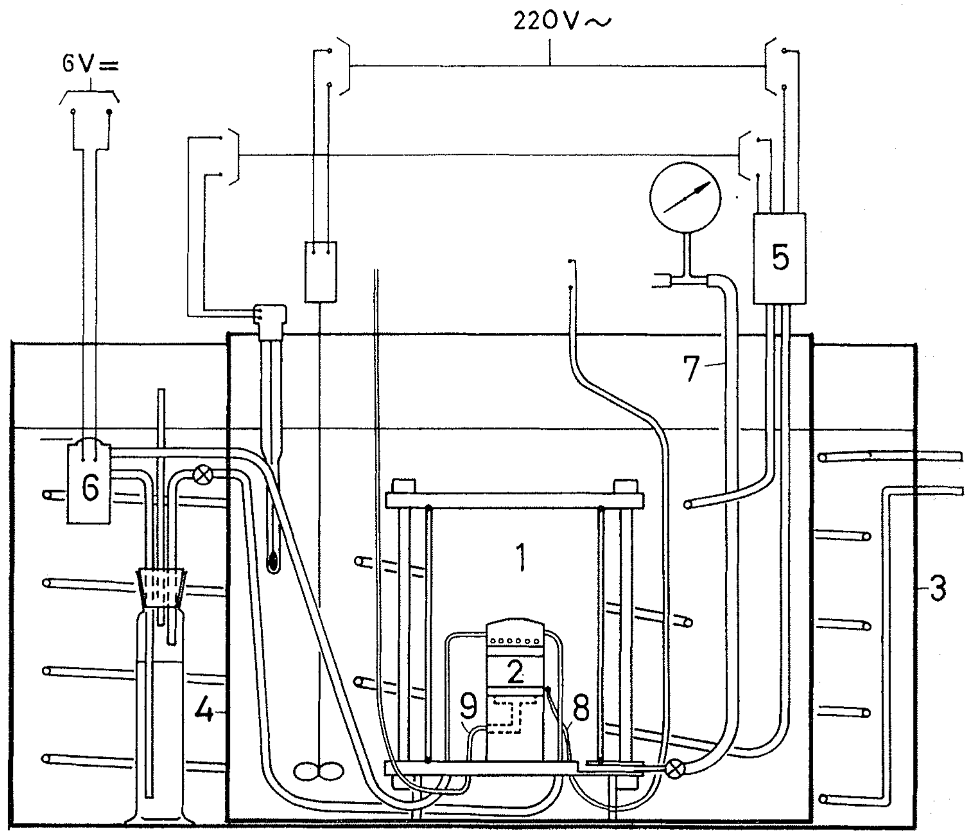
General principle

The aim of the freezing experiments was to determine the pore water pressures at the frost line when this is penetrating through the soil. These determinations were made on various soils and on ceramic filters, and with various total pressures on the sample system. It will be shown also that under appropriate conditions, the pressure on the ice phase is predictable, and thus the quantity $p_i - u_i$ may be obtained.

Measurements of pore pressure changes below a frozen layer have been reported by Beskow (1935), Ruckli (1950), Penner (1957, 1958) and others. In general, mercury manometers have been used to measure the pore-water pressure at the base of a specimen, or within it (Ruckli, 1950). Such manometers cannot be used when the pore pressures are lower than about $-0{\cdot}6$ to $-0{\cdot}8$ kg/sq. cm. Furthermore the permeability of many soils is such that pore pressures measured at a point some distance from the frozen layer may often be substantially different from those at the frost line.

In the present experiments the specimen was placed on a filter of substantially different pore size to that of the specimen; the specimen and filter were saturated with water and the water in the filter connected to a pressure measuring device. The sample was frozen from the top down, and as the frost line penetrated downwards the observed water pressure fell. The permeability of the finer grained samples was such that the observed pressure was initially somewhat higher than that at the frost line. An abrupt change in the observed water pressures ensued when the frost line reached the base of the specimen and ice–water interfaces entered the filter. This was, of course, due to the different pore size of the filter and was in accordance with Eqn (2). The water pressure immediately before this abrupt change is taken as that at the penetrating frost line; as the frost line is then closely adjacent to the filter there is no significant difference between the observed pressure (that in the filter) and that in the soil pores adjacent to the frost line.

It was also possible to determine values of $p_i - u_i$ for soils when the pore-water pressures would have fallen to less than $-0{\cdot}8$ kg/sq. cm—the limit to which direct observations could be made with the techniques of Beskow, Ruckli and Penner. In such cases the total pressure on the soil system was initially raised an appropriate amount above atmospheric pressure by raising the gas pressure round the sample. At the same time the pressure on the water in the pore-water pressure measuring system was raised by an equal amount (if this were not done drainage of the sample, of course, commenced). Even after the fall produced by freezing, the



Key

1. Modified triaxial cell with steel walls, filled with compressed air or nitrogen
2. Sample on pedestal, freezing device in cap (see Fig. 2)
3. Thermally insulated outer tank with anti-freeze mixture, cooled by coil from external refrigeration compressor
4. Perspex-walled inner tank with thermostatically controlled anti-freeze mixture (+0.1° C)
5. Temperature regulating device for inner tank, with contact thermometer, relays, stirrer and heating coil
6. System for circulating freezing fluid through sample top cap, with reservoir, pump and thermally insulated tubing
7. Intake for compressed air or nitrogen with Bourdon gauge (0-12 kg/sq. cm)
8. Thermistor and/or thermocouple with connexion to recording instrument
9. Connecting tubing to pore-water pressure measuring device

Fig. 1. Apparatus for measurement of pore-water pressures at penetrating frost line

water pressure is still higher than that at which the water column in the connexion to the measuring device would rupture.

Apparatus

The specimen was mounted on the filter on a pedestal in a cell (Figs 1 and 2) constructed from a 'triaxial' cell (Andresen *et al.*, 1957) as used in conventional strength tests. Freezing was brought about by circulation of an alcohol–glycerol mixture at about $-8°C$, in a coiled channel in a perspex and dural cap placed on top of the sample (Fig. 2). In this way a fairly steady cooling occurred, the rate of freezing depending on the rate at which the fluid circulated through the coil. The circulation was achieved with a 12 V vehicle fuel pump and the rate of flow adjusted by a pinch clamp on the nylon connexion tube (Fig. 1). The basal filter usually had significantly coarser pores than the sample, care being taken that in the latter case the pores were not so coarse as to permit passage of soil particles. In some tests a filter with significantly finer pores was used.

The cell was placed in a bath filled with an alcohol-glycerol mixture, thermostatically controlled at $+0{\cdot}1°C \pm 0{\cdot}05°C$. The bath itself was placed in a tank of the same liquid, cooled to about $-12°C$, and thermostatic control was achieved by an electric heater and contact thermometer in the bath. The cell was filled with air or nitrogen, the pressure of which could be increased from a commercial cylinder. As a safety precaution, the original perspex wall of the cell was replaced by one of steel. The small size of the temperature gradients occurring between the cell wall and side of the sample, together with the gas-filled space between them, effectively minimized lateral heat exchange with the sample. The frost line was, accordingly, plane and horizontal. Temperature changes in the lower part of the

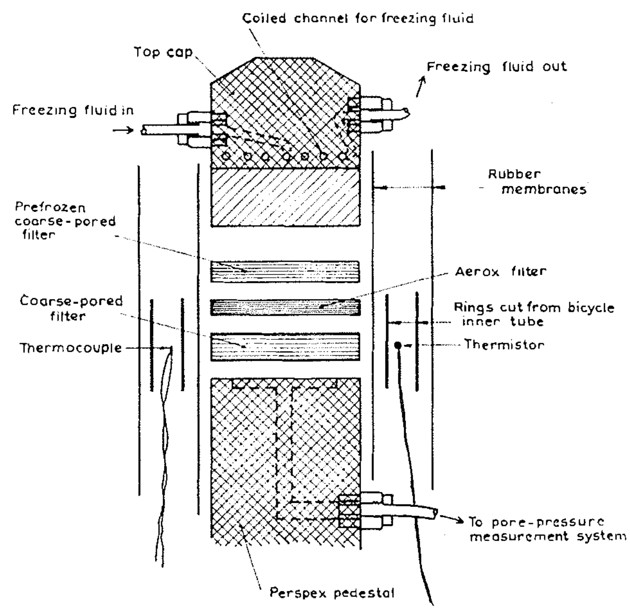


Fig. 2. Enlarged view of pedestal assembly for freezing test with Aerox filter. A similar arrangement was used for tests on soils, except that the rubber rings did not bear on the sample, and in general a single thick membrane surrounded the sample

specimen, due to external factors, were reduced by replacing the original brass pedestal by one of perspex (which has a much lower thermal conductivity).

Tests were carried out under either 'undrained' conditions, in which no water was allowed to enter the sample as the pore pressure changed, or under 'drained' conditions when a small quantity of water entered the sample from the measuring system. In the former case, pressures were measured either with a Bourdon gauge, or with a mercury manometer and, as the pressure fell, movement of water into the sample was prevented by repeated adjustment of a screw control and observation of a 'no-flow' indicator (see Andresen, *et al.*, 1957).

When changes of pore pressure were expected to be small, a mercury or water manometer was used. Only 'drained' conditions were satisfactory in the case of soils giving a small pore pressure change (i.e. $<0{\cdot}2$ kg/sq. cm) because a sufficiently responsive no-flow indicator could not be found. The manometer tube was 4·5 mm in diameter, thus avoiding any significant effects due to adhesion or the presence of the meniscus in the tube.

The sample was contained in one, or two, rubber membranes of the usual type. Expansion of the sample was thus permitted, and in most cases any confining pressure exerted by the membranes was too small to significantly affect the freezing process. In order to ensure that leakage of gas at the base of the specimen into the pore pressure measuring system did not occur as the water pressure fell during freezing, an arrangement of rubber rings (Fig. 2) was necessary. In addition, silicone grease was applied between the membranes, and rings. The temperature at the side of the basal filter was observed with a thermocouple, or thermistor, and could be recorded continuously. Although sensitive to changes of temperature of 0·001°C and possibly less, the thermocouple arrangement was such as to give only an indication (to perhaps $\pm 0{\cdot}05$°C) of the absolute temperature of the filter.

Nature and preparation of specimens

Natural soils, graded fractions and rigid ceramic filters (Aerox celloton VI, $\frac{1}{8}$ in. thick)[2] were used as specimens. The grain size composition and general characteristics of the natural soils are shown in Fig. 3. The graded fractions were prepared by elutriation from the glaciofluvial silt ØA (Østre Akers vei, Oslo). The grain-size limits given were calculated from Stokes' law. Examination of the 1·7 μ to 6·4 μ fraction under the electron microscope showed that a high degree of sorting was achieved. Application of ultrasound to the electron microscope specimen resulted in the appearance of a number of clay size particles (about 0·1 μ in 'diameter'). These particles (which represented an extremely small part of the fraction) were therefore normally closely attached to the surface of the quartz particles.

Preliminary tests showed that reproducible results could not be obtained if air was present, either free or in solution. Such air (which tends to be accumulated ahead of the frost line) affects the observed pore pressures in a variety of ways. Accordingly, in the tests on ceramic filters, these were thoroughly boiled before placing on the cell pedestal and basal filters.

In the case of the graded fractions and the natural soils it is necessary not only to de-air them completely, but also to make certain that no sedimentation occurs while, or after, the sample is placed on the pedestal. Such sedimentation would, of course, result in somewhat larger particles accumulating towards the bottom, and the observed pore pressures might then not be typical of the sample as a whole. It was found that the most satisfactory procedure was to boil the material and place it on the basal filter (the supporting membrane being in place already) as a homogeneous slurry. The specimens were 1 to 2 cm in height. The two clay soils (Leda clay KNB (remoulded) and Winnipeg clay B) were cut from saturated samples and dissolved air could not be removed. The results obtained in these cases are therefore approximate. These specimens were about 2·5 cm in height.

[2] The filters are of the type commonly used in triaxial testing of partly saturated soils.

Procedure. In all tests the basal filter was first freed of air by boiling, and the pore-water pressure measuring system flushed out with boiled water. After placing the sample on the pedestal the triaxial cell was closed and placed in the thermostatic bath. When the temperature of the sample assembly had fallen to approximately the same temperature as the bath, the cell was reopened and a coarse-pored stone filter, previously boiled and then frozen, was placed on top of the specimen. The top cap was placed on this, and connexions made to the circulation system. The cell was reclosed and replaced in the bath. The use of a prefrozen filter in this way was necessary to ensure nucleation of ice in the specimen. It was helpful to have a small quantity of free water on top of the specimen before placing the filter. This ensured that ice growth in the pores of the specimen did not start before the pore pressures could be observed. The cell gas pressure and pore pressure were then raised (if appropriate) and circulation of freezing fluid through the top cap started.

For samples where a very small pore pressure change was expected on freezing, the cell was vented to the atmosphere to avoid errors due to thermal contraction and pressure changes of the air in the cell. As freezing progressed the pore pressure was generally noted about every three minutes, the gas pressure in the cell (if above atmospheric) being checked at the same time. In tests carried out with slow freezing, undrained tests (involving a no-flow indicator) were impractical because of the frequent adjustment required, and the level of the mercury in the manometer was observed at longer intervals.

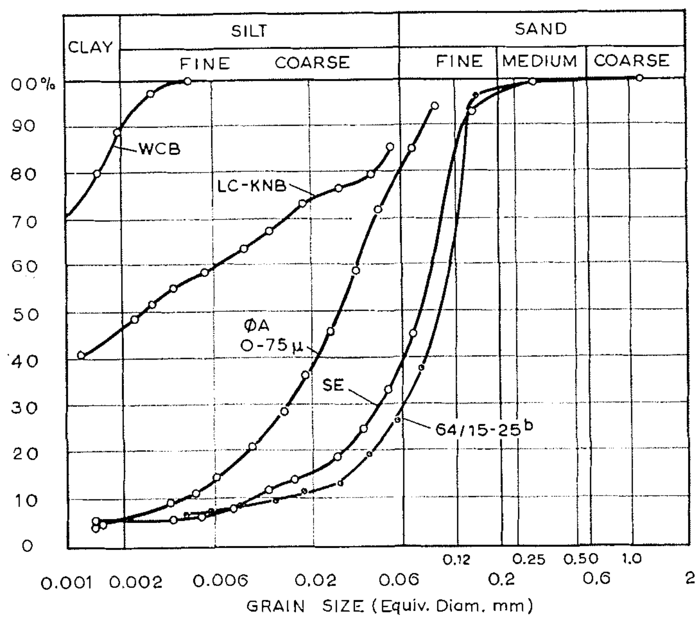


Fig. 3. Grain size composition and other characteristics of soils investigated:

WCB	**Undisturbed, mottled, montmorillonitic clay, Winnipeg**
LC-KNB	**Remoulded Leda (marine) clay, mainly illite**
SE	**Silt, mainly quartz and felspar particles thinly coated with iron hydroxides, 5% dark mineral content, mainly hornblende with some magnetic minerals. Colour of material 5 Y 6/2 (R.C.C.C. 1963)**
ØA	**Glaciofluvial silt (mainly quartz). Particles >75 μ removed by sieving**
64/15-25b	**Silt, mainly quartz and felspar. 5% dark minerals. Colour of material 6 Y 7/1 (R.C.C.C. 1963)**

After the ice had penetrated through the soil, the cell was reopened to check that freezing had taken place in the manner intended. In some cases the ice penetrated to the basal filter at night, so that the mercury manometer was first observed the following day when considerable ice had formed in the basal filter. The expansion on formation of this ice resulted in a slight displacement of the mercury for which a small correction could be made to give the pore pressure established at the time of ice penetration to the bottom of the specimen.

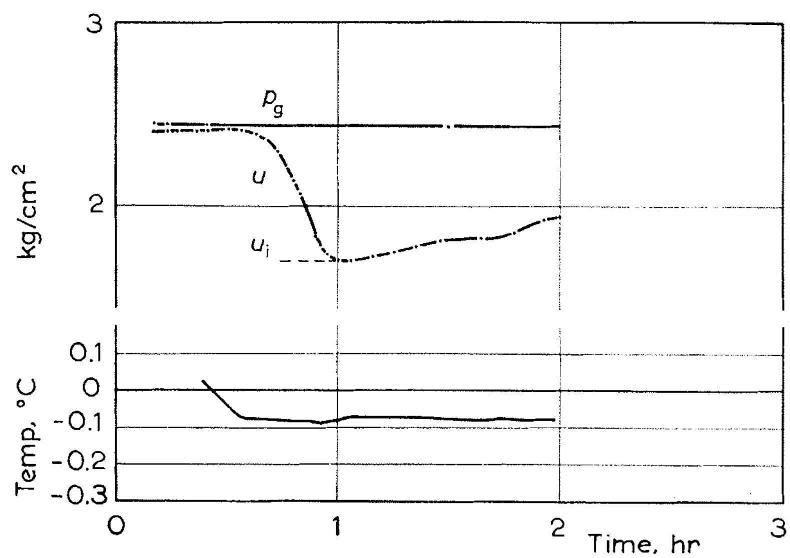
It may be noted that it was not necessary to make an airtight seal between the top cap and supporting membranes, because the fall in pore pressure induced by freezing is never great enough to draw air into the pores of the soil. The falling pore pressure also draws the membranes firmly against the sides of the sample.

Results of freezing tests

Observations made during tests with ceramic filters are shown in Figs 4a–c. The gas pressure in the cell (p_g) and the pore-water pressure are shown as a function of time. The pore pressure is seen to fall, as freezing progresses, and then to rise after the value u_i is attained, as ice advances into the basal filter. Observations from a test on Leda clay are shown in Fig. 5a, and from a test on the 6·4–23 μ silt fraction, using a finer filter below in the sample in Fig. 5b. In this case the pore-water pressure shows a further fall as the ice penetrates this filter. A test on the 73–75 μ silt fraction is illustrated in Fig. 5c (this was an undrained test).

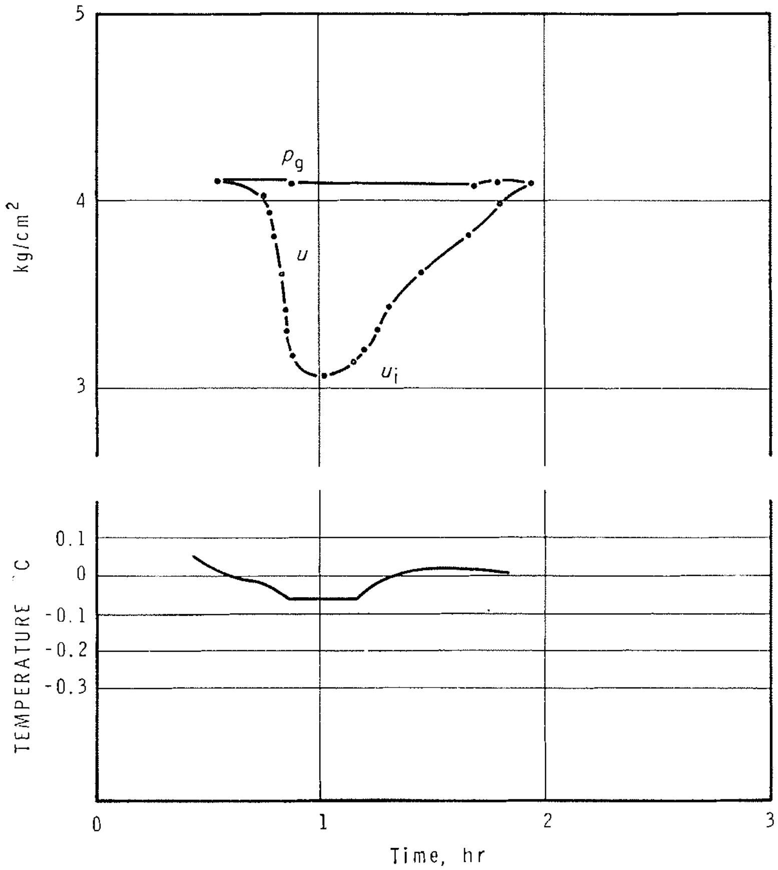
The results of the tests are summarized in Tables 1 and 2, where the pore pressure at the penetrating frost line is expressed as a difference from the gas pressure (a 'confining' or 'all-round' pressure) on the sample p_g.

The temperature recorded at the side of the basal filter shows a rise associated with the commencement of ice growth in the basal filter. The magnitude of the temperature rise is related to the difference in pore size of the soil and basal filter and is therefore larger for soils with smaller pores, or for coarser filters (see Williams, 1966(c)).



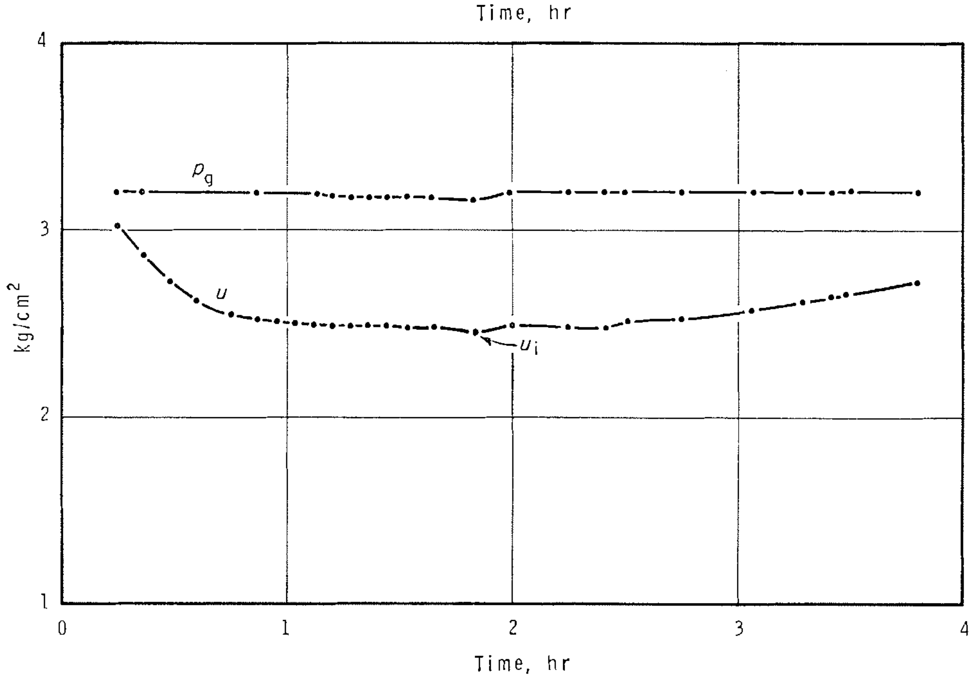
(a)

Fig. 4(a), (b), (c). Pore pressure (u) at the base of specimen and cell gas pressure (p_g) measured during penetration of frost line through Aerox filters. Also shown is the temperature recorded by a thermistor at the side of the basal filter (see Fig. 2)



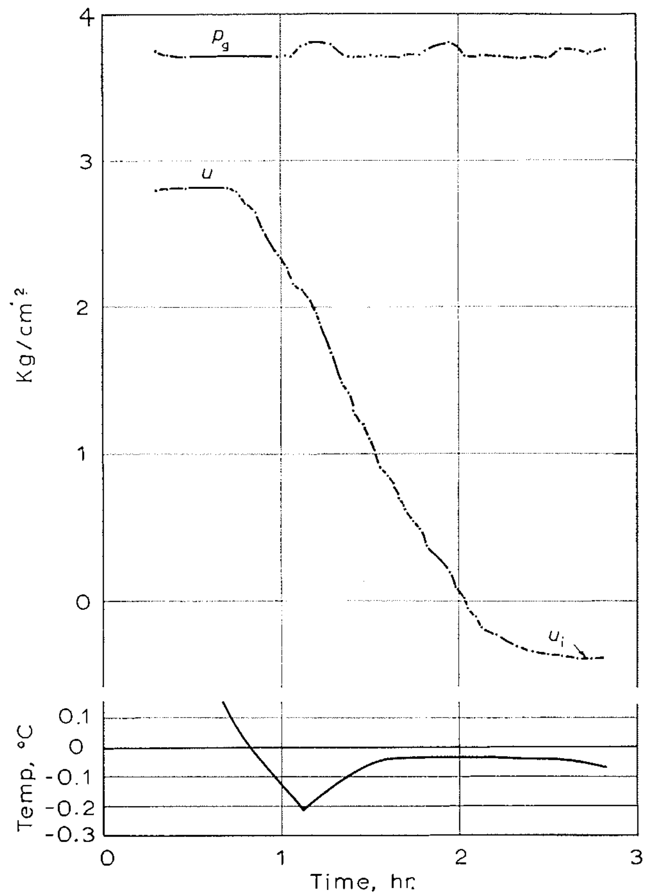
5
4
3
kg/cm²
p_g
u
u_i
0.1
0
-0.1
-0.2
-0.3
TEMPERATURE °C
0
1
2
3
Time, hr

(b)



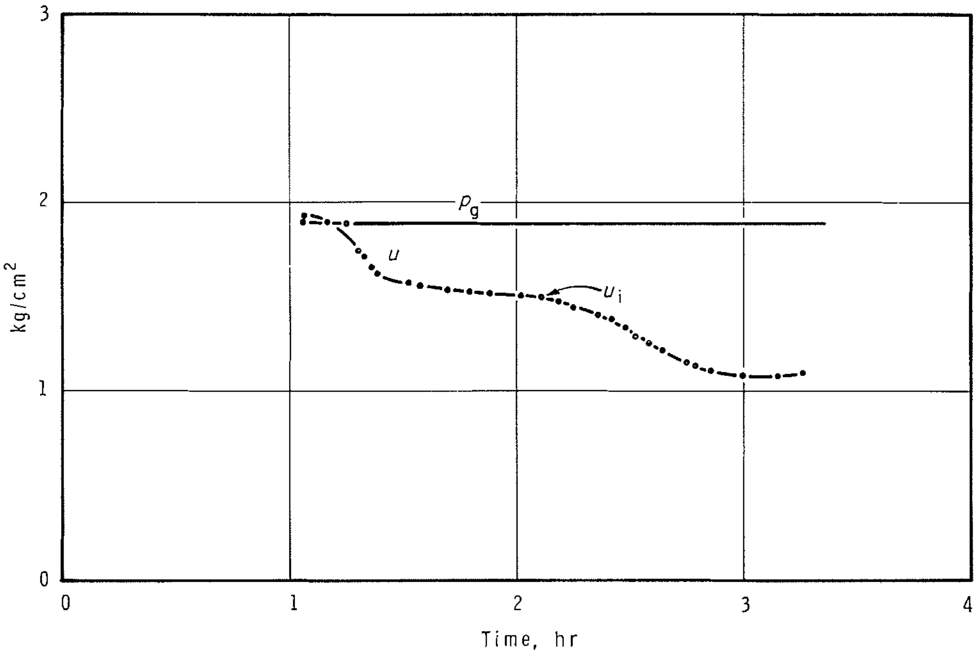
4
3
2
1
kg/cm²
p_g
u
u_i
0
1
2
3
4
Time, hr

(c)

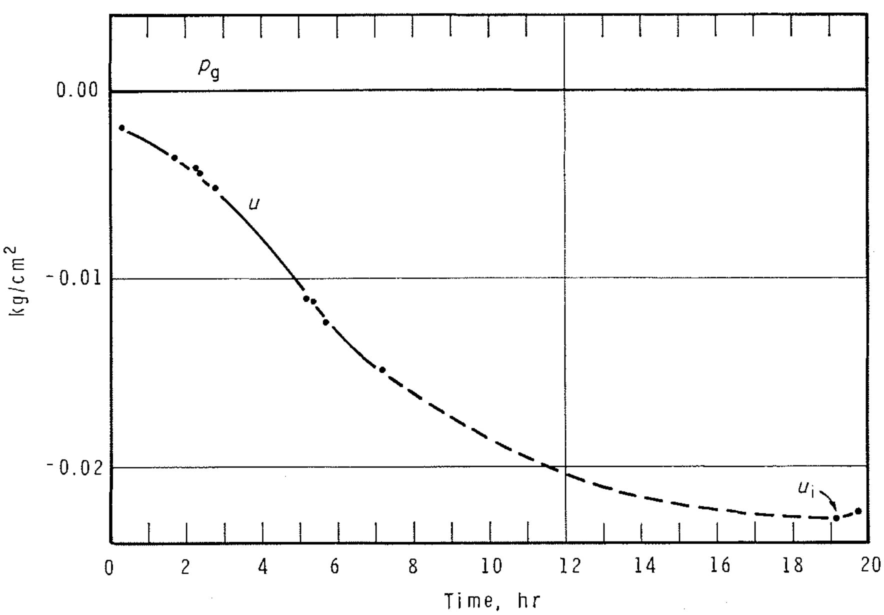


(a)

Fig. 5. Pore pressure (u) and cell gas pressure (p_g) measured during penetration of frost line. (a) Leda clay—the temperature recorded at the side of the basal filter is also shown. (b) 6·4–23 μ silt fraction. A second fall in pore pressure occurs when the frost line reaches the basal filter, which has finer pores than the specimen. (c) 73–75 μ silt fraction


3
2
1
0
kg/cm^2
p_g
u
u_i
0
1
2
3
4
Time, hr

(b)


0.00
-0.01
-0.02
kg/cm^2
p_g
u
u_i
0
2
4
6
8
10
12
14
16
18
20
Time, hr

(c)

Table 1

Pore-water pressures at a penetrating frost line (u_i), expressed as a difference from pressure on the ice, $p_i(=p_g+0{\cdot}02)$ kg/sq. cm

Sample	Rate of freezing (time required for penetration of frost line to base of specimen)		Water intake during freezing *cu. cm*	Gas pressure in cell p_g *kg/sq. cm*	$(p_g+0{\cdot}02-u_i)$ *kg/sq. cm*
Graded fractions	*hrs*	*mins*			
73 μ to 75 μ[1]	19		1·64	0	0·043[3]
	15 (ca.)		0·16	0	0·046[3]
	2	0	0·42	0	0·027
	2	45	0·36	0	0·027
49 μ to 73 μ	20 (ca.)		0·71	0	0·134[3]
23 μ to 49 μ	15 (ca.)		0·76	0	0·143[3]
	2	10	0·15	0	0·069
	0	40	0·021	0	0·101
6·4 μ to 23 μ	1	15	0·0	1·88	0·30[2,3]
	1	10	0·0	1·89	0·40[2,3]
	1	30	0·0	0·94	0·30[2,3]
	1	0	0·0	1·49	0·26[2,3]
1·7 μ to 6·4 μ	1	15	0·0	1·83	0·84[3]
0 μ to 75 μ[1]	2	0	0·0	1·17	0·66
Natural soils					
Silt E	15 (ca.)		0·34	0	0·075[3]
Leda Clay KNB	2	10	0·0	3·74	4·16[3]
Winnipeg Clay B	2	0	0·0	8·40	1·70[4]
Silt 64/15–25[b]	15 (ca.)		0·24	0	0·056[3]

[1] Wet sieved through 75 μ sieve.
[2] Basal filter used of finer pore size than sample.
[3] Value used in preparation of Fig. 9.
[4] Ice did not pass through clay pores.

Table 2

Pore-water pressure at a penetrating frost line. 'Undrained' tests with ceramic filters, type Aerox Celloton VI

Filter	p_g *kg/sq. cm*	u_i *kg/sq. cm*	$(p_g+0{\cdot}02-u_i)$ *kg/sq. cm*	Rate of freezing (time taken for penetration of frost line to basal filter) *mins*
A	1·90	1·18	0·74	64
A	1·51	0·90	0·63	83
A	3·20	2·48	0·74	125
A	2·93	2·28	0·67	18
A	5·52	4·77	0·77	38
B	4·11	3·32	0·81	33
B	4·41	3·61	0·82	24
B	2·95	2·46	0·51	15
B	4·11	3·09	1·04	25
C	4·08	3·59	0·51	16
C	2·43	1·71	0·74	28

Interpretation of results of freezing tests

In the discussion at the beginning of this Paper, it appeared that the parameter of interest was $p_i - u_i$ (i.e. $p_i - p_w$, Eqn 2). It can be assumed that in many of the tests the pressure on the ice was substantially equal to the gas pressure: $p_i = p_g$. For example, in the case of tests with ceramic filters it is obvious that a layer of ice on top of the filter, covering it more or less continuously, must be under the gas pressure p_g (plus a small amount due to the weight of the top cap, *viz.* 0·02 kg/sq. cm). The incorporation of further water molecules into the ice mass, as this extends into the pores, would be expected to occur in such a way as to most nearly approach thermodynamic equilibrium, i.e. that the ice in the pores would have the same pressure as that of masses larger than pore size. Theoretical reasons for assuming $p_i \simeq p_g$ in many cases are given in Williams, 1966(c), where experimental evidence for this being the case also for ice in the pores of a soil is presented. However, under certain conditions of freezing, discussed below, it appears that $p_i \neq p_g$ for at least part of the ice.

Table 1 shows that in general the observed value of $(p_g + 0{\cdot}02 - u_i)$ kg/sq. cm increased with decreasing grain size (and therefore decreasing pore size of the soil). For two relatively coarse materials an apparently wide range of values occurred. It is uncertain what significance should be attached to this, because the pressure difference represented by this range is in fact so small that were it to occur for finer soils (with higher values of $(p_g + 0{\cdot}020 - u_i)$ kg/sq. cm) it would be quite insignificant. However, the low values are found to be those for the tests which were carried out quickly and with intake of water, and there are several reasons why these might give low values. As noted above no-flow indicators were found not to be sufficiently sensitive to the small falls of water pressure occurring in these cases. Presumably, changes of pressure are not fully transmitted at once along the connecting tubing, but are to some extent taken up by friction and viscosity together with effects due to constrictions in the connecting line and no-flow indicator. Although not initially considered, similar effects probably occur in the absence of the no-flow indicator and, because of the greater flow of water involved, to an even greater extent.

An alternative explanation is that for these tests p_i is not equal to $(p_g + 0{\cdot}020)$ kg/sq. cm but is somewhat greater. The additional volume of ice due to the water taken in by the sample requires expansion of the sample (frost heave) which might be resisted by the strength given by ice already present. This would result in an increment of pressure, C, on the ice such that:

$$p_i - u_i = (p_g + 0{\cdot}020 + C - u_i) \text{ kg/sq. cm}$$

It could further be reasoned that the value of C is substantially reduced if freezing (and the associated expansion) occurs slowly. It is well known that the resistance to deformation of ice and frozen ground is largely dependent on the rate of application of the deforming stresses.

While this component of pressure produced in the ice by its own expansion may be of considerable importance in broader consideration of frost heave, it appears unlikely that it has much significance for the present tests. The increase in volume of ice due to water intake only varies between about 0·2 to 1·5 times that associated with the normal expansion on freezing of the water already present in the sample.

A further possible explanation for the observation, that $(p_g + 0{\cdot}02 - u_i)$ kg/sq. cm was lower for rapid tests with water intake, is that ice penetrated downwards between the membrane and sides of the specimen causing premature ice growth in the basal filter. Temperature observations showed that under such conditions there was a tendency for the membranes and sides of the sample to cool more rapidly than the interior and also for small vertical ice lenses to form between the membrane and sample. However, it is doubtful whether heat extraction upwards at the sides of the sample would be rapid enough to cause sufficient ice growth in the filter to overcome the reduction of pore pressure associated with ice-water interfaces within the sample.

The temperature observations shown in Figs 4(a) and (b), and Fig. 5(a) are in agreement, within the limits of experimental accuracy, with those to be expected from theoretical considerations. The temperature T at an interface between ice and water is given by (Williams, 1966(c)):

$$\ln \frac{T}{T_0} = \frac{(p_i - u_i)V_l}{L}$$

where V_l = specific volume of water

L = latent heat of fusion

T_0 is the freezing temperature where there is no significant curvature of the interface. It is therefore 273·18°K (=0°C) if the externally applied pressure were atmospheric. It decreases by 0·0075°C per kg/sq. cm of additional pressure. The equation above is equivalent to one put forward by Schofield (1935). The observed temperature should be at a minimum as the frost line reaches the base of the sample. In the case of the test illustrated in Fig. 4(a) the temperature should then be −0·077°C (calculated using appropriate values in the equation). The observed value is in close agreement. On entering the pores of the coarse filter, the ice interfaces are less restricted in size so that the pressure difference $p_i - u_l$ decreases and the temperature starts to rise. Both effects apparently occur only slowly, perhaps because a significant quantity of ice must form in the coarse filter before near-equilibrium conditions are re-established. The rise in observed temperature may be limited by the situation of the thermocouple on the cold side of the frost line at this stage.

A tendency was observed for the temperature rise to commence somewhat before the rise in pore pressure. It is to be expected that while the rise in temperature would commence as soon as any ice forms in the coarse filter, the rise in pore-water pressure would not commence until a considerable proportion of the ice–water interfaces had advanced from the sample pores into the filter. This effect is marked in the case of the Leda clay (Fig. 5a) and this supports the above suggestion that ice may in some cases penetrate to the filter at an early stage through a crack or discontinuity in the sample considerably larger than pore size, but this does not necessarily prevent attainment of the value u_i of the pore pressure.

The unexpectedly low value of $(p_g + 0{\cdot}02 - u_i)$ kg/sq. cm observed for the montmorillonitic clay Winnipeg B (in which the pores are extremely small) was found to be due to substantial ice penetration to the basal filter along bedding planes (at a slight angle to the filter) not visible in the specimen before freezing. Between the small ice layers in these planes, the clay remained ice-free.

In Table 2 the results of a number of tests with ceramic filters are shown and it is apparent that $(p_g + 0{\cdot}02 - u_i)$ kg/sq. cm is substantially independent of the value of $(p_g + 0{\cdot}02)$ kg/sq. cm. This is also shown by a number of tests in Table 1 where p_g was other than atmospheric. The relatively large scatter for the results from the tests with ceramic filters is probably due to the fact that the extreme importance of de-airing the sample and pore-water pressure system was not realized at the time the tests were carried out. Because the freezing process resulted fairly frequently in fracture or chipping of the ceramic filter, it was necessary to use three filters from the same production batch to complete the tests.

AIR-INTRUSION VALUE DETERMINATIONS

Apparatus and procedure

Following determination of $p_i - u_i$ $(= (p_g + 0{\cdot}02 - u_i)$ kg/sq. cm) for the samples, values of the quantity $p_a - u_a$ were required to verify Eqn (4). For the determination of $p_a - u_a$ (Eqns 3 and 4) the apparatus shown (Fig. 6) was used. The sample, prepared in the same manner as for the freezing test, is placed in a perspex ring, and is separated from the underlying filter and water column by a membrane filter.[3] The air pressure in the cell is raised by a series of increments applied over several minutes. Some drainage from the sample occurs due to compression as the air pressure is raised, and then at a certain pressure there is a sharp acceleration of drainage.

This is due to the air–water menisci at the surface of the soil being overcome, such that air spreads throughout the sample displacing water. The membrane serves the same purpose as in conventional pressure membrane apparatus in that the pores of the membrane, because of their small size, remain waterfilled in the range of air pressure/water pressure difference involved.[4] Direct passage of air into the water volume measuring arrangement is thus prevented.

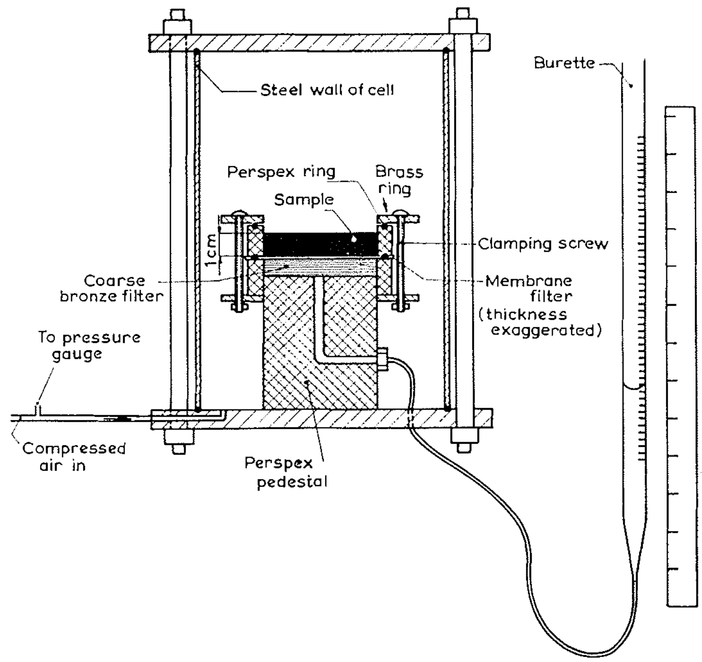


Fig. 6. Apparatus for determination of air–intrusion values

The apparatus is quite similar to a conventional pressure membrane apparatus, but with the essential difference that the membrane filter has a permeability of 5×10^{-6} cm/sec (corresponding to a flow rate through the membrane of 3×10^{-3} cu. cm/(sq. cm/sec) with a pressure drop of 0·010 kg/sq. cm through the membranes)[4] compared with 2×10^{-10} cm/sec (a flow rate of 6×10^{-7} cu. cm/(sq. cm/sec) with a pressure drop of 0·050 kg/sq. cm through the membrane) for a dialysis membrane commonly used in pressure membrane apparatus. This high permeability means (for the soils for which the test is appropriate) that following

[3] Membrane Filtergesellschaft GmbH filter series MF.

[4] Preliminary tests with Filter MF 30, pore diameter 0·3 μ, showed that (as would be expected from Eqn (3)) air could not be blown through the water saturated pores by air pressures in excess of 2 kg/sq. cm.

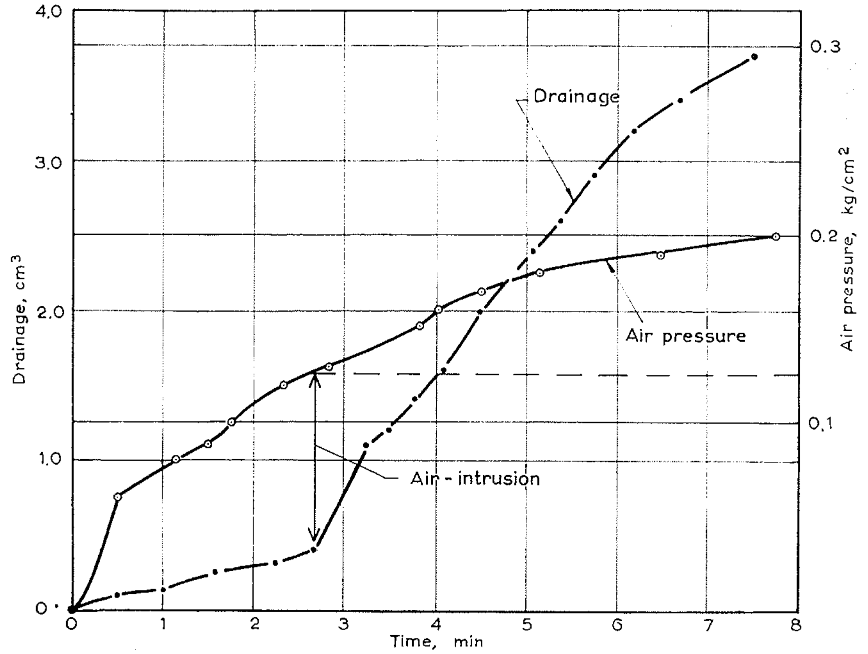


Fig. 7. Test observations during measurement of air-intrusion value of silt E

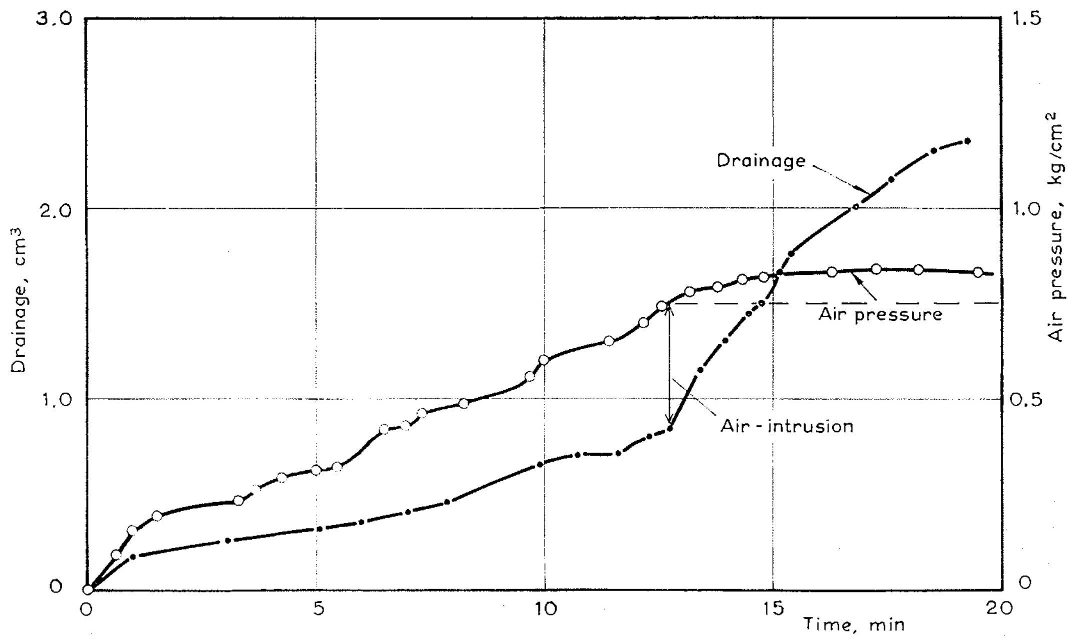


Fig. 8. Test observations during measurement of air-intrusion value of 6·4 μ–23 μ fraction

each increment of air pressure, the pore-water pressure throughout the specimen assumes an approximately uniform pressure in a few minutes at most, the high permeability of the membrane allowing rapid passage of water from the specimen.

The difference in pressure between the air and pore-water at which acceleration of drainage occurs, $p_a - u_a$, is called the air-intrusion value. The value of u_a was given by the level of the water in the burette in relation to that of the sample surface, except for a small correction which might be necessary to allow for a pressure gradient in the pore water due to permeability effects.

The fact that the air pressure may be raised quite rapidly (while the pore-water pressure remains definable) accounts for the well-defined point of air-intrusion (Figs 7 and 8). The apparatus as described can be used for soils having air-intrusion values up to about 2·5 kg/sq. cm; soils having higher air-intrusion values (those containing a high proportion of clay size particles) have too low a permeability unless very thin samples are used.

In pressure membrane tests such a well-defined point of air-intrusion is not found. Because of the low permeability of the membrane drainage is slowed down considerably. During the time involved, it is suggested that a time-dependent process of bubble formation and enlargement occurs in pores which are isolated from others by narrower necks (remaining water filled at the value of $p_a - u$ in question). Further drainage ensues on account of this and obscures the sudden fall in moisture content associated with achievement of $p_a - u_a$ and air intrusion through the largest continuous openings. Furthermore the time required for pressure membrane tests (where in practice a number of determinations are required for the establishment of a particular point on the pressure–moisture content curve) makes them unsatisfactory for the present purpose.

The Statens Väginstitut capillarimeter, widely used in Scandinavia, measures a basically similar air-intrusion value to that obtained with the present apparatus, but suffers from the disadvantages that it is only usable for a very limited range of air-intrusion values. It also suffers from serious inaccuracies resulting from the passage of air through eventual discontinuities in the specimen and between the specimen and the glass walls of its container (cf. Graton and Fraser, 1935). Such air is prevented by the membrane from entering the water drainage volume measuring system in the present apparatus.

Theoretical considerations in the use of the present apparatus are discussed in another paper (Williams, 1966(b)) where a form of the apparatus suitable for use in field determinations is described.

In using the apparatus (Fig. 6) it is generally useful to first place a small quantity of free water on top of the specimen, and to draw this through the specimen by lowering the burette such that the water level in the burette is 10 or 20 cm lower than that of the sample surface. The observed rate of passage of water then indicates the rate at which the air pressure may be raised. It is not necessary to wait until drainage ceases after each increment of air pressure. So long as the drainage resulting from raising the air pressure does not occur faster than that for the 10 or 20 cms water head, then the pressure gradient in the water and associated error in the air-intrusion value will not exceed a corresponding amount. Correction can, of course, be made for this error. The permeability of the sample may also be calculated from this preliminary test (account being taken of the effect of the permeability of the membrane).

About 0·4 cu. cm of water is drained by compression of the membrane filter onto the bronze filter and by compression of the bronze filter onto the pedestal, before the air-intrusion value is reached. Some drainage also occurs if the soil is compressible because the air pressure acts as an effective stress. Such drainage may be reduced if necessary by a preliminary consolidation of the sample. For this, a rubber membrane is placed over the sample and held by the brass ring (Fig. 6). An air pressure somewhat below that expected for the air-intrusion value is then applied. After drainage has occurred, the cell is reopened and the rubber membrane cut away carefully. The test is then carried out in the usual manner.

Results of air-intrusion value tests and their interpretation

The results of air-intrusion value tests on the materials used for the freezing tests are shown in Table 3. As is to be expected, materials with finer pores (associated with finer particles) show higher air-intrusion values. The temperature at which each test is carried out is also noted. The surface tension of water σ_{aw} (cf. Eqn (3)) is only slightly temperature-dependent (varying from 72·75 dynes/cm at 20°C to 75·6 dynes/cm at 0°C) such that the air-intrusion values would not be expected to show a marked temperature dependence. The permeabilities, however, are more temperature dependent as a consequence of viscosity changes.

Also included are results obtained with natural soils having a wide range of grain size. Although, perhaps, it is not immediately apparent, these soils also show well-defined air-intrusion values (e.g. Fig. 7). The existence of a characteristic size of largest continuous opening (as discussed earlier) in such soils may seem surprising but it is understood when the small size of soil particles is considered. Even a piece of such soil a few cubic millimetres in volume contains many tens of thousands or hundreds of thousands of particles and is thus statistically representative of the sample (assuming it to be homogeneous in a macroscopic sense). Correspondingly there will be a characteristic size of largest continuous opening, repeatedly occurring in such a material, and it is this which determines the air-intrusion value. However,

Table 3

Air-intrusion values ($p_a - u_a$) and permeabilities

Sample	Air-intrusion value ($p_a - u_a$) *kg/sq. cm*	Permeability[1] of sample, *cm/sec.*	Temperature °*C*
Graded fractions:			
73 μ to 75 μ[2]	0·14	3×10^{-4}	20·0
	0·16	3×10^{-4}	0·7
	0·14		20·0
49 μ to 73 μ	0·27	5×10^{-5}	0·7
	0·32	9×10^{-5}	0·7
	0·27	6×10^{-5}	0·7
23 μ to 49 μ	0·35	2×10^{-4}	20·0
	0·32	2×10^{-4}	20·0
	0·34	8×10^{-5}	0·7
6·4 μ to 23 μ	0·73	3×10^{-5}	0·7
	0·71		20·0
1·7 μ to 6·4 μ	2·00	1×10^{-6}	0·7
Natural soils			
Sandy Silt E	0·13	2×10^{-4}	0·7
Silt 64/15–25[b]	0·08	1×10^{-4}	20·0
ØA 0 μ to 75 μ[2]	0·89	8×10^{-6}	20·0
	0·88	4×10^{-6}	0·7
Leda clay KNB[3]	4·6		20·0
Ceramic filter, Aerox Celloton VI	1·85		20·0
	1·73		20·0
	1·62		20·0

[1] The values given show variations due to differences in packing of the particles related to the time the sample was standing before the test.
[2] Wet sieved through 75 μ sieve.
[3] From suction moisture content test and oedometer test.

for such soils, the volume of water drained from the sample immediately after the air-intrusion value has been reached is less than in some graded fractions.

The air-intrusion value for the Leda clay KNB is based on an interpretation of a suction-moisture content test and an oedometer test, and is taken from Williams (1966(a)). Direct measurement of the air-intrusion value was not possible with this material because of its very low permeability, and also because the apparatus could probably not be operated without leakage at the high air pressure that would be involved.

DISCUSSION

Comparison of pore pressure at the penetrating frost line, and air-intrusion value

As discussed above, it appears that for freezing tests carried out slowly, or under undrained conditions,

$$(p_g + 0{\cdot}02 - u_i)\ \text{kg/sq. cm} = p_i - u_i.$$

Using the values of $(p_i - u_i)$ from such freezing tests, and the observed air-intrusion values, $(p_a - u_a)$, for the same materials, the experimental observations have been plotted in Fig. 9,

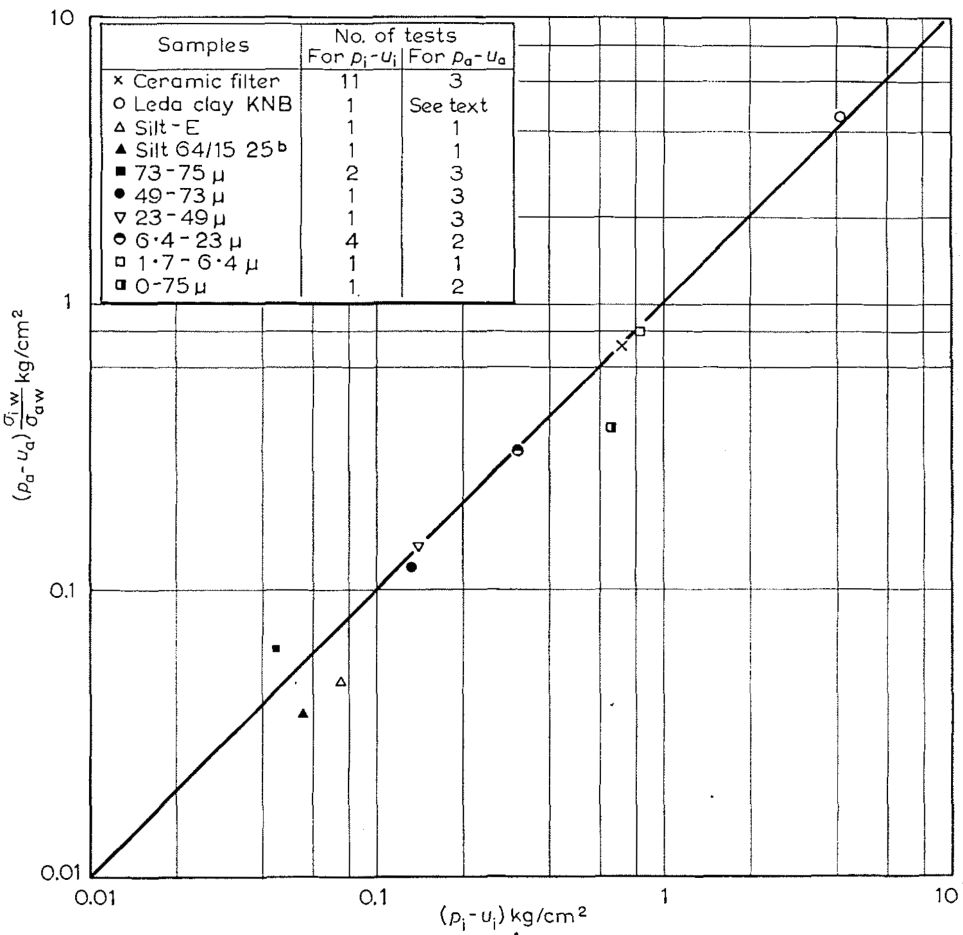


Fig. 9. Relationship between air-intrusion value ($p_a - u_a$) and pore pressure (expressed as $p_i - u_i$) at a penetrating frost line. One or more tests were carried out on each sample to give average values shown

where $(p_a - u_a)$ is multiplied by the factor σ_{iw}/σ_{aw}. A value of 30·5 dynes/cm was used for σ_{iw} (after Hesstvedt, 1964), and 75·6 dynes/cm or 72·75 dynes/cm for σ_{aw}, depending on whether the air-intrusion value test was carried out at +0·7°C or 20°C.

It is apparent that the experimental evidence substantially confirms Eqns (2) and (4). The fact that $(p_i - u_i)$ is independent of p_i in the freezing tests is further confirmation that Eqn (2) correctly describes the freezing process. According to that equation the only effect of changing p_g (and thus p_i) could be through its effect on the interfacial energy σ_{iw} and the value of this is practically independent of pressure in the range of pressure here represented.

Use of air-intrusion value as a criterion of susceptibility to frost heave

The establishment of the relationship between air-intrusion value and pore pressure at the penetrating frost line suggests the use of air-intrusion value tests as a guide to the susceptibility of a given material to frost heave.

Frost heave as generally understood by highway engineers, is the volumetric increase of the soil which occurs as the result of uptake of water (which generally appears as ice lenses) by the freezing soil. The expansion associated with the freezing of in-situ moisture is generally of relatively little significance. It may be important under certain specific conditions but it is not considered here. The uptake of water occurs as a result of a hydraulic gradient through the unfrozen soil towards the frost line, established by a reduction of pore pressure at the frost line. It is this reduction in pore pressure which can be predicted by the air-intrusion value test.

For small samples of negligible thickness, the pore pressure at the penetrating frost line may be predicted from the product of the air-intrusion value and the factor σ_{iw}/σ_{aw} (=0·4 approx.) according to Eqn (4) as illustrated in Fig. 9.

Under field conditions the pore pressures at the penetrating frost line will also depend on the overburden pressure. Thus whether or not the material will show frost heaving will depend on the overburden pressure, and also on the pore-water pressures in the underlying unfrozen layers. This fact, that the susceptibility to frost heaving is not alone a property of the material *per se* but also of the situation under which it occurs, is often overlooked in the use of, for example, grain size distribution as a criterion. Frost susceptibility criteria are reviewed by Townsend and Csathy (1963(a)) and the same authors (1963(b)) have also discussed the possible significance of pore size characteristics in relation to observed frost damage to highways.

The situation is best illustrated by an example. We assume that a silty material is available and suggested as being satisfactory for use as highway sub-grade which must not be subject to frost heave. The air-intrusion value (at 20°C), is determined to be 0·34 kg/sq. cm. The pore-water pressure u_i, at the penetrating frost line is then given by

$$(p_i - u_i) = 0{\cdot}34 \,.\, 0{\cdot}42 = 0{\cdot}14 \text{ kg/sq. cm} \qquad (5)$$

If p_i were atmospheric pressure (0 kg/sq. cm according to conventional soil mechanics terminology), then u_i would be $-0{\cdot}14$ kg/sq. cm. If frost heave occurs the ice, as explained earlier, must bear the pressure due to the weight of overburden $\gamma . x$, where γ = bulk density, and x = depth. $\gamma . x = p_i$, the pressure on the ice.[5] Taking the bulk density of the frozen soil as 0·002 kg/cu. cm for a penetrating frost line at depth x cm, then at that depth $p_i = 0{\cdot}002x$ kg/sq. cm, and the pore pressure u_{i_x} is given by:

$$u_{i_x} = (0{\cdot}002x - 0{\cdot}14) \text{ kg/sq. cm} \qquad (6)$$

The pore pressure at the penetrating frost line in this material is thus defined as a function of depth. Whether or not frost heave actually occurs will depend on whether or not higher

[5] The rate of freezing in the ground is such that additional stresses would not in general be expected in the ice. However, under certain conditions (for example on sloping ground) they may occur and have an effect equivalent to an increased overburden pressure.

pressures occur in the neighbourhood of the frost line such that a hydraulic gradient is established towards it.

Information as to the pore pressures to be expected in the soil before freezing is therefore required. For the present example, we assume that a free water table occurs at 100 cm depth. The pore pressure (before freezing) as a function of depth x is:

$$u_x = (-100+x) \text{ cms of water} = \frac{(-100+x)}{1000} \text{ kg/sq. cm} \quad . \quad . \quad . \quad (7)$$

where u_x = pore-water pressure before freezing at depth x cm. Water migration to the frost line occurs when:

$$u_x > u_{i_x}$$

i.e. the maximum depth x_m to which frost heave can occur is that at which the pore pressure $u_{x_m} = u_x = u_{i_x}$. From Eqns (6) and (7):

$$0{\cdot}002x_m - 0{\cdot}14 = \frac{-100+x_m}{1000} \text{ kg/sq. cm}$$

or

$$x_m = 40 \text{ cm.}$$

This material is therefore only satisfactory for use at greater depths than this. This simple example is included here to illustrate the manner in which the field conditions of pore-water pressure and overburden pressure, together with the pore size characteristics (characterized by the air-intrusion value) determine whether or not frost heave will occur for a given material.

In practice, some additional considerations are likely to be necessary. Calculation of the initial pore-water pressure u_x at various depths from considerations of hydrostatic equilibrium and depth to the water table only is not fully justified in many cases, because of transient effects due to weather, the freezing process itself, etc. Secondly, while the agreement between the two sets of experimental results as a whole is good, in individual cases there are discrepancies which would leave room for significant variation in the predicted value of x_m. These discrepancies may in fact be mainly due to the experimental observations in the relative difficult freezing tests being of poorer quality than in air-intrusion value determinations. Further studies of the latter (Williams, 1966(b)) indicate that it is relatively easy to obtain quite closely reproducible results for the air-intrusion value. It seems likely that under field conditions as well the pore pressures developed at the frost line may to some extent be influenced by several barely predictable factors. The natural variability of soils over quite small distances must be remembered. The procedure outlined above is somewhat conservative in so far as frost heave will often be so slight as to have no importance at a depth somewhat less than that at which $u_{i_x} = u_i$. If u_{i_x} is only slightly less than u_x then the water migration to the frost line will often be so small that very little heave occurs.

It appears that the air-intrusion value gives a quantitative and clearly definable evaluation, both theoretically and experimentally justifiable, of the susceptibility to frost heaving of a given soil material. It thus appears to have considerable advantages over conventional methods of appraisal. The extent to which it may be necessary to use some safety factor to allow for uncertainties of the type outlined will depend on the degree of frost heave which could be tolerated, economic factors, experience gained in its use and other factors.

CONCLUSION

(1) The relationship has been established between the pore-water pressure at a penetrating frost line, in a given soil, and the pressure (the 'air-intrusion value') at which air will enter and spread through the pores of the same soil, displacing water from it.

(2) It is probable that the air-intrusion value depends on the size (radius) of the largest

continuous openings through the pore structure of the soil, and that it is this size which determines the pore pressure at a penetrating frost line in a given soil.

(3) Measurement of the air-intrusion value of a soil is performed relatively easily and quickly, and thus provides a convenient method of predicting the pore-water pressures that will be developed at the penetrating frost line.

(4) It has been shown how the occurrence of frost heave depends on this pore pressure and also on depth and ground water conditions.

ACKNOWLEDGEMENTS

Discussions with Dr L. Bjerrum, Director, and other members of the Norwegian Geotechnical Institute are gratefully acknowledged.

REFERENCES

Andresen, A., L. Bjerrum, E. Dibiagio, *and* B. Kjærnsli, 1957. Triaxial equipment developed at the Norwegian Geotechnical Institute. *Norwegian Geotechnical Institute, Publication*, 21.

Beskow, G., 1935. Tjälbildningen och tjällyftningen med särskild hänsyn till vägar och järnvägar. *Statens väginstitut, Stockholm. Meddelande*, 48. (Also published as Sveriges geologiska undersökning. *Avhandlingar och uppsatser. Serie C*, 375, and translated by *Tech. Inst. North-Western Univ., Evanston, Ill.*, Nov., 1947.)

Everett, D. H., 1961. The thermodynamics of frost damage to porous solids. *Trans. Faraday Soc.*, 57: 9: 1541–1551.

Graton, L. C. *and* H. J. Fraser, 1935. Systematic packing of spheres with particular relation to porosity and permeability. *J. Geol.*, 43: 8 (Pt 1): 785–909.

Helmuth, R. A., 1962. Capillary size restrictions on ice formation in hardened Portland cement pastes. *Proc. Int. Symp. on the chemistry of cement*, 4. Wash. D.C., 1960, 2: 855–869.

Hesstvedt, E., 1964. The interfacial energy ice/water. *Norwegian Geotechnical Institute.* Publication, 56, 7–10.

Penner, E., 1957. Soil moisture tension and ice segregation. *Highway Research Board, Wash. D.C.* Bulletin, 168, 50–64. (Also *National Research Council, Canada. Division of Building Research. Research paper*, 67.)

Penner, E., 1958. Pressures developed in a porous granular system as a result of ice segregation. *Highway Research Board, Wash. D.C. Special report*, 40, 191–199. (Also *National Research Council, Canada. Division of Building Research. Research paper*, 81.)

RCCC (Rock-Color Chart Committee), 1963. Rock-color chart. *Geological Society of America, New York.*

Ruckli, R., 1950. Der Frost im Baugrund. *Wien.* 279 pp.

Schofield, R. K., 1935. The *pF* of the water in soil. *3rd. Int. Cong. Soil Sci.*, 2: 37–48.

Taylor, D. W., 1948. Fundamentals of soil mechanics. *John Wiley, New York*, 700 pp.

Townsend, D. L. *and* T. I. Csathy, 1963(a). Compilation of frost susceptibility criteria up to 1961. *Ontario Joint Highway Research Programme. Report*, 14, 27 pp.

Townsend, D. L. *and* T. I. Csathy, 1963(b). Soil type in relation to frost action. Kingston, Ont. *Ontario Joint Highway Research Programme. Report*, 15, 69 pp.

Williams, P. J., 1966(a). Suction and its effects in unfrozen water of frozen soils. *Proc. Int. Conf. on Permafrost*, 1. Lafayette, Ind. 1963. (In press.)

Williams, P. J., 1966(b). Air intrusion value in soils and its measurement. *Norwegian Geotechnical Institute*. (In press.)

Williams, P. J., 1966(c). Unfrozen water in frozen soils; pore-size-freezing temperature, pressure relationships. *Norwegian Geotechnical Institute.* (In press.)

REPLACEMENT OF WATER BY AIR IN SOIL PORES

By P. J. WILLIAMS*

SYNOPSIS

Soils composed of minerals having the same refractive index as water are translucent when saturated. Any air in the pores is fully visible. Such soils have been used to observe the manner in which air may replace water in soil pores. Where the external air and water pressures are established such as to cause rapid drainage of water, the air suddenly spreads through the pore structure at a certain air/water pressure difference. Where drainage occurs more slowly, a progressive diffusion of air in solution occurs, giving enlargement of bubbles, as well as a progressive breakdown of menisci not easily explainable from capillary theory.

On peut preparer en laboratoire des sols dont les minéraux ont le même indice de réfraction que l'eau et sont translucides lorsque saturés. On peut facilement voir l'air occlu dans les espaces interstitiels. On s'est donc servi de ces sols pour étudier le remplacement de l'eau par l'air dans les interstices.

Pour certaines différences entre la pression de l'air ambiant et celle de l'eau, il est possible d'établir un état tel que l'évacuation de l'eau et la propagation de l'air sont rapides. Toutefois, si les conditions sont telles que le taux d'évacuation de l'eau est réduit, il se produit une diffusion progressive de l'air dans la solution, avec comme résultat une expansion des bulles d'air et une modification continue des ménisques que l'on ne saurait expliquer aux termes de la théorie capillaire.

When a soil is not completely saturated, the interfaces between the air and water in the soil are confined by the soil pores. Because of capillarity, a pressure difference then exists between the water and the air; this is represented by the capillary equation:

$$p_a - p_w = \frac{2\sigma}{r} \cos\theta \qquad (1)$$

where

σ = interfacial energy

r = the radius of curvature of the interface.

In the case of water and soil minerals the contact angle θ is generally assumed to be 0°, and the radius of curvature r is equivalent to the radius of the pore in which the interface lies. The lower the moisture content of the soil the greater is the difference $p_a - p_w$, because the interfaces then lie in smaller pores and constructions of pores. This constitutes the well-known suction - moisture content relationship.

This relationship can be determined in pressure membrane apparatus consisting of a cell with a drainage outlet covered by a membrane. The soil sample is placed on the membrane and the air pressure, p_a, in the cell raised. The membrane has pores so small that they remain water-filled at all values of p_a in question and the air cannot pass through it. Water, however, drains out of the soil until the water pressure, p_w, in the soil

* Soil Mechanics Section, Division of Building Research, National Research Council, Ottawa, Canada.

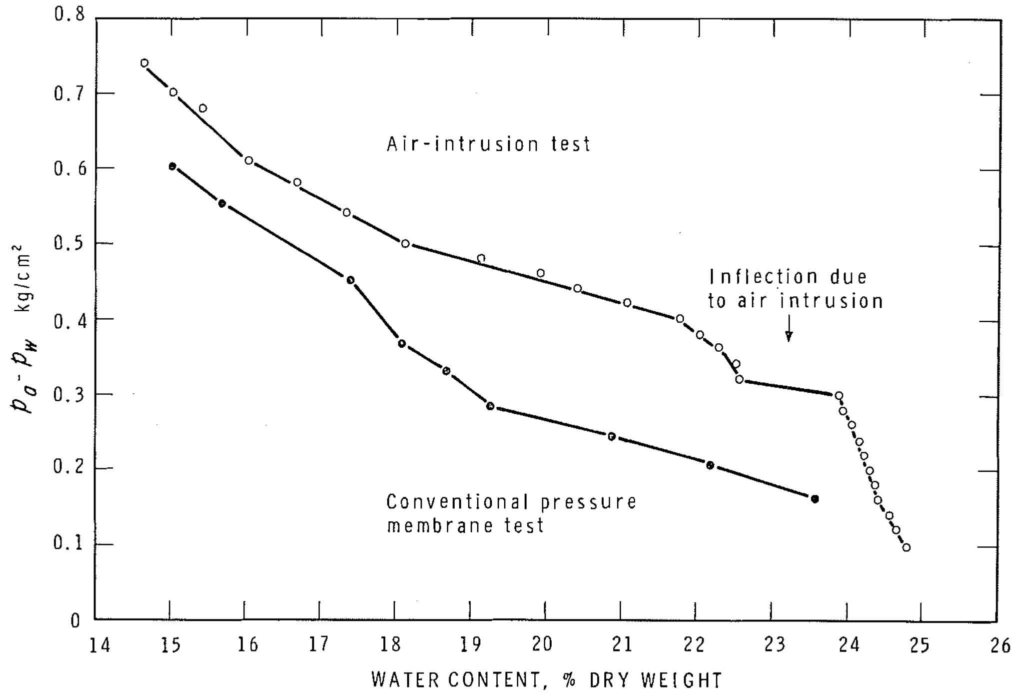


Fig. 1. Suction - moisture content relationships obtained for a silt containing some clay, by a conventional pressure membrane test, and by an air-intrusion value test in which each air pressure increment was applied as soon as perceptible drainage had ceased.

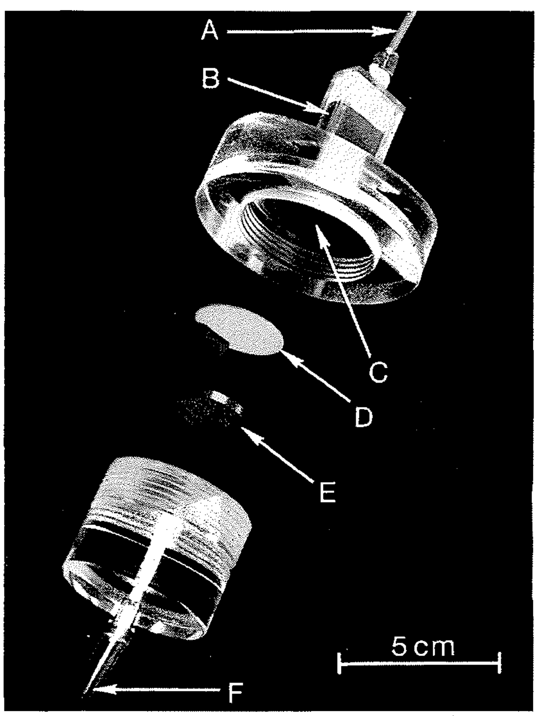


Fig. 2. Pressure-membrane cell used for tests with low microscope magnification.

A – Port for compressed air.
B – Part of sample viewed under microscope.
C – Base of sample.
D – Membrane.
E – Bronze filter.
F – Drainage (to burette).

is in equilibrium with the atmospheric pressure. The moisture content of the sample is then determined. The test is repeated for a number of different values of p_a.

Several days are required for the moisture content of even small samples to attain equilibrium. The low permeability of the membrane and of the soil itself (as the moisture content is reduced) is generally thought to be the main cause. In recent years membranes have become available which have substantially greater permeability. When using these membranes with quite permeable soils it would be expected that drainage should be completed in a few minutes or hours, *i.e.*, that the water would assume a uniform pressure throughout the sample within that time.

In tests using such membranes it was found that the drainage occurring after the application of an air pressure increment rapidly decreased, becoming imperceptible after a few minutes. A suction - moisture content relationship could be obtained on the basis of a series of pressure increments applied over, for example, one hour. The relationship so obtained differed, however, from that obtained by conventional procedures (Fig. 1); in particular there was a marked inflection, due to the release of a considerable quantity of water at a certain value of $p_a - p_w$, (Williams, 1966 (*a*)). For a range of values of $p_a - p_w$, the water content was lower in the conventional test, implying that some drainage continued over a long period of time. The presence of the inflection in the rapid test can be more easily explained than can its absence in the slow test.

The pore system of a soil consists of interconnected openings of various sizes, giving a network of channels with innumerable variations of diameter. Consider a soil homogeneous in a microscopic sense and initially saturated. For relatively low values of $p_a - p_w$, only a few large pores, either opening directly to the surface or in direct connection with the external air channels nowhere narrower than of radius r, will be emptied of water. As $p_a - p_w$ is increased somewhat, the radius r is decreased. Strings of interconnected air-filled pores extend a little deeper into the sample, but their penetration is limited by occurrence of constrictions smaller than r. When the value of $p_a - p_w$ is such, however, that these strings extend through perhaps ten to 100 pores (representing a layer perhaps 0·5 mm thick) it becomes likely to the point of certainty that there will always be additional openings of appropriate radius, through which the air may advance further. There is, therefore, a critical pressure difference, $p_a - p_w$, at which air-filled channels, instead of being confined to a thin surface layer, can spread through the sample. This value of $p_a - p_w$ is referred to as the air-intrusion value and is associated with a substantial drainage of water. The corresponding value of r equals r_c, which is the radius of the largest continuous opening through a soil sample big enough to contain a statistically representative assemblage of particles.

The term air-entry value has been used (Bishop and Henkel, 1962) to describe the air-water pressure difference at which air can be forced through initially-saturated fine-pored filters used in soil strength testing. Brooks and Corey (1964), in a theoretical treatment, refer to a similar quantity, "bubbling pressure . . . a measure of the maximum pore size forming a continuous network of flow channels within the medium". Topp, Klute and Peters (1966) have made observations on the influence of duration of test and other factors on the form of suction - moisture content curves.

The explanation of the air-intrusion value already given leaves open the question of why this kind of suction - moisture content curve differs from that obtained by the slower conventional test procedure. Several authors believe that the form of the curve obtained by the latter procedure is a measure of pore-size distribution (Donat, 1938; Schofield,

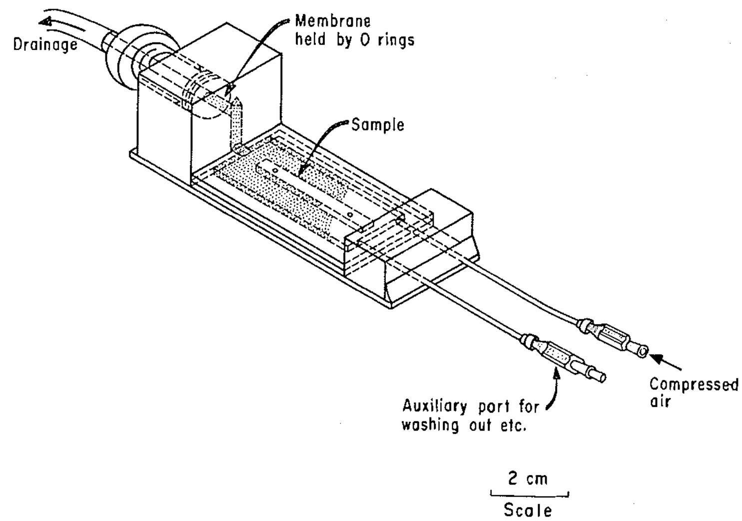


Fig. 3. Pressure-membrane cell used for tests under high microscope magnification.

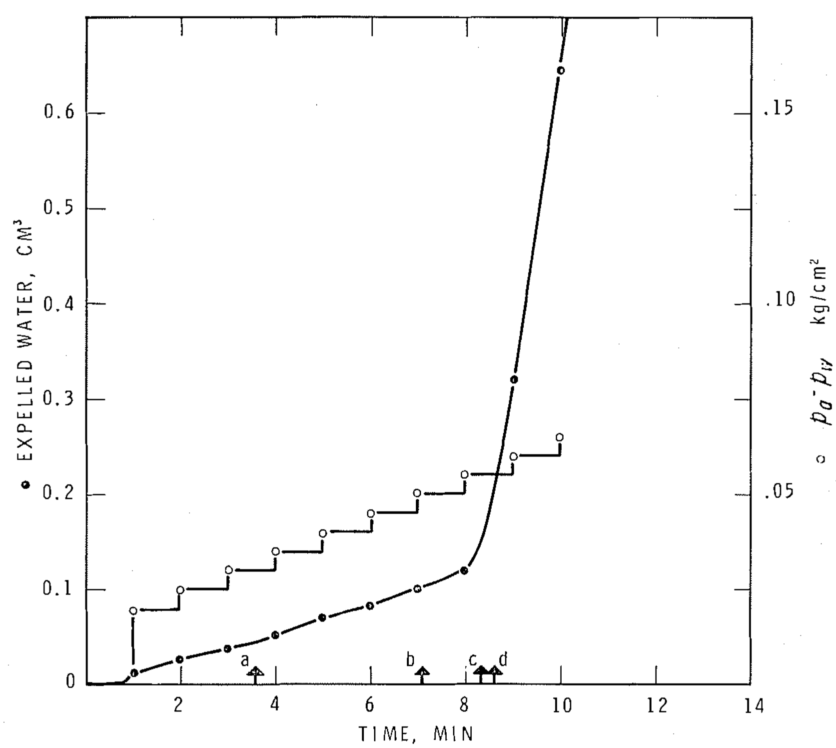


Fig. 4. Air-intrusion test on chiolite, 74μ–250μ particle diameter fraction. The drainage and applied air pressure ($= p_a - p_w$) are shown as a function of time. Also shown are the times at which the photographs in Fig. 5, were taken.

1938) and that for any value of $p_a - p_w$ the remaining water content is situated only in pores of equal or smaller radius than that given by equation (1). This would require nucleation and enlargement of air bubbles in pores of this size but which are totally isolated by smaller pores. Powers (1962) believes a process of this type to occur in cement.

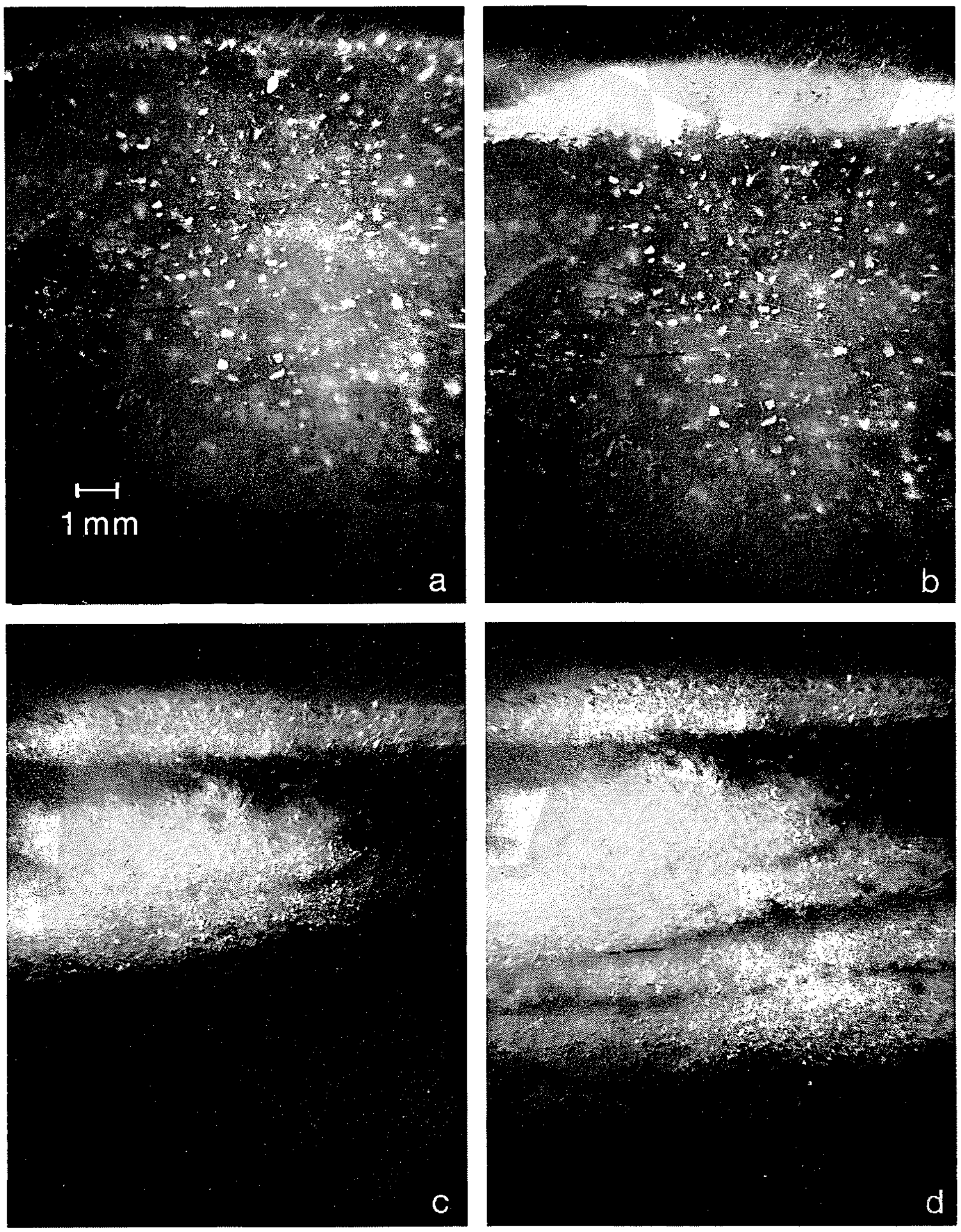


Fig. 5. Photographs of air intrusion (compare fig. 4) in part of sample.

Experiments are now reported that enabled direct observations of the manner of water replacement by air under various test conditions.

EXPERIMENTAL WORK

Soils were prepared by crushing the minerals chiolite (5 $NaF.3AlF_3$) or cryolite (Na_3AlF_6). These translucent, smoky-white minerals have a refractive index similar to that of water (in which they are substantially insoluble). The crushed material is white and opaque, but when mixed with water it is transparent in thin layers. Air within the pores is then fully visible.

Small pressure membrane cells were constructed of Perspex, such that the sample in the cell could be viewed through a microscope. Depending on the magnification desired and the grain-size composition of the sample, the microscope could be focused on a plane up to about 2 mm within the sample. One type of cell is shown in Fig. 2 and another in Fig. 3. The cells were so designed that there is a sufficient ratio of membrane area to sample height and volume for drainage from the sample to lead to hydrostatic equilibrium throughout the water in the sample in a short time. In addition, the dimensions of the cell are determined by the working distances of the microscope at the magnifications desired.

The following are examples of the four kinds of test carried out:

Observations of drainage from pores during rapid test

A sample of chiolite consisting of a 74μ–250μ particle diameter fraction was prepared as a slurry, boiled, cooled, and placed in the apparatus (Fig. 2). A preliminary test showed that the permeability of the sample with the membrane in the cell was such that a 20-cm head between the top and bottom of the sample gave a flow of 1·8 cm³/min.

The drainage was measured in a pipette and is shown as a function of time in Fig. 4 (it could also be shown on a similar plot to that in Fig. 1). Photomicrographs taken at the times shown by the arrows in Fig. 4 are reproduced in Fig. 5. Incident lighting was used and the saturated sample appears dark. In the first two photographs air is visible

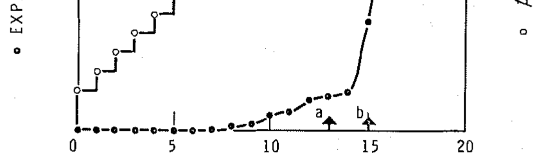


Fig. 6. Air-intrusion test on cryolite, composed of equal parts of <44μ, 44–74μ, 74–149μ, and 149–250μ fractions. The drainage and applied air pressure (= $p_a - p_w$) are shown as a function of time. Also shown are the times at which the photographs in Fig. 7 were taken.

only in a thin surface layer, while slight compression of the sample and membrane assembly gives a small amount of drainage. In the third photograph the air has spread substantially, and in the next taken only ten seconds later, when the air pressure was $0 \cdot 055$ kg/cm^2, the air has spread throughout the sample. This pressure, therefore, represents the air-intrusion value.

A second test of the same type is illustrated in Figs. 6 and 7. The sample consisted of equal parts of 250 μ–149 μ, 149 μ–74 μ, 74 μ–44 μ, and <44 μ fractions. The sample was mixed to a slurry and not boiled. Entrapped air bubbles are visible and the presence of the finer grain-size fractions also reduces the transparency of the material. The intrusion of air gave a somewhat different pattern of air-filled regions in this test.

In some experiments with fine-grained and relatively compressible material, the paths taken preferentially through the soil by the air were in fact observed to be fully-developed cracks. As the air entered down these paths the soil separated, the crack finally having a width perhaps ten times the radius of the air-water interfaces. Associated with such cracking is an increment of drainage, which is distinct from that due to intrusion of air into pores.

Intrusion of air apparently took place only by continuous extension of the air-filled regions: in the boiled samples no new isolated air bubbles arose and in the unboiled samples no significant enlargement of isolated bubbles already present was observed. In Fig. 10 a string of air-filled pores is seen (in transmitted light) after air-intrusion in a sample consisting of equal parts of <44 μ, 44 μ–74 μ, 74 μ–149 μ fractions, and ½ part of 149 μ–250 μ. The dark regions are shadows of air-filled pores. Air-intrusion occurred in jumps, in which strings of perhaps twenty or even fifty or more pores empty apparently simultaneously. At the height of the air-intrusion process these jumps occur at short intervals of time or simultaneously, in various parts of the sample.

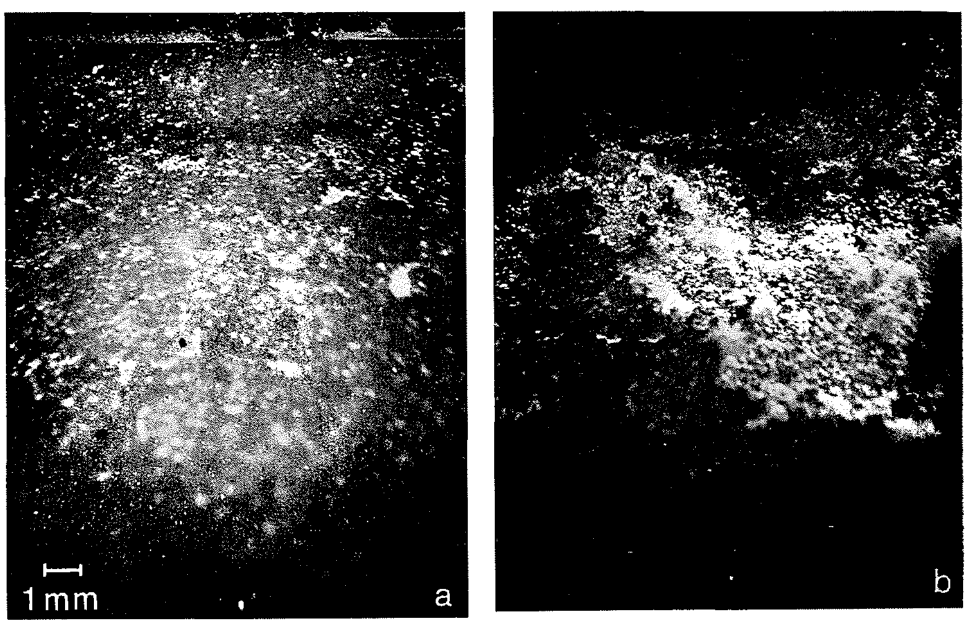


Fig. 7. Air intrusion (compare fig. 6) in part of fine-grained sample.

Observations of slow enlargement of entrapped bubbles

In these tests the value of $p_a - p_w$ was raised to just below the air-intrusion value, which had been determined previously. Bubbles trapped some millimetres in from the sample surface, following its preparation as a slurry, were then observed. In Fig. 8(*a*) bubbles are shown about twenty minutes after the application of the air pressure, in this case $0 \cdot 22$ kg/cm^2 (the air-intrusion value was $0 \cdot 25$ kg/cm^2 for this material which was similar to that described in the previous paragraph). By this time volume changes of the bubble associated with transient pressure changes in the water were complete. The bubbles enlarged slowly during the next two or three days, and in Fig. 8(*b*) the same bubbles are seen after twenty-six hours. Because of poor control the air pressure varied slowly by up to ± 5 % in this case. Other tests showed that this has no significance for the observation.

Attempts to observe nucleation of air bubbles

A third series of tests also involved samples where $p_a - p_w$ was raised to a value just below the air-intrusion value. The highest value of $p_a - p_w$ used was $0 \cdot 3$ kg/cm^2 using a $<44\,\mu$ fraction with more than 80 % by weight medium and coarse silt, and with an air-intrusion value of $0 \cdot 35$ kg/cm^2. $p_a - p_w$ was established both by raising the air pressure, and in other tests by lowering of the water pressure. The sample was repeatedly examined by traversing with the microscope to ascertain whether new air bubbles arose during a period of days. In some cases photographs were taken repeatedly. In no cases were bubbles observed to have formed.

Changes at air-water interfaces during tests of long duration

It was noticed that although no new bubbles were formed when the value of $p_a - p_w$ was maintained for long periods somewhat below the air-intrusion value, there was a progressive extension in the amount of air-filled regions of the sample. Observations were therefore made under high magnification, over periods of days, of interfaces between the soil water and external air. The progressive advance of such interfaces into the soil is seen in Fig. 9. In this example $p_a - p_w$ was established at $0 \cdot 30$ kg/cm^2, using the material described above having an air-intrusion value of $0 \cdot 35$ kg/cm^2.

DISCUSSION

The experiments show that there are at least three distinct processes by which air may replace the water in the soil pores. The simplest process is demonstrated by experiments of the first type and is sufficiently described by equation (1). Air-water interfaces retreat inwards from the soil surface until they are confined by pores such that their radius is that given by equation (1), for the appropriate pressure conditions $p_a - p_w$. When $p_a - p_w$ is equal to or greater than a certain critical value – the air intrusion value – there are interconnected air-filled pores through the sample. Fig. 10 is a projection, but it is possible to make an estimate of the radius of the interfaces as $5\,\mu$–$15\,\mu$. In this case the value of $p_a - p_w$ was $0 \cdot 24$ kg/cm^2, and the value of $6\,\mu$ is, therefore, predicted by equation (1).

Fig. 5 shows that in a soil composed of material in a narrow grain size range, and which is not susceptible to significant compression, the spreading of air occurs rather uniformly at the air-intrusion value. The somewhat banded appearance is due to some sorting (sedimentation) that appears to be inevitable when samples are placed. The slurry has to be inserted in small quantities through the air port or base of the cell;

tapping the cell is necessary to ensure filling of the space above the membrane.

The second material, illustrated in Fig. 7, was somewhat compressible in bulk and tended to decrease slightly in volume under the raising of the air pressure. This compressibility is largely responsible for the tendency for air-intrusion to occur preferentially along certain paths (incipient cracks) or along the sides of the pressure membrane cell. The air-intrusion value is also well-defined, however, for this material.

The formation of true cracks, observed in some tests with compressible materials, is not to be regarded as a replacement of water in pores by air. It is a consolidation due to the effective stress set up in the soil skeleton by the establishment of the air-water pressure difference. This cracking commonly occurs for values of

$$p_a - p_w < \frac{2\sigma}{r_c},$$

i.e., before the air-intrusion value is reached. The drainage associated with this volume change must not be confused with that occurring as air replaces water in the pores.

A second process involving replacement of water in pores by air, demontrated by experiments of the second type, occurs much more slowly. It depends on the rate of diffusion of air through the water to bubbles. This occurs if the interface radius of entrapped bubbles is greater than that of the interfaces to the external air. The solubility of air in water depends on the pressure of the air (according to Henry's Law the mass of dissolved gas in a definite mass of the liquid at a given temperature is very nearly proportional to the (partial) pressure of the gas). If the pressure of the water is uniform, then the air pressure will be lower where the interface radius is greater, according to equation (1). There is thus a concentration gradient for air in solution from the vicinity of small interfaces towards larger interfaces. Diffusion occurring along this gradient

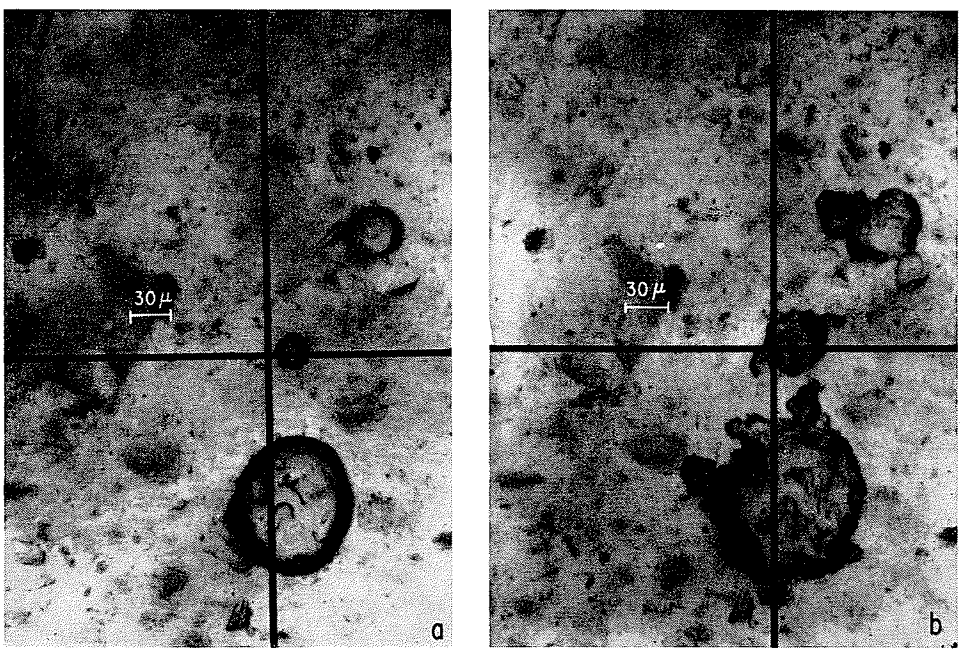


Fig. 8. Enlargement of entrapped air bubbles: (a) bubbles 20 min after establishment of $p_a - p_w = 0{,}22$ kg/cm²; (b) 26 hr later.

results in air coming out of solution, when the saturation concentration for the air pressure at the larger interface is exceeded. Entrapped bubbles can thus increase in size. As bubbles grow they will tend to penetrate smaller pores until the interfaces have a radius equal to that of the interfaces to the external air.

The general equation for diffusion between two locations is:

$$\frac{dQ}{dt} = K \frac{dc}{dx} dy\, dz \qquad (2)$$

where

$\frac{dQ}{dt}$ = rate of diffusion, cm³/s*)

K = diffusion coefficient, cm²/s.

dc = difference in concentration between the two locations, cm³/cm³

dx = distance between the locations, cm.

$dy\, dz$ = cross-sectional area for diffusion, cm², and the volumes of air, cm³, are reduced to S.T.P.*).

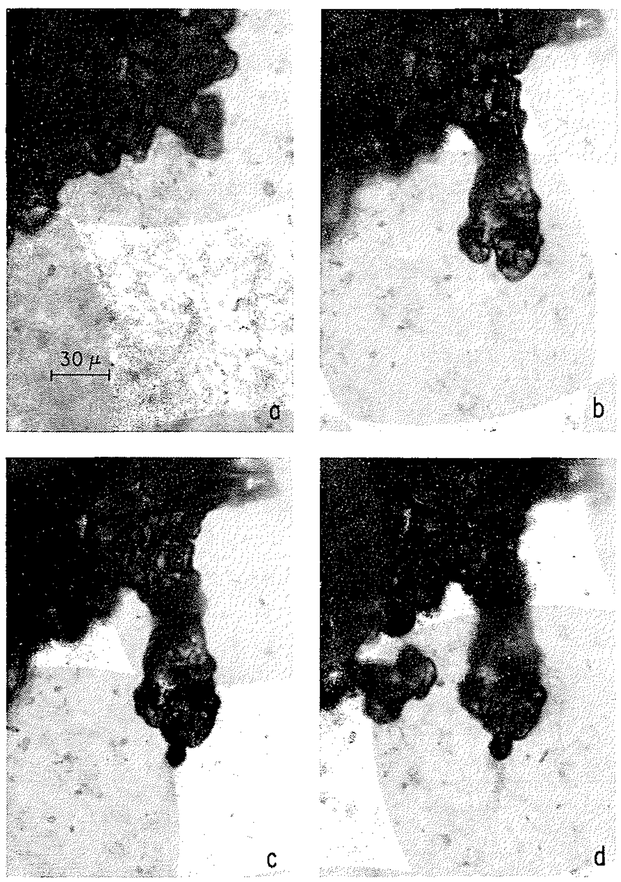
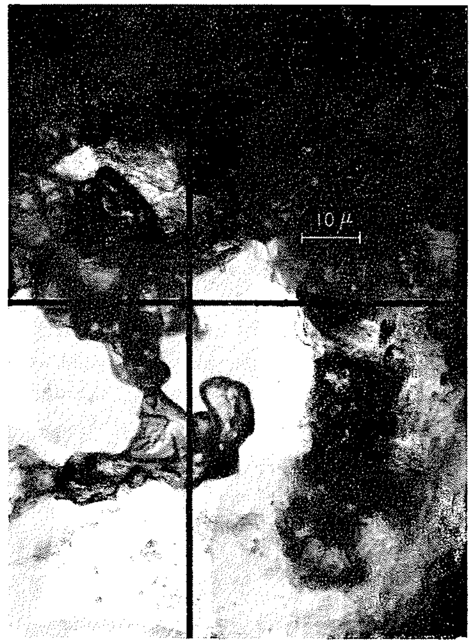


Fig. 9. Progressive advance of air into soil by breakdown of menisci under constant $p_a - p_w$ of 0,30 kg/cm². (a) after 24 hr; (b) after 12 days; (c) after 14 days; (d) after 19 days.

* For dimensional conformity, the mass of air dissolved in a given mass of water is here converted to the volume at S.T.P. dissolved in unit volume.

Fig. 10. Photograph of air intrusion into string of pores. The dark areas are air-filled.



The diffusion coefficient of air in water is $2 \cdot 10^{-5}$ cm^2/s (Liebermann, 1958). The geometry of the system under consideration is complicated but insertion of appropriate values into equation (2) indicates that the observed rate of growth of the bubbles is compatible with that to be expected. The third process of drainage observed in the experiments of the fourth type described above, occurs very slowly. There is no immediate explanation for the gradual advance of interfaces through the soil. This occurred in the test illustrated (Fig. 9) with $p_a - p_w$ about 14 % less than the air-intrusion value for the material in question. It implies that the right-hand term in equation (1) is lower by this amount than in the case of the "rapid" procedure under which the air-intrusion value is measured.

It is possible that the radius of the pore openings is changed by local particle rearrangement, especially in the neighbourhood of the air-water interfaces. Because of the irregular shapes of particles the local stresses arising from interfacial energy effects are complex (such stresses due to ice-water interfaces are discussed by Everett and Haynes, 1965), and might result in such particle movements. The consolidation process described above also presumably involves particle reorientation.

The contact angle of the air-water interface to the particle surface has been assumed to be 0 and has been ignored in equation (1). In fact there may be a small contact angle θ such that the right-hand term must be increased by a factor cos θ. The contact angle might vary with time. Topp (1966) investigated the effect on the interfacial energy air-water of the proximity of various materials. In the present apparatus the Perspex could be responsible for a decrease of perhaps 3 %. This appears insufficient to account for the breakdown of the menisci.

An alternative possibility is that there must be a certain minimum hydraulic gradient ("threshold" gradient) in the pore water for drainage to continue steadily to completion. The last, very small increments of drainage to establish equilibrium might only occur sporadically at long intervals because the gradient is then less than the threshold gradient.

The capillary equation, equation (1), may be insufficient to explain the process by which a slow breakdown of menisci occurs. The equation does not take into account, for example, the presence of properties of the adsorbed water layers surrounding the particles. Slow movement of water may take place in these layers and there may be changes in contact angle or air-water interface radius in connection with this.

The formation of new bubbles apparently did not occur in the pressure range covered by the experiments. If a new bubble were to arise it would have to have a certain minimum size to persist. The radius would have to be such that the pressure of the air in the bubble was in equilibrium with the concentration of dissolved air in the vicinity of the bubble. For a sample in which interfaces already present have assumed a uniform radius r as given by equation (1), then a new bubble must have the same or greater radius. An exception to this would be bubbles occurring in association with hydrophobic sites (discussed below), although such bubbles were not observed in the present experiments. It is probable that the concentration of dissolved air was insufficient to allow the necessarily nearly instantaneous formation of such a bubble.

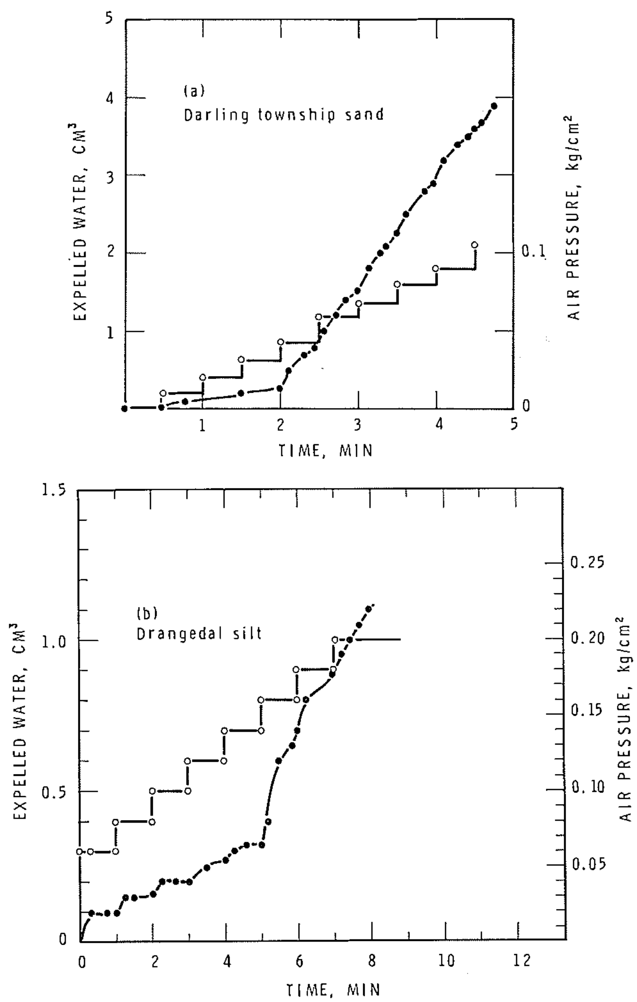
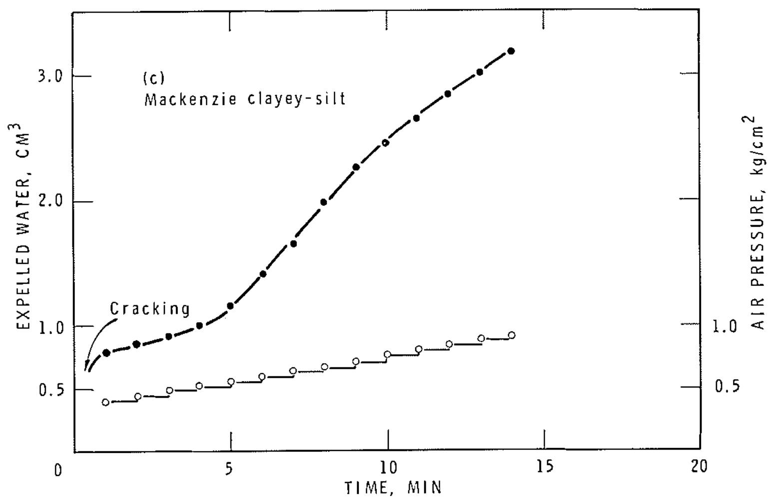
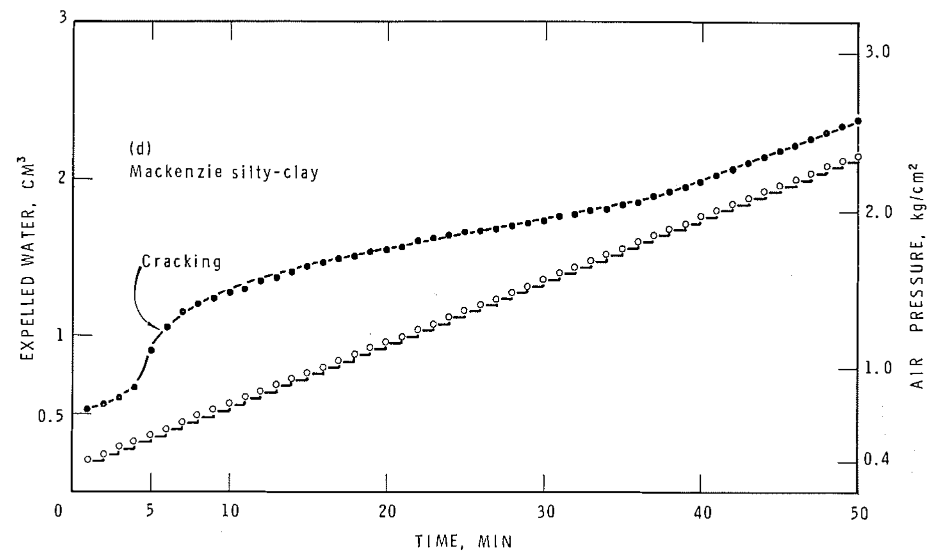


Fig. 11. Results of air-intrusion tests on four soils. (a) Air intrusion occurred at an air pressure of 0,04 kg/cm²; (b) Air-intrusion at 0,15 kg/cm²; (c) Air-intrusion occurred at 0,52 kg/cm². Cracking occurred at about 0,4 kg/cm²; (d) Air-intrusion at 1,95 kg/cm². The exact air-intrusion values are obtained after small corrections as outlined in the text.

The formation of vapour bubbles would not be expected in the present experiments. Vapour in such bubbles would have to have a pressure as low as the vapour pressure of water at the temperature in question, about 0·024 kg/cm² absolute (*i.e.*, –0·976 kg/cm² when atmospheric pressure is taken as 0). From equation (1), substituting p_v, the pressure of the vapour, for p_a, it is apparent that the pore water pressure, p_w, would then have to be lower than this by an amount depending on the bubble radius r.

Although the formation of new air bubbles was not observed in the cryolite soil, it is possible this occurs in soils of other minerals (especially quartz). Liebermann (1957) has shown that bubble nuclei of extremely small size occur on chemically-cleaned glass. These nuclei are believed to be air attached to a hydrophobic site; the air-water interface at such a site would be convex towards the air and the pressure of the air therefore sufficiently low that there would be no tendency for the minute bubble to dissolve. They would increase in size when the appropriate concentration of dissolved air occurs.





DETERMINATION OF AIR-INTRUSION VALUE

The air-intrusion value, the difference in pressure between soil air and water at which the air becomes continuous through the soil, is a single-valued characteristic of a soil's pore structure. It is of significance in the study of partially-saturated soils, for both geotechnical and agronomic purposes. Knowledge of the air-intrusion value of a soil permits evaluation of susceptibility to frost heave (Williams, 1966 (*a*)). The intrusion of ice into a soil on freezing is in many respects analogous to the intrusion of air.

An apparatus (now available commercially*) has been designed and constructed, and a procedure developed to facilitate determination of the air-intrusion value of natural soils. More than fifty different soils, other than more or less pure clays, have been tested and in all cases it has been possible to measure an air-intrusion value. Because of their extremely low permeability, clays require special consideration and have not been investigated in detail. In developing a procedure for the test which would have general application, regard was paid to various factors varying from soil to soil, and which might influence the results obtained.

The procedure for the test has been described in detail (Geonor, 1966). The factors considered and the design of the instrument are outlined here. The apparatus consists basically of a pressure membrane cell which is rapidly assembled. The sample completely covers the membrane which is of the permeable type, and is contained in a Perspex ring. The sample is 1 cm to 2 cm high and 20 cm^2 in cross-sectional area. The proportion of the surface area of the sample in contact with the Perspex containing-ring is small. The small height of the sample in relation to its volume results in short flow paths for the water draining from the sample, and equalization of pore-water pressure occurs relatively quickly after each increment of air pressure. Drainage is observed in a burette.

In the case of substantially compressible samples, it is desirable to consolidate the sample after placing it in the Perspex containing-ring, before carrying out the air-intrusion test. Drainage due to consolidation during the course of the test may otherwise be so substantial as to obscure drainage due to air intrusion. To allow such preliminary consolidation, there is provision for placing a rubber membrane directly over the sample. The consolidating load is applied by raising the air pressure, causing the rubber membrane to bear on the sample with the same pressure. After drainage has ceased, the cell is opened and the rubber membrane removed. The test is then carried out in the usual way.

In carrying out the test, the sample is generally prepared as a slurry. Tests can also be carried out, with some modification, on undisturbed samples. After the slurry is placed, a small quantity of water is added on top of the sample and a measurement of the permeability of the sample and membrane made by lowering the drainage burette and observing the flow.

The permeability measurement can be used to determine the appropriate rate of application and size of air pressure increments. For the apparatus in question the relationship between observed permeability, and appropriate manner of application of air pressure, was determined by analysis of a large number of trials on different soils. In plotting the results of a test (drainage as a function of time), it is necessary to choose scales for the axes which reveal clearly the acceleration of drainage associated with intrusion of air. The amount and rate of drainage vary substantially with soil type. The

* Geonor A/S, Oslo, Norway.

permeability observation is also used as a general guide to the choice of axis scales.

Drainage must necessarily involve a slight difference in water pressure p_w from the top to the base of the sample. Any correction necessary on this account can also be determined from comparison of the permeability measurement and the rate of drainage at the moment of air intrusion.

In relatively coarse-grained soils the point of air intrusion is usually very clearly defined. For finer-grained soils, it is often less conspicuous and more care is necessary in selecting the rate of increase of air pressure and appropriate scales for plotting of results. Fig. 11 (*a* to *d*) shows typical examples of tests carried out on various soils.

As in the case of the experiments described earlier, cracking of the sample may occur giving a smaller preliminary period of accelerated drainage. This has been investigated by tests in which the sample was visible through a window in the cell. If cracking occurs it is prior to, or possibly simultaneous with, air intrusion. The accelerated drainage as a result of cracking is always less than that occurring in association with air intrusion.

CONCLUSIONS

There are three distinct processes by which air may replace water in the pores of a soil when a difference in pressure p_a-p_w is established between the air and water:

(*a*) A process explained by the normal "capillary" equation, equation (1):

$$p_a-p_w=\frac{2\sigma}{r}\cos\theta$$

and the radius r of the air-water interfaces. If this process alone occurs it gives a curve of moisture content versus p_a-p_w in which there is a conspicuous air-intrusion value of p_a-p_w, corresponding to an interface radius r_c equal to that of the largest continuous openings through the soil.

(*b*) A process which occurs more slowly, involving diffusion of air (from air external to the sample) to entrapped air bubbles, with a resulting enlargement of these.

(*c*) A process of slow advance of interfaces into the soil, possibly due to a change with time, of interfacial energy, contact angle or interface radius, or to a process not explained by the capillary equation, equation (1). This occurs under constant p_a-p_w.

Additional drainage may occur because of consolidation due to the stresses on the soil skeleton, arising from the air-water pressure difference. This drainage may occur suddenly with the development of cracks in the soil.

Under appropriate experimental conditions the air-intrusion value may be measured for all soils in a homogeneous state, with the possible exception of those largely composed of clay particles.

ACKNOWLEDGEMENTS

Discussions with Dr. Clarke Topp, Central Experimental Farm, Ottawa, were especially helpful. This paper is a contribution from the Division of Building Research, National Research Council, Canada, and is published with the approval of the director of the division.

REFERENCES

Bishop, A. W., and Henkel, D. J., (1962) *The Measurement of Soil Properties in the Triaxial Test,* 2nd ed., Arnold, London, 228 pages.

Brooks, R. H., and Corey, A. T., (1964) *Hydraulic Properties of Porous Media,* Hydrology Papers, No. 3, Colorado State Univ., 24 pages.

Donat, J., (1938) *Das Gefüge des Bodens und Dessens Kennzeichen,* Int. Soc. Soil Sci., Commission 6, Zurich 1937, Transactions, Vol. B, pages 423–439.

Everett, D. H., and Haynes, J. M., (1965) *Capillary Properties of Some Model Pore Systems with Special Reference to Frost Damage,* RILEM Bull., New Ser. 27.

Geonor, (1966) *Air Intrusion Value in Soils and its Measurement,* (duplicated).

Liebermann, L., (1957) *Air Bubbles in Water,* Jour. Appl. Phys., Vol. 28, No. 2, pages 205–211.

Powers, T. C., (1962) *Physical Properties of Cement Paste,* Proc. Fourth Int. Symp. Chemistry of Cement, 1960, No. 2, pages 577–609.

Schofield, R. K., (1938) *Pore Size Distribution as revealed by the Dependence of Suction (pF) on Moisture Content,* Int. Soc. Soil Sci., Commission 1, Bangor, 1939, Transactions, Vol. A, pages 38–45.

Topp, G. C., (1966) *Surface Tension and Water Contamination as Related to the Selection of Flow System Components,* Soil Sci. Soc. Amer., Proc., Vol. 30, No. 1, pages 128–129.

Topp, G. C., Klute, A., and Peters, D. B., (1966) *Private communication.*

Williams, P. J., (1966) (*a*), *Pore Pressures at a Penetrating Frost Line and their Prediction,* Géotechnique, Vol. XVI, No. 3, pages 187–208.

Williams, P. J., (1966) (*b*), *Movement of Air Through Water in Partly Saturated Soils,* Nature, Vol. 212, No. 5069, pages 1463–1464.

THE NATURE OF FREEZING SOIL AND ITS FIELD BEHAVIOUR

Contents

THE NATURE OF FREEZING SOIL AND ITS FIELD BEHAVIOUR*

By P. J. Williams

INTRODUCTION

In the preceding six papers experiments have been described in detail which show the relationships existing between the temperature of freezing soil, and the stresses or pressures in the water and ice phases, and in the soil skeleton itself. These relationships explain many of the characteristic properties and behaviour of freezing soil; the purpose of this final article is to present as simply as possible, the picture of the soil freezing process presented by the experimental and theoretical work as a whole. It is shown how a few basic equations, the relevance of which has been established by the experimental work, may be used to give a quantitative analysis of many of the phenomena observed in the field by engineers, geologists and other field workers concerned with freezing soil.

Even this picture must include certain concepts with which those largely involved in field studies of frost action may be unfamiliar. These concepts belong in the general fields of physics, chemistry or thermodynamics: They will be described in the simplest terms in this Introduction, as an approach to their utilization in the following pages to describe the behaviour of freezing soils.

General role of interfacial tension.

Small bubbles in water are spherical because of surface tension. This can be ascribed to attractive forces which occur between the molecules at a surface or interface. The surface tension causes the bubble to assume the most compact form – sphere – and also causes the gas in the bubble to have a higher pressure than the water.

If an open-ended glass capillary tube is placed vertically with its lower end in water, water rises ('is sucked') up the tube to a certain height, the 'height of capillary rise', and remains there. The interface (the meniscus) between the air and the water in the tube is curved, because of surface tension. Attractive forces between the glass and the water cause the water surface immediately adjacent to the wall to lie parallel to it (there is a 'contact angle' of 0°). At a point in the centre of the tube the water surface is level, and because of surface tension the interface as a whole accordingly has a form which approaches hemispherical for tubes of small diameter. As in the case of the bubble there is an associated difference in pressure between the air and water at the interface, the water having a lower pressure relative to that of the air which causes the water to rise up the tube, until the weight of the water column is just equivalent to the lowered pressure. The difference in pressure is greater, the greater the curvature. Hence water rises higher up capillary tubes of smaller diameter. A soil is a collection of, albeit very irregular, capillaries. Suction and other capillary effects due to air-water interfaces confined in the pores are familiar.

---

* Contents List, see p. 90.

The phenomenon of a pressure difference across a small interface is by no means limited to combinations of a gas (*e.g.* air) and a liquid (*e.g.* water). The idea of a pressure difference between for example, a solid and a liquid, necessarily occuring if their interface is small, may seem strange. It has been well known to physicists and chemists for many years.

In the case of freezing soils, one is largely concerned with interfaces between ice and water, which are small because they occur within the soil porous system. Ice is a crystalline material and for precision in the following pages is frequently referred to as 'ice crystals'. However, to a substantial extent the term 'ice' would be sufficient – the ice behaving in a similar manner to an amorphous material. The different pressures of the ice and water associated with their curved interface is of fundamental importance when considering the mechanical and other properties of frozen soil.

Insofar as one is considering a solid (ice) and liquid (water), and to a degree a gas (which is water vapour with air), which are all the same compound, water, there is an added complication of basic significance. Liquid water and ice can normally only co-exist (without a change in their relative amounts) at a certain temperature, the melting (or freezing) point of water. It is only a 'point' in a limited sense, because as is well-known, if the pressure on the ice and water is changed there is a change in the temperature at which they can co-exist. However, the 'normal' equation (familiar from elementary texts) relating freezing point of water and pressure often does not apply in the case of water in soils. The 'normal' equation essentially applies only to ice and water subjected to a uniform pressure change. Because ice-water interfaces in soil are generally small, the pressure of the two phases differs. The variation of freezing point with interface size (associated with these pressures) has been established (pp. 39–42). Because interfaces can have a wide range of size in any soil (there being a wide range of pore size), freezing and thawing of a soil occurs over a range of temperatures.

All these phenomena are considered in the following pages.

I. FUNDAMENTAL CONSIDERATIONS

I.1 *Pressure relations for small crystals*

Small crystals in their own melt have, because of surface tension and their large surface to volume relationship, a pressure higher than that of the melt (see for example Adamson 1960). If the crystal is represented by a sphere of radius r, then the pressure difference between the two phases in the case of ice and water, is given by:

$$p_i - p_w = \frac{2\sigma_{iw}}{r_{iw}} \qquad \qquad (1)$$

where p_i = pressure of ice
p_w = pressure of water
σ_{iw} = surface tension ice-water
r_{iw} = radius of interface

This equation also applies with minor qualifications, to ice developing inside a capillary tube. In such a case the ice is polycrystalline and may have a considerable length; the interface between the ice and remaining water is curved (due to the surface tension), and when as is usually assumed the contact angle with the walls of the tube is 0°, it has a radius approximately that of the tube.

A similar phenomenon is the rise of water up a capillary tube, which is a result of the lower pressure of the water immediately below the curved meniscus compared to that of the air above it.

The pores of soils can be regarded as a series of interconnected capillaries, and give rise on freezing to effects of the kind described by eqn. (1).

I.2 *Depression of freezing point*

The pressure relationships described above also influence the freezing point which falls as the pressure difference (eqn. 1) increases, that is, as the size of a crystal, or the radius of the ice-water interface decreases. The equation applicable to soils, describing freezing point as a function of the radius r is (p. 41):

$$\ln\frac{T}{T_0} = -\frac{V_l 2\sigma_{iw}}{r_{iw}L} \qquad (2)$$

which may be written in the finite difference form:

$$T-T_0 = -\frac{V_l 2\sigma_{iw} T_0}{r_{iw}L}$$

where T = freezing point °K
T_0 = normal freezing point (*i.e.* when the pressure is uniform on the two phases)
V_l = specific volume of water
L = latent heat of fusion

T_0 has the value 273,15° K (0° C) if the pressure on the ice phase is atmospheric. This is approximately the case for initially saturated soils not subject to confining pressures, but in many field situations the pressure on the ice is greater than atmospheric, and the value of T_0 must be modified (section I.6 below)*.

Equations (1) and (2) provide the basis for interpreting a wide variety of phenomena characteristic of soils undergoing freezing.

I.3 *Progressive extension of ice with falling temperature*

When a soil is uniformly cooled below 0° C ice formation generally commences on the surface and at a temperature close to 0° C (provided that the sample is saturated or nearly so). Substantial supercooling may occur if the soil as for example in laboratory tests, is shielded from ice nuclei. In such cases, once nucleation has occurred the temperature rises rapidly to just below 0° C. Under natural conditions it is unlikely that nucleation is delayed to any comparable extent, such that little supercooling occurs.

In accordance with equation (2) ice growth first occurs in large pores or openings – these being able to freeze at the highest temperatures. As the temperature falls ice is able to penetrate into smaller openings, and spread through the soil. Subsequently the soil ice mass can be regarded as having a honeycomb structure, possibly with some layers of ice, the 'holes' being occupied by mineral particles and by water in spaces too small for ice-water interfaces to exist in them at the temperature in question. Many pores remain water-filled because all the openings to the pore are too small to allow the interfaces to grow through them. Such water is supercooled but to such a slight

* If salts occur in solution in the pore water, these will also give a small component of freezing point depression (pp. 16–17).

extent that spontaneous nucleation of the ice will not occur. During thawing there is usually somewhat less unfrozen water present (Williams 1963) than during freezing, presumably because *all* frozen pores thaw at the equilibrium temperature predicted by eqn. (2).

Thus the process of freezing of the water in a soil is a gradual one, occurring over a range of temperature. The amount of water remaining unfrozen decreases with falling temperature, but varies greatly from soil to soil because of the different distribution of pore sizes in different soils. Many examples have been given (pp. 15, 18).

The progressive freezing of the soil water as the temperature is lowered involves the liberation of the latent heat of fusion. Over a range of temperatures the heat to be removed to cool the soil by a further increment of temperature, consists not only of the specific heats of the substances present, but also of some heat of fusion. The resulting *apparent specific heats* have been determined, as a function of temperature for various soils (pp. 4–9).

I.4 *Changes in pore water pressure caused by freezing*

The progressive extension of the ice with falling temperature into smaller and smaller openings, must result in the development of smaller ice-water interfaces, and in accordance with equation (1) an increasing pressure difference between the ice and water is established. Equations (1) and (2) may be combined to give this difference in pressure as a function of temperature:

$$p_i - p_w = -\frac{(T-T_0)L}{T_0 V_l} \qquad \qquad (3)$$

Equation (3) is valid for all soils (see p. 42). For situations where the ice phase is substantially under atmospheric pressure (*i.e.* a gauge pressure of 0) the pore water must have a negative pressure, or 'suction' of the magnitude given by this equation. Where the pressure on the ice is other than atmospheric, the value to be taken for T_0 is slightly different (see section I.6), and the pressure on the water modified accordingly.

I.5 *Conditions at the frost line; formation of ice lenses*

In the foregoing sections, the basic equations describing the freezing of soils have been considered, as well as the manner in which the quantity of ice present in a soil is gradually increased as the temperature is progressively lowered. Of special significance is the freezing occurring at the frost line, that is, the boundary between frozen, and ice-free soil.

Frost heaving is caused by the development of layers or lenses of more or less pure ice, much larger than pore size, by migration of water to the frost line, and its accumulation there as ice. The expansion of water on freezing is only about 10 % and the magnitude of observed frost heaves shows that this expansion alone cannot be responsible. The migration of water to the frost line occurs as a result of a fall in pore water pressure in the vicinity of the ice-water interfaces.

If the frost line is to advance through the pores of the soil, the ice-water interfaces must have a size r_c sufficiently small to do so. The pressure on the ice p_i and water p_w are then related by (following eqn. 1):

$$p_i - p_w \geqq \frac{2\sigma_{iw}}{r_c} \qquad \qquad (4)$$

The value r_c for a given soil is an important characteristic; it determines the pore pressure relative to that of the ice, occurring at the ice-water interfaces at the penetrating frost line. Its determination is described in section II.6.

The pressure on the ice is generally equal to the total or confining pressure (see section I.6). When p_i is known, the maximum water pressure p_w that may occur at the interfaces when the frost line is penetrating downwards through the pores can be determined from eqn. (4). Depending on the ground water conditions the pore water pressure may already have this or a lower value. On the other hand it will frequently be greater such that:

$$p_i - p_w < \frac{2\sigma_{iw}}{r_c} \quad \ldots\ldots\ldots\ldots \quad (5)$$

While this is the case, the radius r_{iw} of the ice-water interfaces is somewhat greater than r_c and the ice is unable to penetrate through the soil pores. Instead freezing results in growth of an ice lens. The migration of water to the lens (occurring as a result of the water pressure at the interface of radius r_{iw}), results in a fall in pore water pressure such that the conditions of eqn. (4) are approached. As soon as they are achieved, the ice lens ceases to grow, the ice advancing through the pores instead.

However, the pore pressure will then usually start to rise again (since the removal of water to the ice lens has ceased) so that the condition for development of an ice lens will again arise. It is for this reason that frost heaved soil normally consists of alternate layers of frozen ground and ice in lenses. Furthermore, variations in thickness and frequency of the lenses would appear to be explainable in terms of the permeability of the soil, and the rate of heat extraction (*i.e.* the rate at which water can be transferred to lenses). Very thick lenses are known to be associated with a low rate of freezing and relatively high permeability – conditions where the tendency for lens formation to lower the pore water pressure in the underlying layer would be at a minimum (see also section III.1).

In the case of clay soils the concept of r_c as a radius determined by the size of continuous pore openings, apparently requires some modification. In such soils it appears that the ice at the penetrating frost line does not advance so uniformly through the soil but advances preferentially through scattered paths (probably cracks or other discontinuities) which are for example some cm apart (see p. 64). Ice lenses arise periodically as described above, and between the lenses will be layers or pockets of clay which are substantially free of ice. The existence of these still plastic layers in frozen clays at temperatures near 0° C has often been observed (see *e.g.* Beskow 1935, p. 31). As the temperature falls further, ice will enter the pores also in these layers, which will become rigid.

I.6 *Pressure of the ice, p_i; absolute values of porewater pressure, p_w*

Equations (1), (3) and (4), express the pressure on the water in the soil as a *difference* from the pressure on the ice. To establish the absolute value of the water pressure, as a function of the temperature T, or interface radius r_{iw}, requires a knowledge of the ice pressure.

If the pressure of the ice at the ice-water interfaces changed as the interfaces advanced and changed in size, this would result in closely adjacent parts of the ice being under different pressures. Such a situation is unlikely to occur as it would represent a change away from thermodynamic equilibrium. In the case of water, changes in pressure initiated at the interfaces are immediately spread through the water in the vicinity by fluid flow, so that comparable pressure discontinuities do not occur (see also Haynes 1964).

It is therefore to be expected that all the ice would have the same pressure as that initially formed, at least for samples subject to uniform confining stresses. In the case of a saturated sample at atmospheric pressure the ice would be expected to be substantially at atmospheric pressure (p. 42). Experiments measuring the consolidation of clay layers between ice lenses (pp. 27–35), demonstrated this to be so. The experiments showed that the pore water pressures developed on freezing to various temperatures, were negative and equal to the expression on the right in eqn. (3). It follows that the pressure on the ice remained atmospheric.

If the sample were under a confining pressure other than atmospheric, then the ice would have that pressure. The value of T_0 in eqns. (2) and (3) would then be less by 0,0073° C per kg/cm^2 of confining pressure (see p. 22). For soils freezing at some depth in the ground, the overburden pressure represents such a confining pressure. That the ice in the lenses (which are much larger than pore size) will have a pressure equal to the overburden is immediately obvious, and the reasoning given above leads to the conclusion that also the ice in the pores, at that depth, will have the same pressure. It should be noted that the occurrence of frost heave results in an increase in confining pressure (the 'heaving pressure', see sections I.5, and II.7).

I.7 *Effect of air in soils*

In the foregoing sections it was implicit that the soil was saturated. Under natural conditions however, soils frequently contain significant quantities of air, and this must be taken into account in discussing properties of such soils when frozen. There is a further important reason for studying phenomena associated with the two phases air and water in porous systems: The behaviour of the two phases ice and water in such systems, is in many important respects analogous. In both cases the dimensions of the interfaces between the two phases, as imposed by the size of pores, and the associated stresses and pressures, are of basic importance. Several experimental procedures involving displacement of water by air in soil allow an analogical approach to the behaviour of the same soil undergoing freezing.

When the water content of a soil is reduced below saturation, air enters the pores such that the size of the interfaces between air and water is determined by the pores. As the quantity of air present in the soil increases, the interfaces come to lie in progressively smaller pores. At water contents near saturation, the air is localized in a few large pores, or even merely in such large pores as open directly to the soil surface, but at lower water contents a mass of interconnected air-filled pores extends throughout the soil.

The existence of a small interface as in a pore, results in the water having a lower pressure than the air according to the equation:

$$p_a - p_w = \frac{2\sigma_{aw}}{r_{aw}} \qquad (6)$$

where p_a = pressure of air

p_w = pressure of water

σ_{aw} = surface tension air-water (= 72,75 dyn/cm at 20° C. Hdbk. Phys. Chem. 1965)

r_{aw} = radius of interface

This equation, which is a form of the well-known 'capillary' equation (giving for example, the height of rise of water in a capillary tube), is analogous to eqn. (1).

The progressively lower pressure on the water phase (relative to the air pressure) associated with decreasing water content constitutes the well-known 'suction-moisture content' relationship (Croney, Coleman and Bridge 1952) for the soil in question.

Considering now the freezing of unsaturated soils, it is apparent that when freezing commences, the water is already under a reduced pressure. Reference to equation (3) indicates that freezing can only commence at some temperature T below 0° C, T being lower when the moisture content is lower. Application of eqn. (3) to unsaturated soils requires further consideration of the pressure on the ice, p_i. Unlike the situation with a saturated sample under atmospheric pressure, where p_i is essentially atmospheric, in a partly saturated soil the first-formed ice crystals lie within pores or pore openings. An interface between the ice and air which is restricted in size implies a raised pressure in the ice, relative to the air:

$$p_i - p_a = \frac{2\sigma_{ia}}{r_{ia}} \quad \quad (7)$$

where σ_{ia} = surface tension ice-air (= 107,2 dyn/cm, Hesstvedt, 1964)
r_{ia} = radius of ice-air interface ($\approx$ radius of pore)

However the partial pressure of the water vapour in the air adjacent to the interface would also be increased (according to the Kelvin equation, see *e.g.* Defay and Prigogine, 1951, p. 197) and would be higher than that in equilibrium with the water. Unless ice crystals of larger radius than the air-water interfaces are in some way introduced, it is therefore more probable that the first ice occurs *within* the water and with the largest possible radius, *i.e.* $r_{iw} \approx r_{aw}$. From equations (6) and (1) the ice will then have a pressure lower than that of the air by an amount Δp_i. Accordingly, T_0 in equation (3) should be increased by $\Delta p_i \cdot 0{,}0073°$ C, if Δp_i is given in kg/cm^2 (the equation, p. 22, being appropriate since Δp_i for a given r_{iw} equals the lowered pressure of the water relative to the air). As cooling of an unsaturated soil is continued to lower temperatures ice will spread into smaller and smaller pores in a similar manner as for saturated soils. The unfrozen water content curve (compare pp. 15–18 and section IV.3) may be displaced with respect to temperature by the small amount discussed, but in either case the effect is probably too small to be of practical significance.

Whether or not a given unsaturated soil shows frost-heaving will depend on the amount of air in the soil. There are two effects to be considered. Firstly the permeability of the soil and thus the ability of water to migrate to the frost line will be reduced to the extent that voids are air-filled. The magnitude of frost heaving can thus be sharply reduced.

Secondly, the existence initially of a low water pressure must reduce the magnitude of, or prevent the establishment of, a hydraulic gradient towards a frost line. The lowered pressure in the water may be such that the conditions of eqn. (4) are fulfilled immediately when ice forms. In this case ice growth progresses only through the pores and migration of water to produce ice lenses would not be expected. The situation is expressed by:

$$p_a - p_w \geqq \frac{2\sigma_{iw}}{r_c} - \Delta p_i \quad \quad (8)$$

where $p_a - p_w$ = suction occurring in pore water and $\frac{2\sigma_{iw}}{r_c} = p_i - p_w$ = difference in pressure between ice and water (as described in Section I.5).

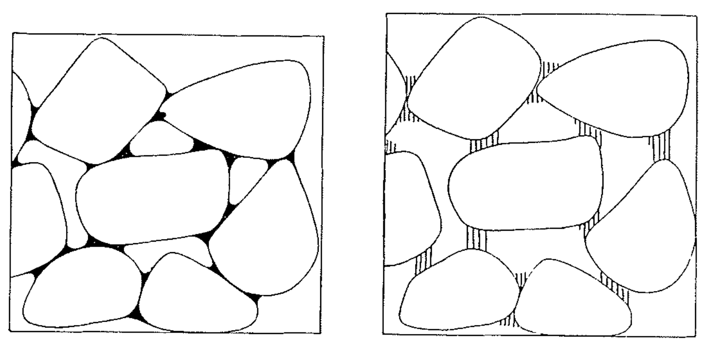
The term Δp_i represents the increment of pressure by which the ice pressure differs from that of the air, due to the curvature of the ice interfaces. Even the development of a very small ice lens would result in the ice having a pressure equal or greater than that of the air, p_a.

Finally it may be noted that if the air has demonstrably spread widely through the the porous structure of the soil, then normal frost-heaving with development of ice lenses cannot occur. Quite apart from the limited water permeability that the soil has, the porewater pressure is then:

$$p_a - p_w \geqq \frac{2\sigma_{aw}}{r_c} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (9)$$

The continuous openings of radius r_c are already empty of water and the condition for ice lens development (equation 10, below) cannot be met.

Although frost heaving would not generally be expected in such unsaturated soils, some change of the soil structure may occur on freezing together with certain pressure effects (see Sections III.3, III.4). In addition, Dirksen and Miller (1966) give detailed evidence for migration of water into the frozen layer in unsaturated soil. The accumulation of ice (including in some cases, lenses) occurs at some distance from the frost line. This is not fully understood, and its practical significance unknown. Beskow (1953) reports an expansion of an unsaturated soil on freezing, that was apparently due to separation of the soil particles by the manner of growth of the ice (see fig. 1). This expansion could be prevented by application of a light pressure.



a b

Fig. 1. Diagram (after Beskow 1935), showing suggested mechanism to explain 'heaving' of an unsaturated sand. (a) before freezing, (b) after freezing (the hatched areas represent ice, in the form of needle-ice crystals).

In contrast, certain soils, when unsaturated, undergo shrinkage on freezing (Hamilton 1966). It is suggested by Hamilton as being due to a process first proposed by Powers and Helmut (1953) for concrete containing air-filled voids. On freezing ice crystals develop in (but probably do not fill) these voids, where their growth is not hindered as it is in the smaller capillary pores of the matrix. The crystals grow by diffusion of vapour from the water in the latter (which has a much reduced freezing point) through the air around the crystal. The loss of water from the matrix pores results in contraction of the material.

II. LIMITING CONDITIONS FOR FROST HEAVE IN FIELD SITUATIONS

II.1 *General case*

Ice lenses develop when:

$$p_i - p_w = \frac{2\sigma_{iw}}{r_{iw}} < \frac{2\sigma_{iw}}{r_c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where p_i, p_w, and $2\sigma_{iw}/r_c$ refer to conditions at the frost line. The pressure on the ice p_i, is generally equal to the total pressure represented by the overburden γx, where γ = bulk density, x = depth. It may be increased somewhat if expansion of the ice lens is resisted by the strength of the soil, but this increment of pressure will be ignored.

The water pressure p_w in the vicinity of the frost line is determined by several factors including, if heaving occurs, the freezing process itself. In predicting whether or not heaving is likely to occur, the latter factor can be ignored. The simplest case is that where (in the absence of freezing effects) hydrostatic conditions may be assumed to exist, such that the porewater pressure p_w is given by the depth to the water table. The water pressure at depth x is then given by $x-z$ where z = depth of water table from ground surface.

The condition for ice lens formation, substituting these values for p_i and p_w into eqn. (10) is:

$$\gamma x - \varrho_w(x-z) < \frac{2\sigma_{iw}}{r_c} \cdot 1{,}0198 \cdot 10^{-3}\ \text{g/cm}^2 \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where γ = g/cm^3
ϱ_w = density of water, 1 g/cm^3
x = cm
z = cm
σ_{iw} = 30,5 dyn/cm (Hesstvedt 1964)
$1{,}0198 \cdot 10^{-3}$ = conversion factor

Insertion of appropriate values into this equation shows for the soil and situation in question whether or not frost heaving (ice lensing) can be expected, and if so the maximum depth to which it is liable to occur (see fig. 2). If heaving does occur, the ice lensing process will tend to lower the porewater pressures from the hydrostatic condition. This only means that there will be a tendency for the maximum depth at which heaving occurs to be somewhat less than calculated. The value $2\sigma_{iw}/r_c$ for a given soil can be determined as described in section II.6. A numerical example of the use of equation (11) to determine susceptibility to heaving is given on pp. 70–71.*

* The value p_w is in the example denoted by the symbol u in accordance with soil mechanics terminology.

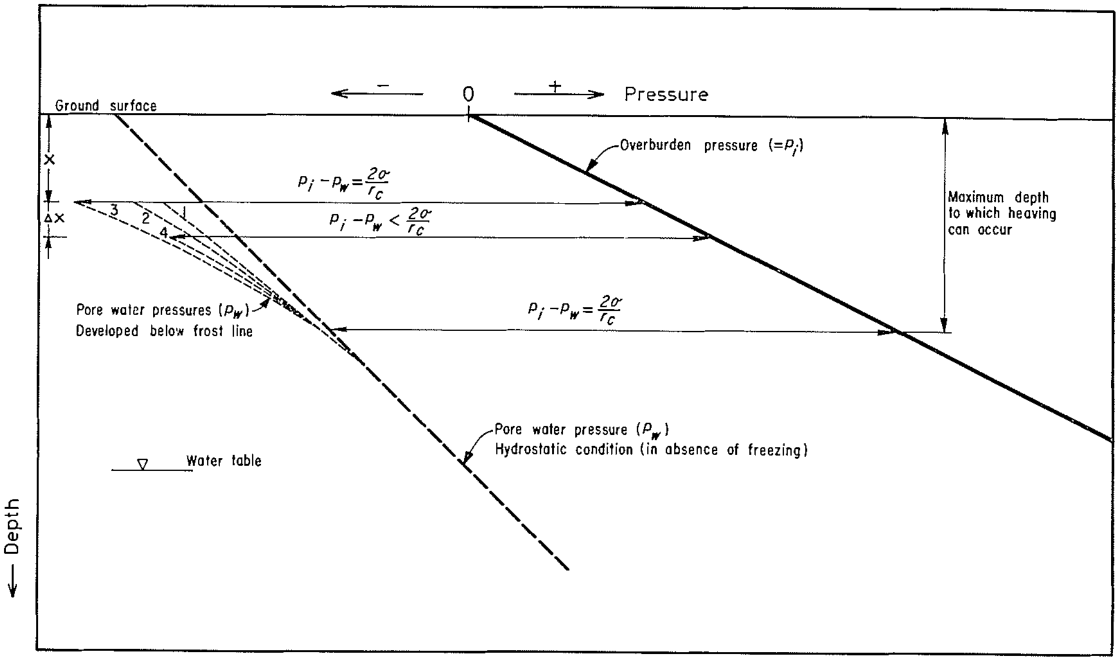


Fig. 2. Diagram illustrating hydraulic gradients in association with frost line during frost heaving. Associated with the growth of an ice lens at depth x is a fall in pore water pressure, giving a changing hydraulic gradient represented by the dotted lines, 1, 2, and 3. When the fall in pore pressure at the ice lens is sufficient that $p_i - p_w = 2\sigma_{iw}/r_c$ the ice lens ceases to grow and the frost line advances through the pores to a depth $x + \Delta x$. During this time the hydraulic gradient changes from 3 to 4, and the pore water pressure again becomes such that $p_i - p_w < 2\sigma_{iw}/r_c$. The next lens therefore commences to develop. Also shown is the maximum depth to which frost heaving occurs. Below this there will be no hydraulic gradient developed towards the frost line.

Equation (11) describes the conditions for frost heave for the simplest situations which may be assumed to occur in the ground. In the following sections various field situations influencing frost heave (see *e.g.* Highway Research Board 1963) are discussed in more detail.

II.2 *Depth of frost line*

Equation (11) demonstrates clearly the effect of depth in determining whether or not ice lens formation will occur. As the depth x of the frost line increases, the first term in eqn. (11), γx, increases faster than the second, $\varrho_w(x-z)$, since the soil bulk density γ is greater than that of water, ϱ_w. At a certain value of x equation (11) will cease to be true, and ice lens formation ceases. Thus, for all soils it is apparent that there is a maximum depth to which lensing can occur. This depth is considerably greater than that of the annual frost penetration in fine-grained soils where the value r_c is large, but is of practical importance especially where 'borderline' soils, with a relatively low value of r_c are being considered. Some examples are shown in table I of soils which are not susceptible to frost heave provided that they are not used in immediately sub-surface layers.

Table I. This table illustrates the pressure difference $p_i - p_w$ that must occur, between the ice and water phases for various soils at the frost line when this is penetrating the soil. The pressure of the ice and of the water also depend on other factors such as overburden and ground water level.

The difference $p_i - p_w$ shows the relative magnitude of the freezing effect for different soils. If there is no overburden or other pressure except atmospheric acting on the ice, then $p_i - p_w$ is equal to the negative pore water at the frost line. If the pore water pressure were atmospheric then $p_i - p_w$ would be equal to the maximum heaving pressure at the frost line.

Note that because the grain size composition is not related in a simple manner to the value r_c, the figures given should be regarded as illustrations only.

General soil type	$p_i - p_w = \dfrac{2\sigma_{iw}}{r_c}$ kg/cm²
Coarse sands, or coarser material only	0
Medium and fine sands, or coarse silty sands	0–0,075
Medium silts, or mixed soils with small amounts $<$ 0,006 mm particle diameter	0,075–0,15
Largely fine silts, or silts with some clays	0,15–0,5
Silty clays	0,5–2,0
Clays	>2

The depth to which ice lensing may occur in fine-grained soils is of importance in regions of permafrost, where the total thickness of frozen ground may be hundreds of metres. Where the soil and porewater pressure conditions were quite uniform with depth prior to freezing (*i.e.* hydrostatic conditions prevailed), it is to be expected that the permafrost will usually be divided into an upper, ice-rich layer containing ice lenses, and a lower part free of lenses. This distribution may be modified by the variations in soil type that are likely within a layer of the thickness of the permafrost. There may also be deviations from the hydrostatic water pressure conditions at certain depths. Few observations of the distribution of ice with depth in permafrost have been made, but results of an analysis of samples from a borehole in the Canadian Arctic (Williams 1968) appears in agreement with the principles outlined here. The lower part of the permafrost contained no visible ice lenses, but higher in the profile ice lenses occurred at intervals, their size and frequency tending to increase towards the surface.

II.3 *Applied loads*

Beskow (1935), and many others have pointed out that frost heave may be prevented by the application of a load. The effect of applying a load to the ground surface is of course similar to that of increasing the depth x, in that the pressure on the ice phase is raised by the amount of the load. If the application of the load occurs in such a way that the porewater pressures remain unaltered, then the load necessary to prevent frost heaving is that which satisfies eqn. (4), see also figure 3. The effect of load in reducing the maximum depth to which frost heave occurs is easily calculated from the equations pp. 70–71.

Application of a load may also cause a change in the value appropriate for the porewater pressure term (eqns. (4) and (11)), if time is not allowed for dissipation of excess pore pressure by consolidation.

II.4 *Drainage*

The value of drainage as a means of preventing frost heave has also been known for many years. Drainage reduces the pore water pressure the effect of which is also clearly illustrated by eqns. (4) and (11). If the effect of the drainage is to lower the water table by a determinable amount (cf. fig. 3), with hydrostatic conditions subsequently pre-

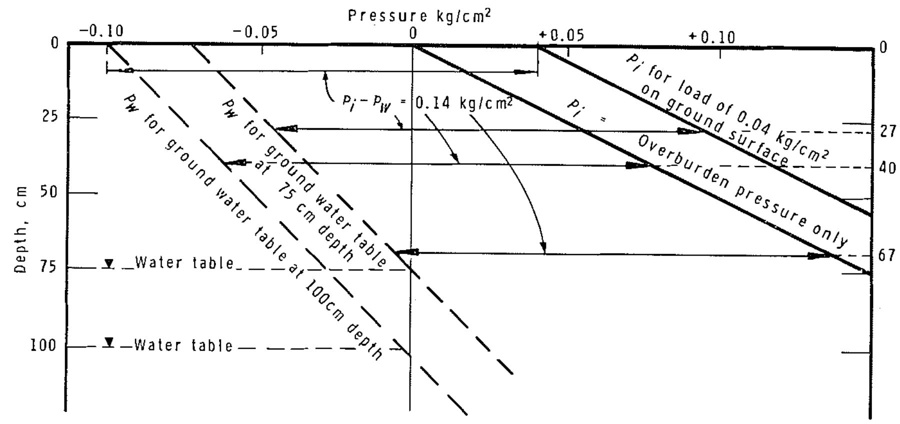


Fig. 3. Diagram illustrating maximum depths, 27, 40, and 67 cm, to which frost heaving will occur for alternative depths of water table, 75 cm and 100 cm, with and without a load of 0,04 kg/cm² applied to the ground surface. The maximum depth to which ice lensing occurs is given by $p_i - p_w = 2\sigma_{iw}/r_c$, assumed in this example to be 0,14 kg/cm² (as might be the case for a coarse silty soil). In the case of the water table at 100 cm depth and the applied load of 0,04 kg/cm², $p_i - p_w = 0{,}14$ kg/cm² at the original ground surface and there is no heaving.

vailing, then the amount necessary to prevent heave is easily calculated (eqn. (11) and pp. 70–71). In practice provision is normally provided for drainage, because of adjacent water courses and associated high ground water levels, or where specially unfavourable conditions may arise, for example, in association with autumn rainfall. Interpretation of the porewater pressure in terms of depth to water table may not then be suitable.

II.5 *Assessment of pore water pressures p_w where hydrostatic equilibrium conditions cannot be assumed*

Rainstorms, snow meltwater, evaporation from the ground surface and other effects produce porewater pressures in the ground which do not change uniformly with depth and cannot be determined on the basis of observations of level of ground water. This will be especially true for relatively impermeable soils, where moisture movement to restore near-equilibrium conditions is slow. Even for relatively permeable soils observations of ground water table level are only satisfactory if made at a time comparable to the period of interest. Pore pressure variations at depths of 1–2 m, equivalent to several metres of water column (1 m water column = 0,1 kg/cm²) are to be expected through the course of a year (see *e.g.* Croney, Coleman and Black, 1958), where the ground surface is subject to infiltration or evaporation. Under a substantially impermeable surface (such as road pavement) near hydrostatic conditions may prevail for most of the year (Russam 1965).

The most precise procedure appears to be direct measurement of the pore pressures. Because the near surface layers which are of main concern in most questions of frost heaving, usually have pore water pressures less than atmospheric, tensiometers (Croney *et al.* 1958 *op. cit.*) with provision for exclusion of air are commonly required. Alternatively observations of water content can be made, and by comparing these with the suction-moisture characteristics of the soil in question a value for pore pressure arrived at. The suction-moisture content characteristics (the variation of suction with moisture content) may be determined in suction plate or pressure membrane tests (Croney, Coleman and Bridge 1952). Although each test requires several days, a number of tests may be run simultaneously. It should be noted that the measured suction is equal to the negative pore pressure that occurs when the soil is under atmospheric pressure. If the soil experiences an overburden pressure, the pore pressure for the observed moisture content is obtainable from the equation (according to Croney, Coleman and Black 1958):

$$u = s + \alpha p \qquad (12)$$

where u = pore pressure
s = suction (from suction-moisture content test)
α = a factor between 0 and 1
p = applied pressure (or overburden pressure)

Examples of value of α for various soils are given by Croney *et al.* (1958 *op. cit.*) and Kassiff and Globinsky (1966). α depends on the compressibility of the soil. For an ideal incompressible soil it is 0. For a compressible soil such as a saturated clay they give the value 1. This is the maximum α can have, and represents the case where the entire applied pressure is carried by the pore water. Whichever method is used to estimate porewater pressures in the ground under non-hydrostatic conditions, substantial errors may be introduced if due regard is not given to changes which may occur between the time of observation, and the onset of freezing.

One situation of particular importance occurs following a temporary warm spell in the winter. A surface layer a few centimeters thick thaws, but is unable to drain freely because of the presence of frozen ground beneath. The thawed layer then has a very high water content (and pore pressures approaching atmospheric or even, locally, greater) which may be further increased by snow meltwater or rain. It is generally believed that this situation is responsible for many cases of damage to highway surfaces, either at the time of the thaw, or after refreezing and subsequent thawing. It is clear from equation (5) that when the overburden pressure is very small so that the pressure on the eventual ice lenses is approximately atmospheric, and the water pressure is also approaching atmospheric, that also soils with a very small value of $2\sigma_{iw}/r_c$ are likely to show ice lens formation. Even fine-sandy materials may thus be unsatisfactory for near surface layers.

II.6 *Determination of the value* $2\sigma_{iw}/r_c$, *by measurement of air-intrusion value*

The value r_c represents the size of the largest continuous openings through the soil pore system. It is not the size of the largest individual pores because these are invariably isolated from the rest of the pore system by smaller openings ('pore necks'). It is the maximum radius which an ice-water interface can have if the interface is to advance through the pore system, and is therefore the size of the interfaces at the advancing frost line.

Although the value $2\sigma_{iw}/r_c$ can be obtained from equation (4) following freezing tests, in which the pressure on the water p_w immediately adjacent to the frost line, and on the ice p_i, are observed, such freezing tests are relatively difficult and time-consuming. A test based on an analogous process, the displacement of water from the soil sample by air, is rapid and convenient (see p. 64). A saturated sample is subjected to an increasing air pressure, over a period of some minutes, while one surface of the sample is in contact with a drainage tube containing water at atmospheric pressure. The water within the soil is therefore also at atmospheric pressure. At a certain pressure, the 'air-intrusion value', a rapid acceleration of drainage occurs, and air spreads generally through the soil. The difference between the air pressure p_a, and water pressure p_w in the pores, has become such that the air-water interfaces can advance through the soil and (Eqn. 9):

$$p_a - p_w = \frac{2\sigma_{aw}}{r_c} \quad \text{. . . . .} \quad (13)$$

where σ_{aw} = surface tension air-water

As the water pressure is atmospheric (*i.e.* 0) the applied air pressure (*i.e.* the air-intrusion value $= 2\sigma_{aw}/r_c$), on multiplying by σ_{iw}/σ_{aw} (= 0,42 approx.), gives directly the difference $p_i - p_w$ that will be established at a penetrating frost line in the soil in question.

The value $2\sigma_{iw}/r_c$ varies from virtually nothing for coarse sand, to several kg/cm^2 for clays (table 1). Silty soils often lie in the range 0,05 to 0,3 kg/cm^2; the value is not easily estimated from observation of grain size.

II.7 *Expansive pressures associated with frost heaving*

The expansive pressures associated with the freezing of frost-heaving soils are of two main types: Those associated with the 'normal' expansion of water on freezing (ice having a volume about 9 % greater than the water from which it is formed), and those associated with increase of water content (*i.e.* as ice) in soils subject to frost heaving. Only the latter is regarded as 'heaving' pressure. Both types of pressure are important in some problems of soil engineering, but the former less frequently so.

The heaving pressure is obtained from Eqn. (4), the pressure of the ice being equal to the heaving pressure:

$$p_i = p_w + \frac{2\sigma_{iw}}{r_c} \quad \text{. . . . .} \quad (14)$$

Heaving occurs only so long as ice is forming outside the pores in ice lenses. The maximum heaving pressure is of course equivalent to the load necessary to just prevent heaving (see Section II.3).

The special significance of heaving pressure is that it is not relieved by small deformations of the confining body. Thus soil heaving against a foundation, lifting foundation walls or piles, or disrupting a semi-rigid road surface continues to exert heaving pressure up to the maximum value, for as long as the freezing conditions are appropriate. Substantial displacements may occur. Pressure caused by the expansion of water freezing to ice, on the other hand, is relieved by deformations corresponding to this small expansion; these deformations are sufficiently small to be unimportant in many situations.

Table 1 illustrates typical heaving pressures for various soil types. The value of the heaving pressure in a particular situation will depend on the pore water pressure, p_w, in the region of the frost line. Assessment of p_w must be made in a manner similar to that described in sections (II.1, II.5). p_w itself may be modified by the development of heaving pressures. Uniform vertical heaving involves development of heaving pressures which are essentially equal to the overburden pressure, and there is thus no increase in pressure felt by the underlying unfrozen layers. Where heaving occurs irregularly and to some extent laterally against rigid structures, the heaving pressure may be transmitted back to the unfrozen soil. This may result in a significant rise in pore water pressure p_w, especially in little-permeable and compressible soils where it would not be quickly dissipated, and in turn leads to heaving pressures higher by an equal amount, since Eqn. (14) must still apply.

The term heaving pressure is not entirely satisfactory because heaving is oriented. The direction of the heaving is largely that of least resistance. However heaving occurs perpendicularly to the ice lenses and their orientation is also affected by the direction of heat flow during freezing; they tend to be oriented perpendicular to the heat flow. Ice lens orientation is also affected by stratification of the soil to which they tend to be aligned parallel. The various factors therefore, which may influence the heaving pressure in a given direction are not fully elucidated. For most practical considerations it seems appropriate to allow for the possible development of heaving pressures as great as those given by Eqn. (14), above*.

Pressures associated with the volume increase of water freezing to ice, are developed where the soil is confined in a manner such that this small volume increase is hindered. Such situations are similar to the cracking of a water-filled, stoppered glass bottle on freezing. The pressure Δp_i developed depends on the temperature below freezing, and, according to the usual relationship (p. 22) $\Delta p_i = 1/0{,}0073 = 137$ kg/cm^2 per $-°$ C.

There is a maximum possible pressure of about 2000 kg/cm^2 (developed at temperatures of about $-22°$ C). This is the highest pressure at which the normal type of ice, ice 1, forms, and at higher pressures a type of ice is formed which does not have a greater specific volume than water (Dorsey, 1940 p. 397).

Pressures of this kind also arise in the freezing of porous materials such as bricks and building stones, with a rigid structure unable to accommodate the volume change where water freezes within pores already closed by ice. Destruction of such materials by frost action seems to be due largely to the expansion associated with the freezing of *in situ* water (Butterworth 1964). The permeability may be sufficiently low that the water itself resists the expansion (Powers 1945). Less high pressures tend to occur if some air is entrapped in the pores, the easily compressed air allowing ice expansion and water displacement. Damage due to freezing is thus reduced. This is utilised in the air-entrainment technique for improving resistance of concrete to damage by frost.

A detailed analysis by Everett and Haynes (1965) has also shown the manner in which differential stresses may be set up within particles and at interparticles contacts, as a result of the irregular shape of the interfaces between ice and water, particles and water, and ice and particles. The localised differential stresses also appear to be substantially independent of the large-scale migration of water and associated ice lens formation.

* Hoekstra (1966) gives evidence for heaving pressures substantially greater than would be predicted by Eqn. (14). The associated heaving is probably occurring within the frozen layer where temperatures are lower and the interfaces of smaller radius than r_c. The magnitude of such heaving would probably be much less than that initiated at the frost line, and the practical significance of the higher pressure is not known.

They probably play a significant role in the breakdown by frost action of rigid porous bodies.

Small deformations and displacements of the type producing failure in building stones do not produce such immediately obvious effects in soils. However displacement of individual particles occurs (Corte 1963) and produces soil structural changes, and thus changes in the strength of the soil after thawing (see Section III.4).

III. ICE FORMATIONS; STRUCTURE OF FROZEN AND NEWLY-THAWED SOIL; STRENGTH AND DEFORMATION OF FROZEN SOIL

III.1 *Distribution and form of ice lenses; magnitude of frost heave*

Two important aspects of the soil freezing process are the manner of distribution of ice and ice lenses, and the total heave of the ground surface produced in a given time. The former is important especially with regard to the strengths of the frozen and thawing soil, while the latter is significant in engineeering applications (*e.g.* railways where displacement of the ground surface cannot be tolerated).

Ice lenses are normally aligned roughly parallel to the ground surface, but may be arranged quite irregularly and even vertically; the lenses may be simply irregular chunks of ice, or thin and hair like (see illustrations in Beskow 1935), and closely or widely spaced. To some extent the positioning of ice lenses is determined by soil layering or cracks.

The alternation of lens formation and associated depletion of moisture from the immediately underlying soil, with periods of advance of the ice through the soil pores while the water pressure is at a minimum, was described briefly in Section I.5. If the heat flow from the frost line is very small, ice will form slowly. The flow of water to the lenses may be such that the fall of pressure necessary before the ice will advance through the pores occurs only after a considerable period of time. Furthermore, when the lens ceases to grow the water content of the adjacent unfrozen soil will be rapidly replenished, giving conditions for formation of the next lens in close proximity with the former one. The ice lenses will tend to be large and closely spaced. It is often observed that the presence of a snow cover (reducing the rate of cooling of the ground), results in a higher ice content of the frozen soil.

The permeability of the soil may limit the size and/or frequency of ice lenses even when the rate of cooling is slow. When permeability is low (as in clays), the pore pressure falls rapidly as an ice lens grows, since the replenishment of water to the adjacent moisture-depleted layers is then slow. Similarly, the hydraulic gradient towards the frost line may be limiting. It depends on the characteristic r_c of the soil, its permeability, the depth in the ground, the ground water conditions, and also on the rate of freezing and whether or not an ice lens is in process of formation. When frost heaving is occurring, the hydraulic gradient in the vicinity of the frost line is usually not linear, and changes constantly. The manner in which this gradient is presumed to vary with depth is illustrated in fig. 2.

Prediction of the thickness and spacing of ice lenses, even assuming that the limiting factors are correctly interpreted would require a detailed knowledge of the soil, and the *in situ* thermal and moisture conditions. Even where these factors are evaluated the mathematical analysis required would be of considerable complexity. However, the relationships outlined provide a qualitative picture of the ice lens size- and frequency-determining factors. In summary, high ground water level, low overburden pressures (shallow depth), slow rate of freezing, high permeability and a small value of r_c tend to result in large and frequent ice lenses If any of these factors do not occur, depending on the degree to which the others are present, ice lensing may be substantially reduced or even prevented.

The magnitude of frost heaving (the elevation of the ground surface) is governed substantially by the same factors, except that the effect of the rate of freezing must be considered from a somewhat different point of view. The amount of heave occurring will also depend on the length of time during which freezing is occurring. The heave is due to a certain excess of water (as ice) which may be disturbed in a thick or thin layer of soil. The heave therefore does not directly depend on the excess water *per unit volume* of soil.

For an initially saturated soil, the heave is given by (following Ruckli, 1950):

$$h = KJ\, 1{,}09\, \Delta t \qquad (15)$$

where h = heaving in time Δt

K = coefficient of permeability

J = hydraulic gradient to frost line

1,09 = factor for volume increase on freezing of water.

If the underlying soil layers are compressible the heave may be reduced somewhat due to settlement associated with the moisture migration from these layers. Unfortunately the value of J, the hydraulic gradient, as a function of time t, must be defined before a prediction of the amount of heave can be made. This is difficult especially because J varies as the frost line proceeds downwards, as illustrated by fig. 2.

Considerable attention has been given to the possible effect of rate of freezing (*i.e.* rate of heat extraction) on the amount of heave, by Beskow (1935) – who showed that heave was independent of freezing rate, Dücker (1939), Penner (1959) – who showed that heave increased with freezing rate, and others. This divergence of experience must be due to the different experimental arrangements used and their significance with respect to the variation of J. Increase in heave must not be confused with increase of ice content per unit volume: also Penner's experiments showed that ice content *per unit volume* was greater for *low* rates of freezing, although the heave rate was greatest for *high* rates of freezing.

III.2 *Some special cases of ice layers: glaciers, pingos*

There are several geomorphological features which in a simple view consist of a thick layer of ice lying on a bed of soil. Whenever there is a flow of heat away from the interface between ice and soil and the soil is unfrozen, the basic thermodynamic considerations of section I become relevant.

The most widespread and obvious example is that of glaciers. A glacier bed is often soil material (possibly originating as a result of glacier action), and depending on climatic and to a lesser extent other factors, the bed is quite frequently unfrozen.

Only a few specific situations will be considered here. They indicate that the phenomena discussed in this article may be important in connection with for example, the sliding of a glacier over its bed (Weertman 1964, Kamb and LaChapelle 1964), the hydraulic pressures (porewater pressures) in the soil in the neighbourhood of the ice, and within the ice, the transporting of soil materials, their deformation and consolidation, the question of the extension of permafrost below advancing glaciers, and of the thermal regime as a whole of the glacier and underlying material.

A temperate glacier is one defined (Ahlmann 1948, p. 66) as having a temperature at depth equal to it's 'pressure melting point'. This temperature is given by the usual equation (see p. 22) for change of melting point where pressure is applied equally on both ice and water phases. The temperature will therefore depend on depth from the glacier surface:

$$T_x = 0{,}1\varrho_i \cdot x \cdot 0{,}0073^\circ\,C \qquad (16)$$

where T_x = temperature, $-^\circ$ C, at depth x metres
ϱ_i = specific wt. of ice g/cm³
$0{,}1\varrho_i \cdot x$ = pressure on ice, kg/cm² at depth x
0,1 = conversion factor
0,0073 = change of melting point with pressure, $^\circ$ C per kg/cm²

(this equation only approximately describes the situation in so far as there may also be other stresses on the ice than the vertical one associated with overburden).

Ice can only penetrate the soil immediately below the glacier provided a certain pressure difference as given by eqn. (4) exists between the pore water and the basal ice, and associated with this pressure difference is a lower freezing point (eqn. 3). As a first conclusion therefore soil immediately below a glacier at its pressure melting point as defined by Ahlmann, cannot have ice in its pores. Were such ice to occur, in a stable state, this would imply a temperature discontinuity between the base of the ice and the adjacent soil which would be impossible. The only way in which ice could occur in the soil bed would be for it to be located in cracks or openings considerably larger than pore size.

If there is a cooling of the temperate glacier (such as would be expected to occur as a result of climatic change) the soil beneath will not immediately freeze. In order for the ice to develop within the soil pores the necessary difference between the basal ice pressure, and soil water pressure, p_i-p_w (eqn. 4) must be established. This difference may soon occur, but on the other hand in many cases the ground water conditions will be such that the porewater pressures will be maintained at a higher value for a prolonged period. During this time, ice will accumulate on the base of the glacier, by migration of water up through the soil.

The magnitude of porewater pressures in unfrozen soil below glaciers has received limited attention (*e.g.* Mathews and Mackay 1960, Mathews 1964). It appears possible that under certain circumstances they can approach a maximum value equal to the weight of the ice itself. Normally they will be somewhat less, the weight of the ice being borne in part by the soil skeleton. There is likely to be a relatively steady and complete replenishment of water to the subglacial layer (bearing in mind that the accumulation of basal ice would in any case occur quite slowly, because of the slow rate of heat loss through the glacier mass). Under these circumstances the ice may only penetrate the pores of the soil (if the pores are fairly small) after there has been a considerable increase in thickness of the glacier ice, in order to give the necessary difference in pressure, p_i-p_w, at the base of the glacier, in accordance with eqn. (4). Until the appropriate thickness is reached there will be a migration of water from the soil to the base of the glacier. Conditions for a substantial 'basal accumulation' of the ice can thus arise. It would be particularly likely where glacier advance over unfrozen soil (*i.e.* without permafrost) occurred in association with a cooling of the climate.

As an example consider a fine silty soil, with a value $2\sigma_{iw}/r_c$ such that (eqn. 4) $p_i - p_w = 0{,}5$ kg/cm². If this soil initially underlies a mass of ice, and the porewater has a pressure about equal to that of the ice (*i.e.* its weight), then the ice thickness would have to increase by at least $\Delta x = 0{,}5/\rho_i$ (where ρ_i = density of ice, 0,0009 kg/cm³), that is, 560 cm or 5,6 metres before freezing of the soil occurs. If the porewater pressure tended to rise as the ice thickness increased, then an even greater amount of ice would be added before the ice could penetrate the soil pores.

Particularly with fine-grained soils, under certain circumstances it is conceivable that ice could penetrate downwards through cracks or rupture planes. Isolated blocks of unfrozen soil might thus become incorporated into the moving base of the glacier. Being unfrozen they might be subject to substantial deformation. The manner in which freezing occurs at the base of the glacier thus has importance in connection with basal transport and deformation of sedimentary material.

Pingo: Pingos are, typically, conspicuous conical mounds, that may be 30 m or more in height, occurring in cold climates. Geographically they appear limited to certain regions (*e.g.* Northeast Greenland, Mackenzie Delta in N.W. Canada, parts of Arctic Russia *etc.*) where the permafrost does not extend particularly deep. In the centre of the pingo is a core of ice, often tens of metres thick.

They have attracted the attention of many explorers and others. Two important papers, Müller 1959, and Mackay 1962, ascribe their formation to the freezing of subterranean water, accumulating under a high enough pressure to raise a surficial frozen layer. In ...se of the N.E. Greenland pingos, the artesian pressure is believed by Müller to be due to water moving down between impermeable layers (frozen ground or rock) from higher elevations.

Mackay believes that, in the Mackenzie Delta, beneath each pingo there is a column or bulb of unfrozen ground being steadily reduced in size by freezing from the sides and to some extent from below (the ground thermal conditions are such that the existence of these columns is likely). This encroachment by the frozen region, Mackay believes, causes a concentration of water in the column and rising water pressures. The existence of water pressures of this type appears to be well documented in Russian literature (Shumskii, 1964 pp. 223–229); being artesian pressures they are likely to lift an impermeable layer higher in the profile. The surficial frozen layer represents such an impermeable layer.

Both Müller and Mackay discount the role of the frost heaving process in the development of pingos. If the ice core lies upon unfrozen soil, however, this must constitute one of the situations referred to in the opening paragraph of this section, where the dependence of $p_i - p_w$ on r_c cannot be ignored. The nature of the soil in the vicinity of the underside of the pingo ice core is not accurately known, but in the case of the Greenland pingos, silty, sandy and clayey soils occur in the area. So long as the soil is not very coarse-grained, freezing will cause ice to form outside the pores, *i.e.* as a lens or lenses, or pingo core, until a pressure difference $p_i - p_w$ exists between the ice and water according to eqn. (4). A silty-clay soil for example may have a value $p_i - p_w$ at the frost line of 0,5 kg/cm². In the case of such a soil, the pressure in the base of the core ice will *exceed* the porewater pressure by this amount before the core ceases to grow. The frost heaving process is therefore responsible, in this example, for an increment of uplift pressure which corresponds to the weight of a more than 5 metres thick layer of ice.

In the case of coarse-grained soil there will be only an insignificant, or no such increment. Pingos occurring in such a situation must therefore be ascribed to the water pressure effects alone, as described by Müller and Mackay. In the case of fine-grained soils, however, the frost heaving process will determine the maximum height to which the pingo can grow. In cases where the porewater pressure by itself is not sufficient to lift the overlying material, the increment of pressure in the ice (*i.e.* $p_i - p_w$) may be entirely responsible for the existence of the pingo.

To some extent the growth of a pingo would appear to be self-perpetuating. As the thickness of the ice core increases, some further rise in the porewater pressure in the underlying layers would be expected and this in turn would lead to a greater value of the ice pressure before segregation of the ice ceases.

III.3 *Consolidation effect*

In accordance with the effective stress equation (see Skempton 1961):

$$\sigma' = \sigma - u \qquad (17)$$

where σ' = effective stress
σ = total stress
u = porewater pressure

a change of porewater pressure results in a change of effective stress. Upon the latter, for saturated unfrozen soils, depends the consolidation and shear strength of the soil.

The pressure p_i on the ice of a frozen soil, equals the total stress σ. The pressure difference $p_i - p_w$ established on freezing (eqns. (1) & (3)) is an effective stress according to eqn. (17). There is therefore a consolidation effect associated with freezing.

Immediately below the frost line, in the unfrozen soil, there is a layer subject to an effective stress approaching that represented by $p_i - p_w = 2\sigma_{iw}/r_c$ (eqn. (4)). This hard, consolidated but ice-free layer is often observed in boreholes and excavations in clay-rich (*i.e.* compressible) soils. Its thickness will depend on the rate at which freezing occurs, the soil permeability etc.

After the penetration of ice through the soil has occurred, a progressively higher effective stress is developed as the temperature falls (eqn. 3). In soils which are predominantly silty, ice then fills the pores fairly uniformly and any consolidation occurring would appear to be limited to local, microstructural agradations in which small groups of particles, separated only by small volumes of unfrozen water, are drawn closer together.

In the case of clay soils, the unfrozen clay layers present between ice lenses at relatively high temperatures (section I.5), are substantially consolidated (the effective stress reaching several kg/cm²) before the ice penetrates the pores of these layers (see pp. 27–35). Even after this occurs, structural changes occur within the clay. According to recent investigations by Anderson and Hoekstra (1966), water is removed from the interlamellar spaces of the clay mineral particles at temperatures of −5° C and lower.

After the clay has thawed, the water drains from the soil without becoming reassociated with the particles in the original manner. The void ratio of the clay and its saturation moisture content are decreased, and the freezing process is thus to be regarded as a form of preconsolidation. The presence of voids representing sites of ice lenses commonly produces a higher bulk void ratio but this effect is temporary (see section III.4). Stuart (1964) gives examples of settlements of clay soils attributed to a freeze-thaw cycle experienced by clay never previously frozen. The reported contraction of unsaturated clays (Hamilton 1966) on freezing should also be noted (see section I.7).

The soil structural changes associated with this preconsolidation are not exactly comparable with those occurring under normal loading. Nevertheless, macroscopically the volume changes of clays are of similar magnitude, in relation to the effective stresses developed. As shown by eqn. (3) and on p. 44, the effective stress developed by freezing even to a temperature of $-0{,}5°$ C is large (about 6 kg/cm^2) and the resulting consolidation correspondingly significant.

The well-known 'drying' or 'weathering crust', a layer up to several metres thick immediately below the ground surface in clay soils, is often thought due to the effective stresses developed during drying of the soil. Alternatively processes of chemical weathering may strengthen the soil. It appears that in cold climates, the crust might also be a result of winter freezing of the soil. A close study of the relative importance of drying and freezing in this respect would appear desirable. In regions which at some time in the past have experienced permafrost, preconsolidation of clay layers to considerable depth is to be expected, the degree of preconsolidation at each depth depending on the lowest temperature reached.

III.4 *Structural changes following freezing and thawing*

In addition to the effects of the developed stresses described in the previous section, soil structure is affected in several ways by freezing.

Macroscopically, freezing of a previously unfrozen clay-rich soil results in development of a characteristic shaly, or flaky, structure. The flakes represent the clay layers which lay between ice lenses, while the discontinuities between them represent separations due to the former presence of the ice lenses. Immediately after thawing frost heaved soils usually have a high void ratio and moisture content because of the voids left by thawing ice lenses. Their strength is then substantially reduced (cf. Williams 1959). Subsequently these voids close up, at a rate largely dependent on overburden pressure.

Stones and boulders are often pushed to the ground surface (Viborg 1955, Corte 1961) and may be arranged in various forms of patterned ground (Washburn 1956). Although many processes are involved, differential frost heave, associated in part with the different conductivities of stone and soil is of basic importance.

Microscopically, the development of ice lenses appears to result in some degree of separation of fine particles from a coarser matrix (Corte 1963). This separation is related to differential stresses arising in association with interfacial energy effects on particles of different size and shape (Everett and Haynes, 1965). Changes in grain size composition following freeze-thaw cycles may occur if the grains are themselves porous.

III.5 *Ice-water transitions and their relationship to strength and deformation of frozen ground*

Recent works (*e.g.* Tsytovich 1959, Vialov 1959, Sanger and Kaplar 1966) point out the complex rheological behaviour of frozen soils. Two phenomena of special significance are the observed temperature dependence of the strength of frozen soil, and the dependence of strength on the duration of application of stress. The resistance of frozen ground to deformation by applied loads decreases with time to perhaps 10 or 20 % of its initial value, leading to the concept of 'initial' strength and the much lower 'continuous' strength.

Processes involved in deformation of frozen ground include: an instantaneous, elastic and reversible deformation, a plastic deformation ascribable to plastic properties of the included ice and possibly of other soil components, and deformations in connection with changes of structure and composition associated with the freezing and thawing of ice and water. The work described in this volume, relating ice and water pressures, and temperature, is relevant in the latter respect.

If a load is applied to a frozen soil, while the temperature remains constant, there is a change in the proportions of ice and water; associated with this are changes in the pressure of both ice and water phases. Such effects change the resistance of the soil to load. In so far as changes of pressure in the water phase occur, hydraulic gradients are likely to be set up between the stressed and unstressed areas of the soil. A migration of water, at a rate determined by the permeability of the frozen soil, will also occur. Such migrations themselves are one of the mechanisms by which a time-dependent deformation (creep) occurs. In this section these considerations will be illustrated by analysis of a specific example.

Consider a fine silty soil, at a temperature of –0,5° C, and containing ice lenses, and also ice within the soil pores. The soil is initially not subject to an overburden pressure and the ice therefore has a pressure equal to atmospheric. The water pressure p_w is lower than the ice pressure p_i, by an amount $p_i - p_w = 5{,}97$ kg/cm², as given by eqn. (3) with $T = -0{,}5°$C, (272,6° K) and $T_0 = 0°$ C (273,1° K). If a load of for example 1 kg/cm², is now applied uniformly over a given part of the surface (for example by a circular plate) this load is initially carried by the underlying soil and the ice lenses which it contains (if the soil is saturated the pressure of the water will also initially increase by an equal amount). It is assumed that the pressure experienced by the ice lenses, is also experienced by the ice within pores. If the temperature remains unchanged, with the new values of p_i and p_w the conditions of eqn. (3) are not satisfied. For the loaded condition, the value to be taken for T_0 is as pointed out in section (I.6), lower by an amount $\Delta p_i \cdot 0{,}0073°$ C. The value $p_i - p_w$ for equilibrium (eqn. 3) at -0,5° C for the loaded condition is then 5,88 kg/cm². Melting of some ice therefore occurs, at the ice water interfaces, with the result that they come to lie in pores of slightly larger radius r. The increase in radius of the interfaces soon gives, following eqn. (1), the equilibrium value of $p_i - p_w = 5{,}88$ kg/cm². The value of the water pressure p_w is then $-5{,}88 + 1 = -4{,}88$ kg/cm².

Outside the loaded region the water pressure still has the value –5,97 kg/cm², and a significant hydraulic gradient thus exists. The permeability of frozen ground is small but flow will occur slowly until this gradient is eliminated. In the non-loaded region this flow will result in a rise in p_w, such that $p_i - p_w < 5{,}97$ kg/cm². The equilibrium conditions of equation (3) are disturbed but in this case further freezing will occur, in such a way as to tend to restore the equilibrium value of $p_i - p_w$. Further ice will be added to the ice already present in the non-loaded region, constituting a form of frost heave of the already-frozen soil. This process of transference of ice by melting and refreezing, from the loaded to the non-loaded region of the soil will continue until the water pressures are equalised. Detailed consideration of eqn. (3) leads to the conclusion that this will probably only occur when the ice in the loaded region has ceased to carry any part of the load. This suggests that all the ice lenses (which carry the load) must disappear, their substance being transferred to the non-loaded region. That in fact such a process occurs is illustrated by Russian experiments (Vialov 1959) in which ice lenses below a circular loading plate were shown to disappear with time.

Although no work appears to be reported where the experimental observations of behaviour of frozen ground under load have been analysed in detail with respect to the ice-water transitions involved, it is clear that at least in the range 0 til −2° C such transitions are, together with other processes not considered here, of fundamental importance. At lower temperatures the quantity of water present, at least in other than fine-grained soils (clays) is quite small. Ice-water transitions are then possibly less significant, and in any case, because the interfaces are so small at lower temperatures, these transitions can probably not be analysed in the manner illustrated above.

Although the example above included an assumption of constant temperature, changing temperatures also have effects which may be analysed in a similar way. It appears that merely the existence of a temperature gradient is sufficient to cause a slow redistribution of ice in a soil. Hoekstra (personal communication) has described experiments illustrating this effect in a silt. These gave a 'permeability' of the frozen soil, *e.g.* 3×10^{-5} gm/cm² sec. for a temperature gradient of 1° C/cm. Although in his experiments the applied temperature gradient was substantially in excess of those normally occurring under field conditions, and ice lenses did not occur to hinder water movement, it appears that where a temperature gradient persists over long periods (for example, in permafrost) significant redistribution of ice may occur.

In many engineering considerations, the frozen ground is subject to both temperature and stress changes, while its temperature is within a few degrees of 0° C. It is precisely in this range that ice-water transitions are most marked, and give frozen ground its special characteristic as an engineering material very close to (or 'within') its melting point. At temperatures below about −1° C, the simple analysis given above may require modification, but the occurrence of similar effects is established to temperatures of at least −5° C (Hoekstra *op. cit.* and personal communication).

IV. THERMAL PROPERTIES OF FROZEN SOILS

IV.1 *General considerations*

Many practical problems, such as prediction of depth of frost penetration, calculation of heat loss from pipes in frozen ground *etc.* require a knowledge of the thermal properties of frozen ground. Research studies of strength and deformation behaviour, and other processes in frozen ground may require a detailed understanding of the manner in which the thermal properties vary. The basic thermal relationship of the ground from an engineering point of view are well described by Terzaghi (1952).

The two fundamental thermal properties of the frozen soil are specific heat (calories/g · °C in cgs units) or alternatively heat capacity (calories/cm³ · °C), and thermal conductivity (calories/cm² (°C/cm) sec = calories/cm · °C · sec). Both depend on the content of ice, water and mineral and organic components of the soil. If there is a transition from ice to water, or water to ice, in the relevant range of temperature, the observed specific heat will include a component of latent heat of fusion. The term apparent specific heat is then used. Such transitions normally occur, at temperatures of practical interest. Both the specific heat (apparent specific heat – see examples pp. 4, 7 and 9) and conductivity are functions of temperature. They are also functions of soil pressure, although in most practical cases detailed consideration of this may not be necessary.

In the following section the prediction of unfrozen water content as a function of temperature and pressure, involving relatively simple laboratory procedures is described. This information together with a knowledge of the total moisture content (ice and water) and of the amounts of other soil constituents, permits calculation of apparent specific heats and heat capacity, and thermal conductivity. Kersten (1948) gives specific heats of various soil minerals.

In some practical problems a knowledge of the soil moisture content, and comparison with values obtained for other soils of similar nature, may provide sufficiently accurate approximations to the apparent specific heat and thermal conductivity. Also in such cases however, the relationships discussed in the following sections should be borne in mind.

IV.2 *Prediction of amounts of ice and water present in a frozen soil*

While the apparent specific heats of soils may be determined by direct observation in a calorimeter, of the heat exchange required to raise or lower the temperature of frozen soil, this procedure is difficult and time-consuming. Instead conventional suction-moisture content (or pressure membrane) tests (see *e.g.* Croney, Coleman and Black 1958) may be used as the basis for calculation of the proportions ice and water present at various negative temperatures, and thus for calculation of apparent specific heats.

Suction-moisture content tests give the equilibrium moisture content (at above-freezing temperatures) of the soil for various values of $p_a - p_w$, where p_a is the pressure of air around the sample, and p_w the pressure in the soil water. A difference in pressure between the air and water is established and drainage allowed to occur. The water subsequently remaining occupies pore spaces too small for the air-water interfaces to enter, the radius of the interfaces, r_{aw}, being given by eqn. (6): $p_a - p_w = 2\sigma_{aw}/r_{aw}$. When an equal (unfrozen) water content occurs in the frozen soil the ice-water interfaces have the same size (at least approximately), $r_{iw} \approx r_{aw}$. The pressure difference $p_i - p_w = 2\sigma_{iw}/r_{iw}$, and therefore for a given unfrozen water content:

$$p_i - p_w = (p_a - p_w)\frac{\sigma_{iw}}{\sigma_{aw}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

where $\sigma_{iw}/\sigma_{aw} = 0{,}4$ approx., and $p_a - p_w$ is that corresponding to an equal water content in the suction-moisture content test results. Thus from a series of suction-moisture content determinations on the soil, $p_i - p_w$ can be obtained as a function of unfrozen water content. Finally, use of equation (3) (which relates freezing temperature to $p_i - p_w$) enables determination of the unfrozen water content as a function of temperature, for the soil in question (see also p. 23*).

Although broadly analogous, the two processes of replacement of water by ice, and replacement of water by air (as in the suction test), differ in certain respects. These may necessitate modification of eqn. (18) with σ_{iw}/σ_{aw} being replaced by a factor greater than this, not exceeding one (see pp. 42, 44). The correspondence of the suction-moisture con-

* Note however that the importance of the interfacial energies term was overlooked there.

tent relationship, and the temperature-unfrozen water content relationship has been experimentally and theoretically established for freezing temperatures down to $-1°$ C (pp. 21, 44). Because of the fundamental nature of the relationships, both being an expression of the free energy of the soil water, it is likely that the correspondence is also valid, at least approximately, for prediction of unfrozen water content at temperatures several degrees lower.

Finally, it may be noted that the unfrozen water content of a soil is generally somewhat lower when the soil is warming (thawing) than when it is cooling (see Williams, 1963). Although a similar difference in water content is found between 'wetting' and 'drying' procedures in suction-moisture content tests, the closeness of the relationship between the two types of test has only been established experimentally for the freezing and drying cases respectively.

The amount of ice present at a particular temperature is given by the difference between the unfrozen water content and the total moisture content of the soil. The amount of unfrozen water is substantially independent of total moisture content (Williams, 1963). The total moisture content and thus ice content, of frozen soil can usually only be determined precisely by direct measurement on a sample of the frozen soil, from the site in question. The total moisture content depends on the amount of ice lensing, and thus on the migration of water that occurred on freezing.

The moisture content of the frozen soil is often found to be greater than that for the completely saturated condition in the unfrozen state. The excess is of course, largely water which migrated to the frost line, freezing there (producing frost heave) at a temperature close to 0° C.

IV.3 *Effect of pressure on unfrozen water content*

The effect of a load (such as that due to overburden) is to change the temperature at which a given unfrozen water content will occur. Consider two samples of the same soil, one subject to a load and one not, and at temperatures such that the samples have the *same* unfrozen water content. The ice-water menisci have therefore the same size in both samples, and according to equation (1) the *difference* in pressure between ice and water, $p_i - p_w$, is the same in each case. In the loaded sample the ice will have (see Section I.6) a pressure Δp_i.

The water pressure must also be greater by an equal amount, as $p_i - p_w$ has the same value in both the loaded and unloaded samples. A uniform increase of pressure on both phases gives a change of freezing point ΔT described by the 'usual' equation (see p. 22*) relating freezing point and pressure: $\Delta T = -0{,}0073°$ C per kg/cm^2.

The relationship between unfrozen water content and temperature for a given soil is therefore similar for loaded and unloaded conditions, except that the curve is displaced uniformly with reference to the temperature by the amount given in the case of the loaded condition. Thus the effects of load are particularly significant at relatively high temperatures when the unfrozen water content changes rapidly with temperature.

The situation when the load is applied locally after the soil is frozen was considered in Section III.5.

---

* Edlefsen and Anderson (1943) give the derivation of this equation, from the generalized Clausius-Clapeyron equation, and also those for the situation where the pressures on the two phases are not equal. Equation (3) is of course a particular case of the latter.

IV.4 *Temperature at the penetrating frost line*

This is determined by two factors, the pressure difference (eqn. 4): $p_i - p_w = 2\sigma_{iw}/r_c$ (r_c having the value for the soil in question), and the load on the soil at the frost line.

The freezing temperature corresponding to $p_i - p_w$ is given by eqn. (3): T_0 in that equation is 273,1° K (=0° C) if the pressure on the soil is atmospheric. The effect of load may be allowed for by modifying T_0 by an amount as given in the previous section (IV.3).

The situation at the penetrating frost line is of course, merely a special case of the conditions discussed in sections (IV.2) and (IV.3). The effect of loads (such as overburden) on the temperature at a penetrating frost line is found to be quite small. Even at for example a depth of 100 m, and an assumed $\gamma = 1800$ kg/m³, the temperature will only differ from that for the unloaded case by:

$$-0{,}0073 \cdot 1800 \cdot 100 \cdot 10^{-4} = -0{,}135°\,\mathrm{C}$$

When the frost line is not actively penetrating the pores of the soil, or when thawing is occurring, the temperature will have a value somewhat higher, depending on the amount by which r_{iw}, the radius of the ice water interfaces, is greater than r_c. The temperature will not be higher than the 'normal' freezing point for pure ice and water under a uniform pressure, equal to that on the soil. This corresponds to no curvature of the ice-water interface, *i.e.* $r_{iw} = \infty$.

IV.5 *Calculation of apparent heat capacity*

For most practical problems, the apparent heat capacity (cal/m³ · °C) is required rather than the apparent specific heat. For this the dry density (weight of dry soil per unit volume) must be found, as well as the total volumetric moisture content (volume of water and ice per unit volume of soil). The unfrozen water content as a function of temperature may be determined in the manner described in section IV.2. The unfrozen water content (% of dry weight) so obtained must be converted to the volumetric unfrozen water content (volume of unfrozen water per unit volume of frozen soil) by multiplying by the dry density. If the situation in question concerns thawing (warming) of the frozen soil regard should be paid to the possible significance of the hysteresis described in section IV.2.

The apparent heat capacity is then given by:

$$C_{fsT} = C_s V_s + C_w V_{wT} + C_i V_{iT} + L \frac{\delta V_w}{\delta T} \quad \text{. . . . . . .} \quad (19)$$

where C_{fsT} = apparent heat capacity at temperature T, cal/m³ · °C

C_s, C_w, C_i = heat capacities of soil mineral substance, water and ice (cal/m³)

V_s, V_{wT}, V_{iT} = partial volumes (= volume per unit volume of bulk soil) of mineral soil substance, water and ice at temperature T

L = volumetric latent heat of fusion of water ($\approx 80 \cdot 10^6$ cal/m³)

$\frac{\delta V_w}{\delta T}$ = rate of change of partial volume of water with temperature.

The term $\delta V_w/\delta T$ is obtained geometrically as the tangent of the slope, at the temperature T, of the curve of V_w against T. There is a range of temperature near 0° C where it is difficult to define an apparent heat capacity. This is the region where water is being drawn to the frost line and consequently ice lenses are forming, The heat involved is almost entirely composed of latent heat of fusion and is thus closely related to the total moisture content in the frozen state. Variations in total moisture content often involve only water that freezes in this range of temperature. Thus these heat quantities are frequently best considered as that involved over a finite temperature range (*e.g.* 0° C to –0,3° C, 0° C to –0,5° C), rather than considered in terms of single temperatures (see figs. 2–6, pages 4 to 9, and page 3).

FINAL REMARKS

In this volume it has been shown that the properties and behaviour of freezing soil are largely explainable in terms of a capillary model, in which the effects of interfacial energy at ice-water interfaces confined in the soil porous system are of fundamental importance. Capillary phenomena associated with air-water interfaces are to some extent analogous, and considerations of the replacement of water by air in a given soil may be used to determine the behaviour of the same soil undergoing freezing. In using this approach due regard must be paid to the special characteristics of ice-water interfaces and the appropriate physical quantities.

The experimental work has been confined to the laboratory. If field behaviour in specific situations is to be predicted from laboratory tests allowance must always be made for the complexity of field conditions and especially the variability of natural soils, even those normally considered as homogeneous. No immediate or comprehensive solution for the fundamental engineering problems of frost heave suggests itself, but a better understanding of the soil freezing process cannot but help in their amelioration.

ACKNOWLEDGEMENTS

Discussions with Professor J. Ross Mackay, University of British Columbia, C. B. Crawford (Head, Soil Mechanics Section) and J. K. Latta (Construction Section), Division of Building Research, National Research Council, Canada, are gratefully acknowledged.

REFERENCES

Adamson, A. W. (1960) *The physical chemistry of surfaces.* Interscience, New York, 629 p.

Ahlmann, Hans W:son (1948) *Glaciological research on the North Atlantic coasts.* Royal Geogr. Soc., Research Series No. 1, London, 83 pp.

Anderson, Duwayne M. and P. Hoekstra (1965) *Migration of interlamellar water during freezing and thawing of Wyoming Bentonite.* Soil Sci. Soc. Amer., Proc. Vol. 29, No. 5, pp. 498–504.

Beskow, Gunnar (1935) *Tjälbildningen och tjällyftningen med särskild hänsyn til vägar och järnvägar.* Stockholm, 242 pp. Statens väginstitut, Stockholm, Meddelande 48. (Also published as Sveriges geologiska undersökning. Avh. och Uppsats. Ser. C, 375, and translated into English: Tech. Inst. North Western Univ., Evanston, Ill. November 1947.)

Butterworth, B. (1964) *The frost resistance of bricks and tiles. A review.* Brit. Ceramic Soc. Jour. Vol. 1, No. 2, pp. 203–223.

Corte, A. E. (1961) *The frost behaviour of soils: Laboratory and field data for a new concept. Pt. 1, Vertical sorting.* Research Report 85, U.S. Army Cold Reg. Res. & Eng. Lab, Hanover, N. H., 22 pp.

Corte, A. E. (1963) *Vertical migration of particles in front of a moving freezing plane.* Research Report 105, U.S. Army Cold Reg. Res. & Eng. Lab, Hanover, N. H., 8 pp.

Croney, D., J. D. Coleman and P. M. Bridge (1958) *The suction of moisture held in soil in relation to highway design and performance*. Highway Res. Board Special Rep. 40: 226–252. 226–252.

Croney, D., J. D. Coleman and P. M. Bridge (1952) *The suction of moisture held in soil and other porous materials*. Road Res. Tech. Paper No. 24, (DSIR, RRL, Harmondsworth, Middx.) 42 pp.

Dirksen, C., and R. D. Miller (1966) *Closed system freezing of unsaturated soil*. Soil Sci. Soc. Amer. Proe., Vol. 30, No. 2, pp. 168–173.

Dorsey, N. E. (1940) *Properties of ordinary water-substance*. Amer. Chem. Soc. Monograph Ser. Reinhold, New York, 673 pp.

Dücker, A. (1939) *Untersuchungen über die frostgefährlichen Eigenschaften nichtbindiger Böden*. Forsehungsarbeiten aus dem Strassenwesen, Bd. 17, Volk und Reich-Verlag, Berlin.

Edlefsen. N. E. and A. B. C. Anderson (1943) *Thermodynamics of soil moisture*. Hilgardia, Vol. 15, No. 2, pp. 31–298, Berkeley, Calif.

Everett, D. H. and J. M. Haynes (1965) *Capillary properties of some model pore systems with special reference to frost damage*. RILEM Bull., Paris. New Ser. No. 27, pp. 31–38.

Handbook Chemistry & Physics (1965) 46th Ed. Chem. Rubb. Co., Cleveland, Ohio.

Hamilton, A. B. (1966) *Freezing shrinkage in compacted clays*. Can. Geotechnical Jour., Vol. III, No. 1, pp. 1–17.

Haynes, J. M. (1964) *Frost action as a capillary effect*. Trans. British Ceramic Soc. Vol. 63, No. 11, pp. 697–703.

Hesstvedt, Eigil (1964) *The interfacial energy ice/water*. Norw. Geotechnical Inst., Publ. 56.

Highway Research Board (1963) *Pavement design in frost areas*. II. Design considerations. Highway Res. Record, 33. Washington, 270 pp.

Hoekstra, P., E. Chamberlain, and T. Frate (1965) *Frost-heaving pressures*. Highway Research Record, 101: Frost action and insulation against sub-grade freezing. Washington.

Hoekstra, Pieter (1966) *Moisture movement in soils under temperature gradients with the cold-side temperature below freezing*. Water Resources Res., Vol. 2, No. 2, pp. 241–250. Washington.

Kamb, Barclay and E. LaChapelle (1964) *Direct observation of the mechanism of glacier sliding over bedrock*. Jour. Glaciol., Vol. 5, No. 38, pp. 159–172. Cambridge.

Kassiff, G. and Z. Globinsky (1966) *Apparatus for measuring suction under external loads*. Highway Res. Record, 111, pp. 24–30. Washington.

Kersten, M. S. (1948) *Specific heat tests on soils*. Proc. 2nd Int. Conf. Soils Found. Eng., Vol. III, p. 158. Rotterdam.

Mackay, J. Ross (1962) *Pingos of the Pleistocene Mackenzie Delta area*. Geogr. Bull. No. 18, pp. 21–63. Ottawa.

Mathews, W. H. (1964) *Water pressure under a glacier*. Jour. Glaciol. Vol. 5, No. 38, pp. 235–240. Cambridge.

Mathews, W. H. and J. R. Mackay (1960) *Deformation of soils by glacier ice and the influence of pore pressures and permafrost*. Trans. Roy. Soc. Canada, 3rd Ser., Vol. LIV, 1960, pp. 27–36.

Müller, Fritz (1963) *Beobachtungen über Pingos*. Medd. om Grønl., Vol. 153, No. 3, 127 pp. Copenhagen. (Also in English translation: TT 1073, NRC, Canada.)

Penner, E. (1959) *The importance of freezing rate in frost action in soils*. Proc. Amer. Soc. Test. Mat., Vol. 60, 1960, pp. 1151–1165, (also as NRC, Canada, Publ. 6152).

Powers, T. C. (1945) *A working hypothesis for further studies of frost resistance of concrete*. Jour. Amer. Concr. Inst., Vol. 16, No. 4, pp. 245–271.

Powers, T. C. and R. A. Helmuth (1953) *Theory of volume changes in hardened Portland-cement paste during freezing*. Highway Res. Board. Proc. 32nd Ann. Meeting, Washington, pp. 285–297.

Ruckli, R. (1950) *Der Frost im Baugrund*. Springer, Vienna, 279 pp.

Russam, K. (1965) *The effect of environment on the pore water tension under sealed surfaces*. Proc. 6th Int. Conf. Soil Mech. Found. Eng., Montreal, Vol. 2, pp. 184–187.

Sanger. F. J. and C. W. Kaplar (1966) *Plastic deformation of frozen soils*. Proc. Permafrost Int. Conf., Nat. Acad. Sci. – Nat. Res. Council, Publ. 1287, Washington, pp. 305–314.

Skempton, A. W. (1961) *Effective stress in soils, concrete and rocks*. In: Pore pressure and suction in soils, Butterworths, London, pp. 4–16.

Shumskii, P. A. (1964) *Principles of structural glaciology*. Dover, New York, 497 pp. (transl. by D. Kraus of: Osnovy strukturnogo ledovedeniia, Akad, Nauk SSSR, Inst. Merzlot, 492 pp, 1966).

Stuart, J. G. (1964) *Consolidation tests on clay subjected to freezing and thawing*. Swed. Geot. Inst., Reprints and Preliminary Repts. No. 7, pp. 1–9.

Terzaghi, K. (1952) *Permafrost*. Jour. Boston Soc. Civ. Engineers, Vol. 39, pp. 1–50. Also as: Harvard Soil Mech. Ser. 37.

Tsytovich, N. A. (1959) *Osnovy mekhaniki promerzayushchikh, merzlykh i protaivayushchikh gruntov*. Osnovy geokriologii, Glava III, Akad. Nauk. SSSR pp. 28–79 (Also as: *Basic mechanics of freezing, frozen and thawing soils*. Tech. Trans. 1239, NRC, Canada).

Vialov, S. S. (1959) *Reologicheskaia Svoistva i Nesushchaia Sposobnost Merzlykh Gruntov. Izd. Akad. Nauk SSSR*. (Also as transl.: Rheological properties of frozen ground. U.S. Army Cold Reg. Res. Eng. Lab. Transl. 74, 1965.)

Viborg, L. (1955) *The uplift of stones by frost*. Geogr. Ann. Vol. 37, pp. 164–169.

Washburn, A. L. (1956) *Classification of patterned ground and review of suggested origins*. Bull. Geol. Soc. Am., Vol. 67, pp. 823–866.

Weertman, J. (1964) *The theory of glacier sliding*. Jour. Glaciol. Vol. 5, No. 39, pp. 287–303. Cambridge.

Williams, P. J. (1959) *An investigation into processes occurring in solifluction*. Amer. Jour. Sci. Vol. 287, pp. 481–490. New Haven, Conn.

Williams, P. J. (1963) *Specific heat and unfrozen water content of frozen soils*. in Proc. 1st. Can. Conf. Permafrost, NRC, Canada, Assoc. Cttee. on Soil and Snow Mechs., Tech. Memo No. 76, pp. 109–126.

Williams, P. J. (1968) *Ice distribution in a permafrost profile*. (in preparation).

NORGES GEOTEKNISKE INSTITUTT
NORWEGIAN GEOTECHNICAL INSTITUTE

TECHNICAL REPORTS

The series Norwegian Geotechnical Institute, Technical Reports is intended to comprise a detailed presentation of original data from geotechnical investigations, presented without any interpretation or conclusion.

General description of projects, discussion of research results and articles or monographs on practical or theoretical aspects of soil mechanics will be found in the series Norwegian Geotechnical Institute, Publications.

Technical Reports (and Publications) may be ordered directly from the Norwegian Geotechnical Institute Forskningsvn. 1, Blindern, Norway. The price of each Technical Report will be N. kr. 24,— ($ 3,50) including postage.

1. Measurements at a strutted excavation, Oslo Subway, Grønland 1, km 1,559. 1962. iii + 30 p. + 37 p. tables and figures.
2. Measurements at a strutted excavation, Oslo Technical School. 1962. ii + 16 p. + 24 p. tables and figures
3. Measurements at a strutted excavation, Oslo Subway, Enerhaugen South, km 1,982. 1962. iv + 31 p. + 40 p. tables and figures.
4. Measurements at a strutted excavation, The New Headquarters Building of the Norwegian Telecommunication Administration, Oslo. 1965. v + 133 p. incl. tables and figures.
5. Measurements at a strutted excavation, Oslo Subway, Grønland 2, km 1,692. 1966. viii + 30p. + 94p. tables and figures.
6. Measurements at a strutted excavation, Oslo Subway, Vaterland 1, km 1,373. 1962. iii + 31 p. + 45 p. tables and figures.
7. Measurements at a strutted excavation, Oslo Subway, Vaterland 2, km 1.408. 1962. iii + 24 p. + 32 p. tables and figures.
8. Measurements at a strutted excavation, Oslo Subway, Vaterland 3, km 1,450. 1962. iv + 24 p. + 32 p. tables and figures.
9. Vibrating-wire measuring devices used at strutted excavations. 1962. iv + 151 p. incl. tables and figures.

PROCEEDINGS OF THE GEOTECHNICAL CONFERENCE OSLO 1967

Volume I: Shear Strength Properties of Natural Soils and Rocks, including:

13 papers on Shear Strength of Soft Clay
13 papers on Shear Strength of Stiff Clay
15 papers on Shear Strength of Soil other than Clay
11 papers on Shear Strength of Rocks.
1967.320 p.

Volume II: General Reports, Discussions etc.
Price, Vol. I-II N. kr. 210,— ($30,—)

PUBLIKASJONER
PUBLICATIONS

Publikasjonene kan bestilles direkte fra Norges geotekniske institutt, Forskningsvn. 1, Blindern. Publikasjon nr. 16 forhandles i Norge bare av Teknisk Ukeblad. Prisene er eksklusive porto.

Fullstendig fortegnelse over Instituttes publikasjoner finnes i publikasjonene nr. 1—40.

Bibliografi over Instituttets publikasjoner til og med nr. 30 sendes gratis på forespørsel.

Alle artikler har engelsk resymé.

The publications may be ordered directly from the Norwegian Geotechnical Institute, Forskningsvn. 1, Blindern. Postage is not included in the prisec mentioned below.

Publications prior to No. 31 are listed on the last pages of publications up to No. 40.

A bibliography of the publications Nos 1—30 wll be sent free of charge on request.

All papers are provided with English summary

31. A. CASAGRANDE and P. J. RIVARD: Strength of highly plastic clays. N. SIMONS: Laboratory. tests on highly plastic clay from Seven Sisters, Canada. 1959. 23 + 3 p. Price, N. kr. 6,—.
32. A. W. BISHOP: The principle of effective stress. L. BJERRUM: Norska erfarenheter av stålpålar till berg (Norwegian experiences with steel piles to rock). I. TH. ROSENQVIST: Montmorillonitt fra Skyrvedalen i Hemsedal (Montmorillonite from Skyrvedalen in Hemsedal, S. Norway). 1960. 5 + 15 + 4 p. Price, N. kr. 6,—.
33. Norwegian contributions to the Brussels Conference 1958 on Earth Pressure Problems: O. EIDE and I. J. JOHANNESSEN: Measurements of strut loads in the excavation for Oslo Technical School; I. J. JOHANNESSEN: Test section and installation of test equipment, Oslo Subway; K. ØIEN: An earth pressure cell for use on sheet piles, Oslo Subway; B. KJÆRNSLI: Test results, Oslo Subway; L. BJERRUM, N. SIMONS and I. TORBLAA: The effect of time on the shear strength of a soft marine clay; N. SIMONS: Laboratory determinations of the coefficient of earth pressure at rest (discussion); L. BJERRUM and R. KIRKEDAM: Some notes on earth pressures in stiff fissured clay. 1960. 44 p. Price, N. kr. 12—.
34. A. W. BISHOP and L. BJERRUM: The Relevance of the Triaxial Test to the Solution of Stability Problems. 1960. 56 p. Price, N. kr. 18,—.
35. Norwegian contributions to the American Society of Civil Engineers, Research Conference on Shear Strength of Cohesive Soils, 13—17 June, 1960, Boulder, Colorado, U.S.A.: A. ANDRESEN and N. E. SIMONS: Norwegian triaxial equipment and technique; L. BJERRUM and N. E. SIMONS: Comparison of shear strength characteristics of normally consolidated clays; N. E. SIMONS: Comprehensive investigations of the shear strength of an undisturbed Drammen clay; N. E. SIMONS: The effect of overconsolidation on the shear strength characteristics of an undisturbed Oslo Clay. 1960. 44 p. Price, N. kr. 12,—.
36. L. BJERRUM and N. FLODIN: The development of soil mechanics in Sweden, 1900—1925. SV. SKAVENHAUG: Protection against frost heaving on the Norwegian railways. 1960. 18 + 20 p. + 3 pl. Price, N. kr. 12,—.*)
37. I. TH. ROSENQVIST: Physico-chemical properties of soils: Soil water systems. J. MOUM and I. TH. ROSENQVIST: Hydrogen(protium)-deuterium exchange in clays. J. MOUM and I. TH. ROSENQVIST: Sulphate attack on concrete in the Oslo region. 1960. 16 + 3 + 8 p. Price, N. kr. 12,—.
38. E. M. SERGEJEV: Sandkolmation. L. BJERRUM and T. H. WU: Fundamental shear strength properties of the Lilla Edet clay.*) N. SIMONS: Shear strength parameters of two normally consolidated Norwegian clays. 1961. 9 + 9 + 3 p. Price, N. kr. 6,—.
39. O. EIDE og N. SIMONS: Setninger ved fundamentering på leire. Tilbygg til Rødskog skole ved Drammen (Settlements of a foundation on clay). H. DØHLEN og A. SANDE: Bygging av fyllingsdam ved Bordalsvatn (Construction of an earth fill dam at Bordalsvatn). DRENERING av bygninger (Drainage around buildings). 1961. 11 + 10 + 7 p. Price, N. kr. 6,—.
40. Beretning over Norges geotekniske institutts virksomhet fra 1. januar 1958 til 31. desember 1959 (Report on the activity of the Norwegian Geotechnical Institute from January 1, 1958 to December 31, 1959): L. BJERRUM: Innledning (Introduction); I. TH. ROSENQVIST: Hva er leire? (What is clay?); T. BRÆND: Skredkatastrofen i Sokkelvik i Nord-Reisa 7. mai 1959 (The clay slide at Sokkelvik May 7th 1959); O. KUMMENEJE: Stabilisering av løst lagret finsand (Stabilization of loose sand deposits); O. EIDE: Et industrianlegg på sandgrunn (Foundation of an industrial plant on sand); E. DIBIAGIO: Jordtrykkmålinger for Telgrafverkets nybygg i Oslo (Earth pressure measurements at the new building for Telegrafverket in Oslo); N. E. SIMONS: Sanddren ved den nye rullebane for Fornebu flyplass (Sand drains used for a new runway at the Fornebu air terminal); I. J. JOHANNESSEN: Trykkmålinger i en sementsilo (Pressure measurements in a cement silo); A. SANDE: Preparering av fundamentet for Bordalsdammen, Tokke kraftanlegg (Foundation of the Bordal Dam); I. TORBLAA: Komprimeringsforsøk for fyllingsdammer (Compaction tests for earth dams); [R. KIRKEDAM]: Skredene i Namdalen 1959 (The slides in Namdalen 1959). 1961. 52 p. Price, N. kr. 10,—.
41. L. BJERRUM and I. J. JOHANNESSEN: Pore pressures resulting from driving piles in soft clay. R. E. GIBSON and K. Y. LO: A theory of consolidation for soils exhibiting secondary compression. 1961. 4 + 16 p. Price, N. kr. 6,—.
42. I. TH. ROSENQVIST: Subsoil corrosion of steel. 1961. 6 + 86 p. Price, N. kr. 25,—.
43. A. W. BISHOP und L. BJERRUM: Bedeutung und Andwendbarkeit des Dreiachsialversuches für die Lösung von Standsicherheitsaufgaben. 1961. 4 + 61 p. Price, N. kr. 18.—.
44. J. N. HUTCHINSON: A landslide on a thin layer of quick clay at Furre, Central Norway. L. BJERRUM and O. KUMMENEJE: Shearing resistance of sand samples with circular and rectangular cross sections. 1961. 26 [+ 2 pl] + 7 p. Price, N. kr. 6,—.*)
45. Contributions to the Fifth International Conference on Soil Mechanics and Foundation Engineering, Paris, 1961: L. BJERRUM: The effective shear strength parameters of sensitive clays. L. BJERRUM, S. KRINGSTAD and O. KUMMENEJE: The shear strength of a fine sand; K. Y. LO: Stress-strain relationship and pore water pressure characteristics of a normally-consolidated clay; J. MOUM and I. TH. ROSENQVIST: The mechanical properties of montmorillonitic and illitic clays related to the electrolytes of the pore water; O. KUMMENEJE and O. EIDE: Investigation of loose sand deposits by

*) Avec résumé français.

blasting; O. EIDE, J. N. HUTCHINSON and A. LANDVA: Short and long-term test loading of a friction pile in clay; E. DIBIAGIO and B. KJÆRNSLI: Strut loads and related measurements on contract 63a of the Oslo Subway. 1961. 53 p. Price, N. kr. 6,—.*)

46. SV. SKAVEN-HAUG: Frostsikker fundamenteringsdybde for byggverk og vannledninger (Frostfree depth of foundation for buildings and water mains). I. TH. ROSENQVIST: Korrosjon av armeringsstål i betong (Corrosion of the steel in reinforced concrete). J. MOUM: Korrosjon av støpejernsledning forårsaket av vagabonderende vekselstrøm (Corrosion of cast iron pipeline caused by alternating stray current). 1961. 11 + 3 + 4 p. Price, N. kr. 6,—.

47. Tilbakeblikk ved årsskiftet 1961/62. Artikler tilegnet Olav Folkestad (A retrospect of the years up to 1962. Articles dedicated to Olav Folkestad): L. BJERRUM: Forord (Preface); L. BJERRUM, O. EIDE og B. KJÆRNSLI: Et tilbakeblikk over Norges geotekniske institutts tilblivelse og vekst fra 1947 til 1962 (A retrospect of the etablishment and growth of the Norwegian Geotechnical Institute, 1947—1962; SV. SKAVEN-HAUG: Den geotekniske utvikling i Norge frem til ca. 1950 (The development of geotechnics in Norway up to approximately 1950); J. FRIIS: Den geotekniske utvikling i Norge i de siste 10 år (Development of geotechnics in Norway during the last 10 years); S. AALEFJÆR: Fra betongdammer til fyllingsdammer (From concrete dams to earth dams); O. SVENNAR: Tunnelbanen i Oslo og geoteknikken (The subway in Oslo and geotechnics); O. A. ENGH: Geoteknikkens anvendelse i praksis sett fra en bygningsteknisk konsulents synsvinkel (The use of geotechnics in practice as seen by a consultant); O. ELSRUD: Geonor A/S (Geonor A/S); N. JANBU: Undervisning og utdannelse innen det geotekniske felt (Education within the field of geotechnics at the Technical University of Norway); R. BØHN: Norges geotekniske institutts 10-årige virke i kurver og tall (The 10-years' work of the Norwegian Geotechnical Institute in curves and numbers); F. A. JØRSTAD: Norges geotekniske institutts virksomhet i årene 1952—1961 bibliografisk fremstilt (Bibliography covering the activity of the Norwegian Geotechnical Institute during the years 1952—1961). 1962. 80 p. Price, N. kr. 12,—.

48. B. KJÆRNSLI: Deformasjoner av fyllingsdammer (Deformations of rockfill dams). B. KJÆRNSLI og I. TORBLAA: Asfalt på fyllingsdammer (Asphalt on earth- and rockfill dams). 1962. 11 + 9 p. Price, N. kr. 6,—.

49. B. KJÆRNSLI and N. SIMONS: Stability investigations of the north bank of the Drammen river.*) J. N. HUTCHINSON og L. BJERRUM: Skredet ved Furre 14. april 1959 (The slide at Furre in Namdalen, April, 1959). Å. EGGESTAD: Fundamenteringsforholdene ved Tromsøbroen (Foundation conditions at the Tromsø Bridge). 1962. 21 + 11 + 5 p. Price, N. kr. 6,—.

50. N. JANBU: Karl Terzaghi — grunnleggeren av den moderne geoteknikk (Karl Terzaghi — the founder of modern Soil Mechanics). K. TERZAGHI: Stability of steep slopes on hard unweathered rock.*) 1963. 5 + 20 p. Price, N. kr. 6,—.

51. J. N. HUTCHINSON and E. N. ROLFSEN: Large scale field shear box tests on quick clay.*) O. EIDE og I. J. JOHANNESSEN: Fundamentering av midtblokken på Ullevål Sykehus (Foundation of the new centre building of the Oslo Municipal Hospital — Ullevål Sykehus). A. KJENNERUD: Hydrerte cementmineralers forhold overfor aggressive vanntyper sett i relasjon til betongkorrosjon (Behaviour of hydrated cement minerals under the action of aggressive types of water, considered with reference to concrete corrosion). 1963. 12 + 6 + 12 p. Price, N. kr. 6,—.

52. G. TER-STEPANIAN: On the long-term stability of slopes. G. TER-STEPANIAN: Soil mechanics in the USSR. 1963. 14 + 13 p. Price, N. kr. 6,—.

53. K. Y. LO: Shear strength properties of a sample of volcanic material of the valley of Mexico.*) K. Y. LO: Secondary compression of clays. L. BJERRUM: Technical reports; Announcement of a new NGI publication series. 1963. 16 + 13 + 12 p. Price, N. kr. 6,—.

54. I. TH. ROSENQVIST: The influence of physico-chemical factors upon the mechanical properties of clays. I. TH. ROSENQVIST: Studies in position and mobility of the H atoms in hydrous micas. 1963. 10 + 9 p. Price, N. kr. 6,—.

55. O. EIDE og Å. EGGESTAD: Fundamenteringsforhold ved Telegrafverkets nye administrasjonsbygg i Oslo (Foundation conditions for the new Headquarters Building of the Norwegian Telecommunication Administration, Oslo). O. EIDE: Erfaring med borede peler direkte utstøpt i grunnen; Franki-, Benotopeler og pillarer (Experience with bored piles cast in the ground; Franki and Benoto piles and piers). LISTE over symboler anbefalt til bruk i geotekniske arbeider (List of recommended symbols for soil mechanics). 1963. 9 + 7 + 3 p. Price, N. kr. 6,—.

56. A. LANDVA: Equipment for cutting and mounting undisturbed specimens of clay in testing devices. E. HESSTVEDT: The interfacial energy ice/water. K. FLAATE: An investigation of the validity of three pile-driving formulae in cohesionless material. 1964. 22 p. Price, N. kr. 6,—.

57. L. BJERRUM and K. Y. LO: Effect of aging on the shear-strength properties of a normally consolidated clay. L. BJERRUM, T. L. BREKKE, J. MOUM and R. SELMER-OLSEN: Some Norwegian studies with swelling materials in rock gouges. 1964. 11 + 9 p. Price, N. kr. 6,—.*)

58. Contributions to the European Conference on Soil Mechanics and Foundation Engineering, Wiesbaden, 15.—18. Oct. 1963: J. N. HUTCHINSON: Settlements in soft clay around a pumped excavation in Oslo; N. SIMONS: Settlement studies on a ninestorey apartment building at Økernbråten, Oslo; L. BJERRUM and Å. EGGESTAD: Interpretation of loading test on sand; A. EGGESTAD: Deformation measurements below a model footing on the surface of dry sand; S. FJELD: Settlement damage to a concrete-framed structure. Oslo 1964. 45 p. Price, N. kr. 6,—.*)

59. H. HARTMARK: Geotechnical observations during construction of a tunnel through soft clay in Trondheim, Norway.*) J. MOUM and O. I. SOPP: Saponite from Veo, Jotunheimen. F. JØRSTAD og J. MOUM: Halloysitt i sleppeleire ved Slemmestad (Halloysite found in a clay gouge in Slemmestad, Southern Norway). Oslo 1964. 13 + 7 + 4 p. Price, N. kr. 6,—.

60. Beretning over Norges geotekniske institutts virksomhet fra 1.januar 1962 til 31. desember 1963: Med geoteknikeren på byggeplassen (Report on the activity of the Norwegian Geotechnical Institute from January 1st 1962 up to December 31st 1963): Innledning ved L. BJERRUM (Introduction by L. BJERRUM); Høy fylling på myr på Fornebu flyplass (Soil mechanics at the Oslo Air Terminal); Ny tunnelbane i Oslo (Geotechnical research and measurements in connection with an extension of the Oslo Subway); Prøvetagning i ytre Oslofjord (Geotechnical investigations of the bottom of the Oslofjord); Nytt tetningsmiddel prøvet under Frierfjorden (Development and testing of a chemical grouting mixture); Asfaltdekke på Venemodammen, Tokke kraftanlegg (An earth fill dam with an asphaltic concrete deck); Steinskred i Øksfjord (Geotechnical investigation of the rock above Øksfjord, Finnmark); Opplysningsvirksomhet (Information activity); Administrasjon og økonomi (Administration and economy). Oslo 1964. 53 p. Price, N. kr. 12,—.

*) Avec résumé français.

61. J. N. HUTCHINSON: The landslide of February, 1959, at Vibstad in Namdalen. V. MENCL: Proportions of cohesion and of internal friction in the strength of rocks. J. MOUM: Investigations of some clay materials from rock gouges in Norway. Oslo 1965. 27 p. Price, N.kr. 6,—.
62. T. C. KENNEY: Sea-level movements and the geologic histories of the post-glacial marine soils at Boston, Nicolet, Ottawa and Oslo.* K. FLAATE: A statistical analysis of some methods for shear strength determination in soil mechanics. N. E. SIMONS: Consolidation investigation on undisturbed Fornebu Clay. Oslo 1965. 28 + 8 + 9 p. Price, N.kr. 6,—.
63. T. C. KENNEY: Permeability ratio of repeatedly layered soils.* T. C. KENNEY: Pore pressures and bearing capacity of layered clays. T. C. KENNEY: Stability of cuts in soft soils. Oslo 1965. 9 + 14 + 9 p. Price, N.kr. 6,—.
64. L. BJERRUM, T. C. KENNEY and B. KJÆRNSLI: Measuring instruments for strutted excavations. O. EIDE: Subsoil conditions and the bearing capacity of piles at the site of a new fishery station, Ernakulam, Kerala, South India. I. J. JOHANNESSEN: Observations of the drag load on a steel pile to rock due to settlement of clay.* Oslo 1965. 17 + 8 + 14 p. Price, N.kr. 6,—.
65. Contributions to the Sixth International Conference on Soil Mechanics and Foundation Engineering, Montreal, 1965: A. ANDRESEN, S. SOLLIE and F. RICHARDS: N. G. I. gas-operated, sea-floor sampler; G. AAS: A study of the effect of vane shape and rate of strain on the measured values of in-situ shear strength of clays; I. J. JOHANNESSEN and L. BJERRUM: Measurement of the compression of a steel pile to rock due to settlement of the surrounding clay; G. HOLESTØL, B. KJÆRNSLI and I. TORBLAA: Compression of tunnelspoil at Venemo dam. Oslo 1965. 22 p. Price, N. kr. 6,—.*)
66. B KJÆRNSLI: Beispiele von Verdichtungen an Staudämmen in Norwegen. L. BJERRUM and B. KJÆRNSLI: Design and Construction of Mänika Dam. B. KJÆRNSLI and A. SANDE: Compressibility of some Coarse-Grained Materials. Oslo 1966. 8 + 6 + 7 p. Price, N.kr. 6.—.
67. L. BJERRUM og F. JØRSTAD: Stabilitet av fjellskråninger i Norge (Stability of rock slopes in Norway). F. JØRSTAD: Jettegryter og anleggsarbeide (Potholes and construction work). L. BJERRUM: Problèmes de fondation dans les argiles sensibles de Norvège. Oslo 1966. 17 + 6 + 10 p. Price, N.kr. 6.—.
68. Beretning over Norges geotekniske institutts virksomhet fra 1. jannar 1964 til 31. desember 1965 (Report on the Activity of the Norwegian Geotechnical Institute from January 1st 1964 to December 31st 1965): L. BJERRUM: Innledning (Introduction). E. N. ROLFSEN: Selnes-skredet april 1965 i Ytre Namdalen (The quick clay slide at Selnes, Namdalen, Central Norway). I. FOSS: Setninger i Drammen (Settlements in Drammen). A. SANDE: Ny Tunhovddam (Tunhovd Dam, Nnmedal, Norway). I. TORBLAA: Pore-Trykksmålinger utfort ved Dam Hyttejuvet med forskjellig målentstyr (Pore pressure measuring equipment at Hyttejuvet Dam). J. MOUM: Elektro-osmotisk grunnforsterkning i Ås (Electro-osmotic soil stabilization). G. AAS: Spesielle vingeborforsøk i marken for undersøkelse av våre marine leirers skjærfasthetsegenskaper (Special in situ vane tests to investigate the shear strength characteristics of Norwegian marine clays). I. J. JOHANNESSEN: Påhengskrefter i en stålpel ved setninger i omkringliggende leire (Frictional forces on a steel pile caused by consolidation of surrounding clay). P. J. WILLIAMS: Måling av jordarters telefarlighet: prinsipper, metoder og utstyr (Measurement of the susceptiblity of soil to frost heave: Principles, methods and equipment). T. C. KENNEY: Residual strength of fine-grained minerals and mineral mixtures. L. BJERRUM: Terzaghi-Biblioteket (The Terzaghi Library). R. BØHN: Administrasjon, økonomi og opplysningsvirksomhet (Administration, economy, information activity). Oslo 1966. 71 p. Price N. kr. 12,—.
69. B. KJÆRNSLI and I. TORBLAA: The Venemo Asphalt-Faced Rock-Fill Dam. B. KJÆRNSLI, J. MOUM and I. TORBLAA: Laboratory Tests on Asphaltic Concrete for an Impervious Membrane on the Venemo Rock-Fill Dam. Oslo 1966. 26 p. Price, N. kr. 6,—.
70. L. BJERRUM and A. LANDVA: Direct Simple-Shear Tests on a Norwegian Quick Clay.*) L. BJERRUM, O. EIDE og B. KJÆRNSLI: Geotekniske problemer ved utførelsen av Tunnelbanen fra Jernbanetorvet til Tøyen. (Geotechnical problems during construction of subway through the central part of Oslo). L. BJERRUM og O. EIDE: Anvendelse av kompensert fundamentering i Norge. (Use of compensated foundation in Norway). J. MOUM: Falling Drop used for Grain-Size Analysis of Fine-Grained Materials. Oslo 1966. 20 + 8 + 3 + 5p. Price, N. kr. 6,—.
71. L. BJERRUM—Seventh Rankine Lecture: Engineering Geology of Norwegian Normally-Consolidated Marine Clays as Related to Settlements of Buildings.*) Oslo 1967. 38p. Price N. kr. 6,—.
72. P. J. WILLIAMS: Properties and Behaviour of Freezing Soils.*) Oslo 1967. 119p. Price N. Kr. 20,—.

*) Avec résumé français.