Phoenix model measurement and bifilar swinging

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SUMMARY

This report describes the methods used to measure and describe the Phoenix underwater vehicle model. This included measurements for mass, length, center of gravity, buoyant force, radius of gyration, and moment of inertia. The radius of gyration and moment of inertia values were calculated based on the period of oscillation found from bifilar swinging the model. An uncertainty analysis of the results is also presented here. The method and apparatus described in this report proved to be accurate to within ±2mm and could be repeated for other underwater vehicles in the future.

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PHOENIX MODEL MEASUREMENT AND BIFILAR SWINGING

SR-2005-29

Gerald Hewitt, Erin Waterman

December 2005

Summary

In October 2005 a model of the Phoenix underwater vehicle was tested in the Ice Tank at the Institute for Ocean Technology (IOT). After these test were completed the vessel was reassembled in each of five configurations and its geometrical properties measured in the Autonomous Underwater Vehicle (AUV) Lab at IOT. The method used to measure these parameters and the results of those measurements are presented in this paper.

Measurements of the length and mass of each vehicle configuration were made. The buoyant force of each configuration was found by submerging each vehicle while it was hanging from two load cells. In this manner the load cells could also be used to locate the center of gravity (CG).

The yaw radius of gyration of each configuration was found using bifilar swinging. First the dry model was hung from two filaments and oscillated laterally about it's CG. The period of oscillation was measured and used to calculate the radius of gyration. This procedure was then repeated with the model flooded, hanging in air to determine the radius of gyration of the flooded model. The results of these measurements are presented in this report.

To determine if the swing apparatus was affecting the measurements, a solid aluminum rod was swung from the apparatus as well. The radius of gyration calculated form this swinging was very close (±1 mm) to the theoretical calculated value for that rod. An uncertainty analysis of the results is also presented here. The uncertainty analysis, coupled with the results of swinging the aluminum bar, show that the methods and apparatus used here are valid within ±3 mm, and can be used for other vehicles in the future.

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Background

In October 2005, tests were conducted in the Ice Tank using the Phoenix model. Five different configurations were used for this testing. Each configuration represented a different length-to-diameter ratio, ranging from 8 to 12. These tests consisted of several different maneuvers including arc-of-a-circle. Once the tests were completed the vehicle, and all the parts required for each configuration were brought to the AUV lab for measurements and testing. This involved measuring the geometrical properties and mass of each component, weighing each configuration of the model in air and in water. It also included locating the center of gravity (CG) of each configuration dry, in water and flooded in air. The Moment of Inertia (MOI) for dry and flooded models was also required.

To determine the Moment of Inertia of each configuration, bifilar swinging was employed for each configuration. Bifilar swinging involves hanging the model horizontally from two vertical wires (filaments) equally spaced on either side of the center of gravity. The model is then offset, in yaw, from rest and the period of oscillation measured. In this case we measured the time for 10 oscillations and divided by 10 to get the average period. This was repeated five times for each configuration. Each of the five period measurements was used to calculate a radius of gyration and those radii averaged to get the final radius.

$$K = \frac{TD}{4}\sqrt{\frac{g}{L}}$$

Equation 1 - Radius of Gyration from Period Measurement

Where:
K = Radius of gyration (m)
T = Period of oscillation (s)
D = Distance between filaments (m)
g = Gravitational constant (9.806 m/s^2)
L = Length of the filaments (m)

Configurations

The configurations for the Phoenix vehicle were constructed using the same nose, tail and center body sections. Each configuration then had the required number and length of acrylic pieces (A1, A2, A3, F1, F2, F3) added to make the appropriate length. For increased support, each of these acrylic pieces was supported from the inside using a stainless steel ring. The acrylic pieces were manufactured such that when each piece is assembled in configuration LD12 the labels of each component line up with each other and the labels of the steel rings are visible on the bottom of each piece, as shown in the figure below.

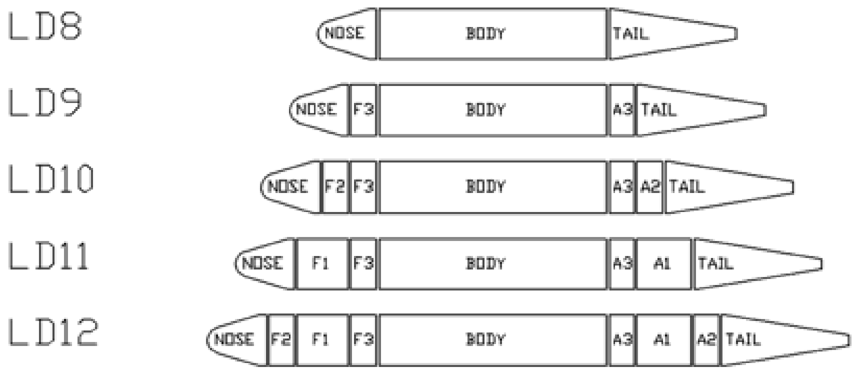

Figure 1 - The five different configurations of Phoenix

Figure 2 - Arrangement of Ring and Acrylic.

Figure 3 - Connection of two adjacent acrylic pieces

Apparatus

All the measurements required were taken at the Autonomous Underwater Vehicle (AUV) Lab at IOT or using the scales located in the Model Prep Shop. The test apparatus for bifilar swinging and in-water testing is shown below.

Figure 4 - Bifilar Swinging Apparatus

A gantry was erected over the trim tank in the AUV lab. This gantry was extended so that its beam was 2.5 metres above the floor. Two pinch clamps were constructed and clamped to the beam for the dry bifilar swinging. These clamps provide an exact point for the origin of the swing and allow the wires to be adjusted so that the vehicle was level during swinging. We attempted to use these clamps for swinging the flooded model, but they failed under the added weight and a system of clamps and slings was used instead.

Figure 5 - Dry Swing Apparatus

Figure 6 - Close up View of Pinch Clamp

For the in-water measurements (buoyant force and CG in water) a different apparatus was used. This involved suspending the models from two load cells hanging from the gantry. This allowed us to locate the center of gravity, position the slings equally on either side and take measurements without taking the model out of the apparatus. The clamp and sling arrangement is shown below.

Figure 7 - Apparatus for Wet Model Measurements

The load cells shown are 100 lb (45 kg) rated load cells that were connected to a data acquisition system. This allowed us to see what the weights were in real-time and record them to file as well.

For the buoyant weight and CG of the model additional slings were added so the entire model was submerged when measurements were taken. Once this was completed, the model was raised from the water using the overhead crane and those slings removed to allow the model to be flooded and swung.

Measurements

All the measurements were conducted on each configuration, then that configuration was disassembled, another assembled and each of the tests repeated. The basic sequence of events was to first assemble and photograph the model. Next the length of the model was measured and the location of the dry CG found.

Next the model was placed in the filaments and hung from the grip clamp apparatus described above. The model was set into oscillation and the period of this oscillation measured. This completed the dry measurements.

The vehicle was then placed in different slings for the wet portion of the tests. It was then submerged in the water. The two loadcells were positioned such that each was in tension and reading a value. The underwater CG was found by moving the slings until the two load cells showed the same reading, thus each loadcell supported half the vehicle's weight; the CG is then at the midpoint of the two slings. In some cases this required positioning the aft sling close to the end of the tail cone and the forward sling near the middle of the vehicle. The vehicle was then carefully lifted from the tank using the overhead crane. Car must be taken during this step, because the CG of the vehicle under water is different then the CG of the flooded vehicle in air. As the vehicle is lifted from the water, the flood water drains from it, changing its CG again.

The sling length was then shortened and the vehicle hung, over the water from the load cells. With the slings located 60 cm apart the vehicle was flooded with water. Provided the seams of the vehicle are sealed with vinyl tape and the tail section plugged, leaking should not be a significant problem. Once the vehicle is flooded, note the readings on the two load cells and use them to locate the CG of the flooded model. Reposition the slings so that they are equally spaced on either side of the CG. While repositioning the sling water will likely spill from the vehicle and the floodwater will need to be topped up and the load cells read again to confirm that the slings are spaced properly. With the slings positioned and the vehicle completely filled with water, the vehicle was again oscillated and the period of oscillation measured.

There was some concern as to the sensitivity of the bifilar measurements to such parameters as, the weight of the filaments, environmental conditions (ventilation fans), and the stiffness of the apparatus. To confirm that the measurements were yielding valid data, a round aluminum bar (1.454 m long, 12.7 kg) was swung from the apparatus. It was oscillated and timed is the same manner as the models. The period was then used to calculate a radius of gyration of 0.4187m.

Since this was a uniform homogeneous body, calculating it's theoretical radius of gyration was straightforward, as shown below,

$$I = \frac{1}{12} m L^2$$

Equation 2 - Moment of Inertia of a rod about its axis

$$K = \sqrt{\frac{I}{m}}$$

Equation 3 - Radius of Gyration - Theoretical

$$K = \sqrt{\frac{L^2}{12}}$$

Equation 4 - Radius of Gyration in terms of length

where:

I = Moment of Inertia (kg m^2)
m = Mass (kg)
K = Radius of Gyration (m)
L = Length of the body (m)

This yields:

$$K = \sqrt{\frac{L^2}{12}} = \sqrt{\frac{1.454^2}{12}} = 0.4197m$$

This shows that the apparatus and environmental conditions have a minimal effect on the overall calculation of the radius of gyration.

An attempt was also made to obtain a period of oscillation for the flooded model in water. The apparatus used for this was the same as the one used to swing the flooded model in air, but with longer slings. With the slings positioned an equal distance on either side of the CG the vehicle was set oscillating. The filaments in this case needed to be located very close to the CG to avoid being placed on the tapered portion of the tail section. This location, combined with the vehicle's weight in water (40 N), produced a very weak restoring force. This caused the vehicle to return very slowly and to translate as well as oscillate about it's CG. This made it impossible to measure the period with confidence.

Results

The results of the measurements are presented in full in Appendix A. A brief discussion of some of those results is presented here.

In general, due to the construction of the model, the center of gravity of the model in water was very near the aft cone piece. This is due to the addition of buoyant foam in the nose cone. The buoyant force of each flooded model ranged from 26 to 45 N.

Listed below are the length and mass, as well as the location of the CG for each configuration measured from the **bow** of the vehicle.

Table 1 - Length and mass measurements for each configuration

Configuration	Length (m)	Mass (dry) (kg)	Mass (flooded) (kg)
LD 8	1.723	24.30	2.656
LD 9	1.927	25.60	3.171
LD10	2.125	27.30	3.792
LD11	2.334	28.20	4.04
LD12	2.535	29.80	4.549

Table 2 - CG locations for each configuration

Configuration	CG (dry) (m)	CG (in water) (m)	CG (flooded in air) (m)
LD 8	0.734	1.287	0.847
LD 9	0.815	1.392	0.939
LD10	0.912	1.504	1.057
LD11	1.011	1.615	1.159
LD12	1.118	1.722	1.256

Based on the period of oscillation measurements the radius of gyration and moment of inertia were calculated for each model. This data is presented in Table 3 and Table 4.

Table 3 - Radius of Gyration for each configuration

Configuration	DRY (in air)	FLOODED (in air)
	Radius of Gyration	Radius of Gyration
	(mm)	(mm)
LD 8	381 ± 1.41	423 ± 1.46
LD 9	419 ± 1.47	490 ± 1.87
LD10	447 ± 1.51	514 ± 1.68
LD11	489 ± 1.58	558 ± 1.73
LD12	529 ± 1.65	648 ± 2.08

Table 4 - Moment of Inertia for each configuration

Configuration	DRY (in air)	FLOODED (in air)
	Moment of Inertia	Moment of Inertia
	(kg m^2)	(kg m^2)
LD 8	3.52	8.82
LD 9	4.49	13.25
LD10	5.44	16.73
LD11	6.73	21.84
LD12	8.34	32.36

Uncertainty Analysis

In an effort to determine the sensitivity of the final radius of gyration calculated from the measurements outlined above an uncertainty analyis was conducted based on the following uncertainty levels in each measurement

Period of Oscillation:	u (T) = ± 0.01 second
Length of Filaments:	u (L) = ± 0.003 meters
Distance between Filaments:	u (D) = ± 0.001 meters

$$K = \frac{TD}{4\pi}\sqrt{\frac{g}{L}}$$

Base on Equation 1 (introduced previously and repeated here) the uncertainty in the radius of gyration value can be described as:

$$u^2(K) = \left(\frac{\partial K}{\partial T}\right)u^2(T) + \left(\frac{\partial K}{\partial D}\right)u^2(D) + \left(\frac{\partial K}{\partial L}\right)u^2(L)$$

Equation 5 - Radius of Gyration Uncertainty

where:

u = Uncertainty in a given measurement
T = Period of Oscillation (s)
K = Radius of Gyration (m)
L = Length of the body (m)

The partial derivatives of the radius of gyration with respect to each measurement are listed below:

$$\frac{\partial K}{\partial T} = \frac{D}{4\pi}\sqrt{\frac{g}{L}} = \frac{K}{T}$$

Equation 6 - Partial Derivative of K with respect to T

$$\frac{\partial K}{\partial D} = \frac{T}{4\pi}\sqrt{\frac{g}{L}} = \frac{K}{D}$$

Equation 7 - Partial Derivative of K with respect to D

$$\frac{\partial K}{\partial L} = \frac{TD}{4\pi}\sqrt{g} * \left(\frac{1}{2}\right) L^{\frac{-3}{2}} = \frac{K}{L}\left(\frac{1}{2}\right)$$

Equation 8 - Partial Derivative of K with respect to L

Equations five to eight can be combined to give the following equation:

$$u^2(K) = \left[\left(\frac{K}{T}\right)^2 u^2(T)\right] + \left[\left(\frac{K}{D}\right)^2 u^2(D)\right] + \left[\frac{1}{4}\left(\frac{K}{L}\right)^2 u^2(L)\right]$$

Equation 9 - Discrete Uncertainty Equation

Based on the uncertainty levels listed above for each measurement, and Equation 9, the uncertainty of each radius of gyration calculation was found. These uncertainties ranged from 1.5 mm to 1.8 mm for the different configurations.

Conclusions

The primary conclusion of the measurements taken from this exercise is the values of the parameters outlined in the 'Results' section above. As outlined above, this procedure is an effective method of determining the radius of gyration of flooded and dry underwater vehicles. This method can be done for either yaw (as outlined here), pitch or roll using the same procedure, but changing the orientation of the vehicle.

Recommendations

Determining the various parameters listed here could be made easier in the future if the following guidelines are followed:

When material is used to fill the void space inside the model, wherever possible use equal amounts in the forward and aft portions of the vehicle. In the case of the Phoenix model the location of the CG underwater (very near the aft cone) made it difficult to take some measurements. This could have been rectified if the foam that was used in the nose cone had been offset with an equal volume of foam in the tail portion.

Where internal bulkheads (stainless steel rings in this case) are used and are not separating watertight compartments, drill holes through the top of those bulkheads. This would allow air to move from one compartment to another when the vehicle is being submerged. This makes it easier to eliminate air bubbles when the vehicle is placed in the water for buoyancy measurements.

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Appendix A: Spreadsheets of Results from Phoenix Model Measurements

LD 8

Overall Length (mm)	Length of Components (mm)	Overall Mass (kg)	Overall Weight (N)
1723.5	1713.5	24.30	238.29

Location of Center of Gravity

From Bow: (mm)	From Stern: (mm)
734	989

Swing in Air

g = 9.806
L = 1.85
D = 0.60
g/L^0.5 = 2.302

Trial	Time for 10 (s)	Period (s)	r (m)
1	34.84	3.484	0.383
2	34.56	3.456	0.380
3	34.59	3.459	0.380
4	34.56	3.456	0.380
5	34.62	3.462	0.381
	Mean :	3.463	0.381
	Median :	3.459	0.380
	Std Dev :	0.012	0.001

Mass in Water (kg)	Weight in Water (N)	Bouyant Force (N)
2.656	26.04	212.24

Location of Center of Gravity in water

From Bow: (mm)	From Stern: (mm)
1287.5	436

LD 8

Flooded Model

Mass in Air (kg)	Weight in Air (N)
49.18	482.26

Flood Water

Mass in Air (kg)	Weight in Air (N)
24.88	243.97

Location of Center of Gravity of Flooded Model in Air

From Bow: (mm)	From Stern: (mm)
847	876

Swing Flooded Model in Air

g = 9.806
L = 1.31
D = 0.500
g/L^0.5 = 2.736

Trial	Time for 10 (s)	Period (s)	r (m)
1	38.84	3.884	0.423
2	39.18	3.918	0.427
3	38.78	3.878	0.422
4	38.75	3.875	0.422
5	38.93	3.893	0.424
	Mean :	3.890	0.423
	Median :	3.884	0.423
	Std Dev :	0.017	0.002

Moment of Iinertia (kg*m^2)

Dry	3.52
Flooded	8.82

LD 8

Uncertainty Calculation

u(T)= 0.01
u(D)= 0.001
u(L)= 0.003

$$u^2(K) = \left[\left(\frac{K}{T}\right)^2 u^2(T)\right] + \left[\left(\frac{K}{D}\right)^2 u^2(D)\right] + \left[\frac{1}{4}\left(\frac{K}{L}\right)^2 u^2(L)\right]$$

Flooded Model	1.19E-06	7.17E-07	2.35E-07
u(K) =	1.46E-03		
Dry Model	1.21E-06	4.03E-07	3.81E-07
u(K) =	1.41E-03		

LD 9

Overall Length (mm)	Length of Components (mm)	Overall Mass (kg)	Overall Weight (N)
1927	1916.36	25.60	251.03

Location of Center of Gravity

From Bow: (mm)	From Stern: (mm)
815	1112

Swing in Air

g = 9.806
L = 1.85
D = 0.60
g/L^0.5 = 2.302

Trial	Time for 10 (s)	Period (s)	r (m)
1	38.01	3.801	0.418
2	38.03	3.803	0.418
3	38.18	3.818	0.420
4	38.21	3.821	0.420
5	38.15	3.815	0.419
	Mean :	3.812	0.419
	Median :	3.815	0.419
	Std Dev :	0.009	0.001

Mass in Water (kg)	Weight in Water (N)	Bouyant Force (N)
3.171	31.09	219.94

Location of Center of Gravity in water

From Bow: (mm)	From Stern: (mm)
1392	535

LD 9

Flooded Model

Mass in Air (kg)	Weight in Air (N)
55.29	542.18

Flood Water

Mass in Air (kg)	Weight in Air (N)
29.69	291.15

Location of Center of Gravity of Flooded Model in Air

From Bow: (mm)	From Stern: (mm)
939	988

Swing Flooded Model in Air

g = 9.806
L = 1.31
D = 0.765
g/L^0.5 = 2.736

Trial	Time for 10 (s)	Period (s)	r (m)
1	29.37	2.937	0.489
2	29.55	2.955	0.492
3	29.46	2.946	0.491
4	29.23	2.923	0.487
5	29.34	2.934	0.489
	Mean :	2.939	0.490
	Median :	2.937	0.489
	Std Dev :	0.012	0.002

Moment of Iinertia (kg*m^2)

Dry	4.49
Flooded	13.25

LD 9

Uncertainty Calculation

u(T)= 0.01
u(D)= 0.001
u(L)= 0.003

$$u^2(K) = \left[\left(\frac{K}{T}\right)^2 u^2(T)\right] + \left[\left(\frac{K}{D}\right)^2 u^2(D)\right] + \left[\frac{1}{4}\left(\frac{K}{L}\right)^2 u^2(L)\right]$$

Flooded Model	2.77E-06	4.09E-07	3.14E-07
u(K) =	1.87E-03		
Dry Model	1.21E-06	4.88E-07	4.62E-07
u(K) =	1.47E-03		

LD 10

Overall Length (mm)	Length of Components (mm)	Overall Mass (kg)	Overall Weight (N)
2125	2119.5	27.30	267.70

Location of Center of Gravity

From Bow: (mm)	From Stern: (mm)
912	1213

Swing in Air

g = 9.806
L = 1.85
D = 0.60
g/L^0.5 = 2.302

Trial	Time for 10 (s)	Period (s)	r (m)
1	40.62	4.062	0.447
2	40.56	4.056	0.446
3	40.75	4.075	0.448
4	40.71	4.071	0.448
5	40.46	4.046	0.445
	Mean :	4.062	0.447
	Median :	4.062	0.447
	Std Dev :	0.012	0.001

Mass in Water (kg)	Weight in Water (N)	Bouyant Force (N)
3.792	37.18	230.52

Location of Center of Gravity in water

From Bow: (mm)	From Stern: (mm)
1504	621

LD 10

Flooded Model

Mass in Air (kg)	Weight in Air (N)
63.21	619.84

Flood Water

Mass in Air (kg)	Weight in Air (N)
35.91	352.13

Location of Center of Gravity of Flooded Model in Air

From Bow: (mm)	From Stern: (mm)
1057	1068

Swing Flooded Model in Air

g = 9.806
L = 1.3
D = 0.600
g/L^0.5 = 2.746

Trial	Time for 10 (s)	Period (s)	r (m)
1	39.18	3.918	0.514
2	39.4	3.94	0.517
3	39.13	3.913	0.513
4	39.12	3.912	0.513
5	39.34	3.934	0.516
	Mean :	3.923	0.514
	Median :	3.918	0.514
	Std Dev :	0.013	0.002

Moment of Iinertia (kg*m^2)

Dry	5.44
Flooded	16.73

LD 10

Uncertainty Calculation

u(T)= 0.01

u(D)= 0.001

u(L)= 0.003

$$u^2(K) = \left[\left(\frac{K}{T}\right)^2 u^2(T)\right] + \left[\left(\frac{K}{D}\right)^2 u^2(D)\right] + \left[\frac{1}{4}\left(\frac{K}{L}\right)^2 u^2(L)\right]$$

Flooded Model	1.72E-06	7.35E-07	3.52E-07
u(K) =	1.68E-03		
Dry Model	1.21E-06	5.54E-07	5.24E-07
u(K) =	1.51E-03		

LD 11

Overall Length (mm)	Length of Components (mm)	Overall Mass (kg)	Overall Weight (N)
2334	2322.9	28.20	276.53

Location of Center of Gravity

From Bow: (mm)	From Stern: (mm)
1011	1323

Swing in Air

g = 9.806
L = 1.85
D = 0.60
g/L^0.5 = 2.302

Trial	Time for 10 (s)	Period (s)	r (m)
1	44.53	4.453	0.490
2	44.31	4.431	0.487
3	44.43	4.443	0.488
4	44.50	4.450	0.489
5	44.46	4.446	0.489
	Mean :	4.445	0.489
	Median :	4.446	0.489
	Std Dev :	0.009	0.001

Mass in Water (kg)	Weight in Water (N)	Bouyant Force (N)
4.04	39.62	236.91

Location of Center of Gravity in water

From Bow: (mm)	From Stern: (mm)
1615	719

LD 11

Flooded Model

Mass in Air (kg)	Weight in Air (N)
70.08	687.20

Flood Water

Mass in Air (kg)	Weight in Air (N)
41.88	410.68

Location of Center of Gravity of Flooded Model in Air

From Bow: (mm)	From Stern: (mm)
1159	1175

Swing Flooded Model in Air

g = 9.806
L = 1.3
D = 0.600
g/L^0.5 = 2.746

Trial	Time for 10 (s)	Period (s)	r (m)
1	42.65	4.265	0.559
2	42.37	4.237	0.556
3	42.78	4.278	0.561
4	42.68	4.268	0.560
5	42.37	4.237	0.556
	Mean :	4.257	0.558
	Median :	4.265	0.559
	Std Dev :	0.019	0.002

Moment of Iinertia (kg*m^2)

Dry	6.73
Flooded	21.84

LD 11

Uncertainty Calculation

u(T)= 0.01
u(D)= 0.001
u(L)= 0.003

$$u^2(K) = \left[\left(\frac{K}{T}\right)^2 u^2(T)\right] + \left[\left(\frac{K}{D}\right)^2 u^2(D)\right] + \left[\frac{1}{4}\left(\frac{K}{L}\right)^2 u^2(L)\right]$$

Flooded Model	1.72E-06	8.66E-07	4.15E-07
u(K) =	1.73E-03		
Dry Model	1.21E-06	6.63E-07	6.28E-07
u(K) =	1.58E-03		

LD 12

Overall Length (mm)	Length of Components (mm)	Overall Mass (kg)	Overall Weight (N)
2535	2526.0	29.80	292.22

Location of Center of Gravity

From Bow: (mm)	From Stern: (mm)
1118	1421

Swing in Air

g = 9.806
L = 1.85
D = 0.60
g/L^0.5 = 2.302

Trial	Time for 10 (s)	Period (s)	r (m)
1	48.15	4.815	0.529
2	48.12	4.812	0.529
3	48.09	4.809	0.529
4	48.03	4.803	0.528
5	48.18	4.818	0.530
	Mean :	4.811	0.529
	Median :	4.812	0.529
	Std Dev :	0.006	0.001

Mass in Water (kg)	Weight in Water (N)	Bouyant Force (N)
4.549	44.61	247.61

Location of Center of Gravity in water

From Bow: (mm)	From Stern: (mm)
1722	813

LD 12

Flooded Model

Mass in Air (kg)	Weight in Air (N)
77.14	756.45

Flood Water

Mass in Air (kg)	Weight in Air (N)
47.34	464.24

Location of Center of Gravity of Flooded Model in Air

From Bow: (mm)	From Stern: (mm)
1256	1279

Swing Flooded Model in Air

g = 9.806
L = 1.3
D = 0.810
g/L^0.5 = 2.746

Trial	Time for 10 (s)	Period (s)	r (m)
1	36.62	3.662	0.648
2	36.53	3.653	0.647
3	36.65	3.665	0.649
4	36.59	3.659	0.648
5	36.53	3.653	0.647
	Mean :	3.658	0.648
	Median :	3.659	0.648
	Std Dev :	0.005	0.001

Moment of Iinertia (kg*m^2)

Dry	8.34
Flooded	32.36

LD 12

Uncertainty Calculation

u(T)= 0.01

u(D)= 0.001

u(L)= 0.003

$$u^2(K) = \left[\left(\frac{K}{T}\right)^2 u^2(T)\right] + \left[\left(\frac{K}{D}\right)^2 u^2(D)\right] + \left[\frac{1}{4}\left(\frac{K}{L}\right)^2 u^2(L)\right]$$

Flooded Model	3.13E-06	6.39E-07	5.58E-07
u(K) =	2.08E-03		
Dry Model	1.21E-06	7.77E-07	7.36E-07
u(K) =	1.65E-03		

Appendix B: Photos of Each Configuration

Figure 1 - LD 8 hanging for dry swing tests

Figure 2 - LD 8 submerged in water

Figure 3 - LD 9 hanging for dry swing tests

Figure 4 - LD 9 submerged in water

Figure 5 - LD 9 flooded for wet swing tests

Figure 6 - LD 10 hanging for dry swing tests

Figure 7 - LD 10 flooded for wet swing tests

Figure 8 - LD 11 hanging for dry swing tests

Figure 9 - LD 11 submerged in water

Figure 10 - LD 11 flooded for wet swing tests

Figure 11 - LD 12 submerged in water

Figure 12 - Nose Cone, Section A1, A2, A3 and Body of LD 12