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COMPUTER PROGRAMS FOR SOLVING NOTCH PROBLEMS USING NISITANI'S BODY FORCE METHOD

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COMPUTER PROGRAMS FOR SOLVING NOTCH PROBLEMS USING NISITANI'S BODY FORCE METHOD

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1. INTRODUCTION

A numerical method for the determination of stress intensity factors and stress concentration factors for cracks and notches has been presented by Nisitani (1). The method, known as the body force method, approximates a plate with a hole (or crack) by a plate without a hole but having body forces distributed along the boundary of the hole.

This method has been implemented on the NRC-IBM 360 system and has been described in a previous report (2). This report describes and lists the computer programs that were used to determine the stress intensity factors or stress concentration factors for the five types of notch and crack problems described in that report.

2. DEFINITIONS OF MAIN VARIABLES

DIST - e - the distance from the edge of the semi-infinite plate to the centre of the crack or ellipse.

EPSIL - ε - the numerical value calculated from FRAC used to define the endpoints of the integral over the singularity.

ETA - η - the y values of the endpoints of the crack intervals.

EVAL
FORCE
FORCE 2 - matrices containing influence coefficients or linear combinations of influence coefficients

FRAC - the fraction of an interval which is to be integrated analytically over a singularity.

INDEX - MM - the number of intervals to be used.

LENGTH - a - half the length of an ellipse along the y-axis.

MIDPT - the value of y for the midpoint of a crack interval, or the value of ϕ for the midpoint of an elliptic interval.

PHI - ϕ - the value of the elliptic parameter at the endpoints of the elliptic intervals.

STRESS - σ_x^∞ - the applied stress at infinity.

THETA - θ - the angle that the normal to the ellipse makes with the x-axis.

WIDTH - b - half the width of the ellipse along the x-axis or the half length of a crack.

X,Y - x,y - the x and y values of the midpoint of the crack or elliptic interval where stresses are being evaluated.

3. CALLING HIERARCHY AND FUNCTION SUBROUTINES

In the calculation of the influence coefficients, most programs use function subroutines to perform numerical integrations using a library routine ROMBRG. The calling hierarchy for these programs is given below.

Program Name	Desciption	Subroutines Called	
BODY 1	crack in semi-infinite plate (analytic integration)	None	
BODY 5	crack in semi-infinite plate (numerical integration)	FUNC	(γ^{XN}_{xM})
BODY 9	edge crack in semi-infinite plate	FUNC 2	(γ^{XN}_{xM})
ELL 5	ellipse in infinite plate	PSIGX	(σ^{XN}_{xM})
		PSIGY	(σ^{XN}_{yM})
		PTAU	(τ^{XN}_{xyM})
		QSIGX	(σ^{YN}_{xM})
		QSIGY	(σ^{YN}_{yM})
		QTAU	(τ^{YN}_{xyM})
ELL8	ellipse or semi-ellipse in semi-infinite plate	XSIGX	(σ^{XN}_{xM})
		XSIGY	(σ^{XN}_{yM})
		XTAU	(τ^{XN}_{xyM})
		YSIGX	(σ^{YN}_{xM})
		YSIGY	(σ^{YN}_{yM})
		YTAU	(τ^{YN}_{xyM})
ELCRA 1	semi-ellipse and crack in semi-infinite plate	XSIGX	(σ^{XN}_{xM})
		XSIGY	(σ^{XN}_{yM})
		XTAU	(τ^{XN}_{xyM})

YSIGX	(σ^{YN}_{xM})
YSIGY	(σ^{YN}_{yM})
YTAU	(τ^{YN}_{xyM})
XCRAC	$(\sigma^{\gamma N}_{xMe})$
YCRAC	$(\sigma^{\gamma N}_{yMe})$
TAUCRA	$(\tau^{\gamma N}_{xyMe})$

3.1 LISTINGS OF FUNCTION SUBROUTINES

```
C
C     THIS FUNCTION CALCULATES THE INTEGRAND FOR NUMERICAL INTEGRATION
C     FOR CRACK IN SEMIINFINITE PLATE.
C     IT IS CALLED BY THE MAIN PROGRAM: SOURCE.BODY5
C
      REAL FUNCTION FUNC*8 (X)
       REAL*8 X
       REAL*8 MIDPT(101),DIST,WIDTH,PI
       COMMON MIDPT,DIST,WIDTH,PI,M
C
       FUNC = 1/PI *(1/(MIDPT(M) - X)**2  - 1/(MIDPT(M) + X)**2
     2 + 12*MIDPT(M)/(MIDPT(M)+X)**3 - 12*MIDPT(M)**2/(MIDPT(M)+X)**4)
     3 * SQRT(WIDTH**2 - (X - DIST)**2)
C
       RETURN
       END
```

```
0005920CC
0006000CC      THIS FUNCTION CALCULATES THE INTEGRAND FOR NUMERICAL INTEGRATION
0006020CC      FOR CRACK AT EDGE OF SEMIINFINITE PLATE.
0006040CC
0006060CC      IT IS CALLED BY THE MAIN PROGRAM: SOURCE.BODY9
0006100CC
0006200C       REAL FUNCTION FUNC2*8 (X)
0006300C        REAL*8 X
0006310C        REAL*8 MIDPT(101),WIDTH,PI
0006340C        COMMON MIDPT,WIDTH,PI,M
0006360CC
0006400C        FUNC2 = 1/PI *(1/(MIDPT(M) - X)**2  - 1/(MIDPT(M) + X)**2
0006500C      2 + 12*MIDPT(M)/(MIDPT(M)+X)**3 - 12*MIDPT(M)**2/(MIDPT(M)+X)**4)
0006602C      3 * SQRT(WIDTH**2 - X**2)
0006610CC
0006630C        RETURN
0006800C        END
```

```
C
C      THIS FUNCTION CALCULATES THE X STRESS AT (X,Y) DUE TO A POINT
C        FORCE P IN THE X DIRECTION AT (A*COS(Z),B*SIN(Z)) IN AN
C        INFINITE PLATE.
C
       REAL FUNCTION PSIGX*8 (Z)
        REAL*8 Z
        REAL*8 PI,A,B,X,Y
        COMMON PI,A,B,X,Y
C
       PSIGX =-1/(4*PI)*(X-A*DCOS(Z))*(3*(X-A*DCOS(Z))**2+(Y-B*DSIN(Z))
      2  **2)/((X-A*DCOS(Z))**2+(Y-B*DSIN(Z))**2)**2  *B*DCOS(Z)
C
        RETURN
        END
```

```
C
C     THIS FUNCTION CALCULATES THE Y STRESS AT (X,Y) DUE TO A POINT
C       FORCE P IN THE X DIRECTION AT (A*COS(Z),B*SIN(Z)) IN AN
C       INFINITE PLATE.
C
      REAL FUNCTION PSIGY*8 (Z)
       REAL*8 Z
       REAL*8 PI,A,B,X,Y
       COMMON PI,A,B,X,Y
C
      PSIGY =1.0/(4*PI)*(X-A*DCOS(Z))*((X-A*DCOS(Z))**2-(Y-B*DSIN(Z))
     2  **2)/((X-A*DCOS(Z))**2+(Y-B*DSIN(Z))**2)**2  *B*DCOS(Z)
C
       RETURN
       END
```

ELL# 3

```
C
C      THIS FUNCTION CALCULATES THE SHEAR STRESS AT (X,Y) DUE TO A
C        POINT FORCE P IN THE X DIRECTION AT (A*COS(Z),B*SIN(Z)) IN
C        AN INFINITE PLATE.
C
       REAL FUNCTION PTAU*8 (Z)
        REAL*8 Z
        REAL*8 PI,A,B,X,Y
        COMMON PI,A,B,X,Y
C
       PTAU =-1.0/(4*PI)*(Y-B*DSIN(Z))*(3*(X-A*DCOS(Z))**2+(Y-B*DSIN(Z))
     2  **2)/((X-A*DCOS(Z))**2+(Y-B*DSIN(Z))**2)**2  *B*DCOS(Z)
C
        RETURN
        END
```

```
0005920CC
0006000CC      THIS FUNCTION CALCULATES THE SHEAR STRESS AT (X,Y) DUE TO A
0006020CC        POINT FORCE P IN THE X DIRECTION AT (A*COS(Z),B*SIN(Z)) IN
0006100CC        AN INFINITE PLATE.
0006120CC
0006200C       REAL FUNCTION PTAU*8 (Z)
0006300C        REAL*8 Z
0006310C        REAL*8 PI,A,B,X,Y
0006340C        COMMON PI,A,B,X,Y
0006350CC
0006360C       PTAU =-1.0/(4*PI)*(Y-B*DSIN(Z))*(3*(X-A*DCOS(Z))**2+(Y-B*DSIN(Z))
0006370C      2  **2)/((X-A*DCOS(Z))**2+(Y-B*DSIN(Z))**2)**2  *B*DCOS(Z)
0006610CC
0006630C        RETURN
0006800C        END
```

```
      THIS FUNCTION CALCULATES THE X STRESS AT (X,Y) DUE TO A PCINT
        FORCE Q IN THE Y DIRECTICN AT (A*COS(Z),B*SIN(Z)) IN AN
        INFINITL PLATE.

      REAL FUNCTION QSIGX*8 (Z)
        REAL*8 Z
        REAL*8 PI,A,B,X,Y
        COMMON PI,A,B,X,Y

      QSIGX =-1.0/(4*PI)*(Y-B*DSIN(Z))*((X-A*DCOS(Z))**2-(Y-B*DSIN(Z))
     2  **2)/((X-A*DCOS(Z))**2+(Y-B*DSIN(Z))**2)**2   *A*DSIN(Z)

        RETURN
        END
```

```
C
C     THIS FUNCTION CALCULATES THE Y STRESS AT (X,Y) DUE TO A POINT
C       FORCE Q IN THE Y DIRECTION AT (A*COS(Z),B*SIN(Z)) IN AN
C        INFINITE PLATE.
C
      REAL FUNCTION QSIGY*8 (Z)
       REAL*8 Z
       REAL*8 PI,A,B,X,Y
       COMMON PI,A,B,X,Y
C
      QSIGY =-1./(4*PI)*(Y-B*DSIN(Z))*((X-A*DCOS(Z))**2+3*(Y-B*DSIN(Z))
     2  **2)/((X-A*DCOS(Z))**2+(Y-B*DSIN(Z))**2)**2  *A*DSIN(Z)
C
       RETURN
       END
```

```
      THIS FUNCTION CALCULATES THE SHEAR STRESS AT (X,Y) DUE TO A
        POINT FORCE Q IN THE Y DIRECTION AT (A*COS(Z),B*SIN(Z)) IN
        AN INFINITE PLATE.

      REAL FUNCTION QTAU*8 (Z)
       REAL*8 Z
       REAL*8 PI,A,B,X,Y
       COMMON PI,A,B,X,Y

      QTAU =-1.0/(4*PI)*(X-A*DCOS(Z))*((X-A*DCOS(Z))**2+3*(Y-B*DSIN(Z))
     2  **2)/((X-A*DCOS(Z))**2+(Y-B*DSIN(Z))**2)**2  *A*DSIN(Z)

       RETURN
       END
```

```
      THIS FUNCTION CALCULATES THE X STRESS AT (X,Y) DUE TO A POINT
      FORCE X AT (A*COS(Z),B*SIN(Z)) IN A SEMIINFINITE PLATE.

      REAL FUNCTION XSIGX*8 (Z)
       REAL*8 Z
       REAL*8 PI,A,B,E,X,Y
       COMMON PI,X,Y /ELLIP/ A,B,E

      XSIGX=-1/(4*PI)*(X-A*DCOS(Z))*((3*(X-A*DCOS(Z))**2+(Y-B*DSIN(Z)
     2  -E)**2)/((X-A*DCOS(Z))**2+(Y-B*DSIN(Z)-E)**2)**2
     3  + (3*(X-A*DCOS(Z))**2+(Y+B*DSIN(Z)+E)**2)
     4  / ((X-A*DCOS(Z))**2+(Y+B*DSIN(Z)+E)**2)**2
     5  + 2*((X-A*DCOS(Z))**4 - 2*Y*(Y+B*DSIN(Z)+E)
     6  * (X-A*DCOS(Z))**2 + 6*Y*(Y+B*DSIN(Z)+E)**3
     7  + 2*Y**2*(X-A*DCOS(Z))**2 - (Y+B*DSIN(Z)+E)**4
     8  - 6*Y**2*(Y+B*DSIN(Z)+E)**2)
     9  / ((X-A*DCOS(Z))**2+(Y+B*DSIN(Z)+E)**2)**3)  *B*DCOS(Z)



       RETURN
       END
```

```
      THIS FUNCTION CALCULATES THE Y STRESS AT (X,Y) DUE TO A POINT
        FORCE X AT (A*COS(Z),B*SIN(Z)) IN A SEMIINFINITE PLATE.

      REAL FUNCTION XSIGY*8 (Z)
       REAL*8 Z
       REAL*8 PI,A,B,E,X,Y
       COMMON PI,X,Y /ELLIP/ A,B,E

      XSIGY =1/(4*PI)*(X-A*DCOS(Z))*(((X-A*DCOS(Z))**2-(Y-B*DSIN(Z)-E)
     2  **2)/((X-A*DCOS(Z))**2+(Y-B*DSIN(Z)-E)**2)**2
     3  +((X-A*DCOS(Z))**2-(Y+B*DSIN(Z)+E)**2)
     4  / ((X-A*DCOS(Z))**2+(Y+B*DSIN(Z)+E)**2)**2
     5  - 2*((X-A*DCOS(Z))**4 + 6*Y*(Y+B*DSIN(Z)+E)
     6  * (X-A*DCOS(Z))**2 - 2*Y*(Y+B*DSIN(Z)+E)**3
     7  - 2*Y**2*(X-A*DCOS(Z))**2 - (Y+B*DSIN(Z)+E)**4
     8  + 6*Y**2*(Y+B*DSIN(Z)+E)**2)
     9  / ((X-A*DCOS(Z))**2+(Y+B*DSIN(Z)+E)**2)**3)   *B*DCOS(Z)


       RETURN
       END
```

```
      THIS FUNCTION CALCULATES THE SHEAR STRESS AT (X,Y) DUE TO A
        POINT FORCE X AT (A*COS(Z),B*SIN(Z)) IN A SEMIINFINITE PLATE.

      REAL FUNCTION XTAU*8 (Z)
        REAL*8 Z
        REAL*8 PI,A,B,E,X,Y
        COMMON PI,X,Y /ELLIP/ A,B,E

      XTAU =-1/(4*PI)*((Y-B*DSIN(Z)-E)*(3*(X-A*DCOS(Z))**2+(Y-B*DSIN(Z)
     2  -E)**2)/((X-A*DCOS(Z))**2+(Y-B*DSIN(Z)-E)**2)**2
     3  +(Y+B*DSIN(Z)+E)*(3*(X-A*DCOS(Z))**2+(Y+B*DSIN(Z)+E)**2)
     4  / ((X-A*DCOS(Z))**2+(Y+B*DSIN(Z)+E)**2)**2
     5  - 2*Y*(-(X-A*DCOS(Z))**4 - 6*Y*(Y+B*DSIN(Z)+E)
     6  * (X-A*DCOS(Z))**2 + 2*Y*(Y+B*DSIN(Z)+E)**3
     7  - (Y+B*DSIN(Z)+E)**4
     8  + 6*(Y+B*DSIN(Z)+E)**2*(X-A*DCOS(Z))**2)
     9  / ((X-A*DCOS(Z))**2+(Y+B*DSIN(Z)+E)**2)**3)   *B*DCOS(Z)



        RETURN
        END
```

```
C
C     THIS FUNCTION CALCULATES THE X STRESS AT (X,Y) DUE TO A POINT
C       FORCE Y AT (A*COS(Z),B*SIN(Z)) IN A SEMIINFINITE PLATE.
C
      REAL FUNCTION YSIGX*8 (Z)
       REAL*8 Z
       REAL*8 PI,A,B,E,X,Y
       COMMON PI,X,Y /ELLIP/ A,B,E
C
      YSIGX =-1/(4*PI)*((Y-B*DSIN(Z)-E)*((X-A*DCOS(Z))**2-(Y-B*DSIN(Z)
     1  -E)**2)/((X-A*DCOS(Z))**2+(Y-B*DSIN(Z)-E)**2)**2
     2  - (Y+B*DSIN(Z)+E)*((X-A*DCOS(Z))**2-(Y+B*DSIN(Z)+E)**2)
     3  / ((X-A*DCOS(Z))**2+(Y+B*DSIN(Z)+E)**2)**2
     4  + 2*(3*Y*(X-A*DCOS(Z))**4 + (Y+B*DSIN(Z)+E)
     5  * (X-A*DCOS(Z))**4 - 5*Y*(Y+B*DSIN(Z)+E)**4
     6  - 6*Y**2*(Y+B*DSIN(Z)+E)*(X-A*DCOS(Z))**2 + 4*(Y+B*DSIN(Z)+E)**3
     7  * (X-A*DCOS(Z))**2 + 6*Y*(Y+B*DSIN(Z)+E)**2*(X-A*DCOS(Z))**2
     8  + 3*(Y+B*DSIN(Z)+E)**5 + 2*Y**2*(Y+B*DSIN(Z)+E)**3)
     9  / ((X-A*DCOS(Z))**2+(Y+B*DSIN(Z)+E)**2)**3)  *A*DSIN(Z)
C
C
C
       RETURN
       END
```

```
0005920CC
0006000CC     THIS FUNCTION CALCULATES THE Y STRESS AT (X,Y) DUE TO A POINT
0006020CC       FORCE Y AT (A*COS(Z),B*SIN(Z) IN A SEMIINFINITE PLATE.
0006100CC
0006200C      REAL FUNCTION YSIGY*8 (Z)
0006300C       REAL*8 Z
0006310C       REAL*8 PI,A,B,E,X,Y
0006340C       COMMON PI,X,Y /ELLIP/ A,B,E
0006350CC
0006360C      YSIGY =-1/(4*PI)*((Y-B*DSIN(Z)-E)*((X-A*DCOS(Z))**2+3*(Y-B*DSIN(Z
0006370C     1  )-E)**2)/((X-A*DCOS(Z))**2+(Y-B*DSIN(Z)-E)**2)**2
0006380C     2  - (Y+B*DSIN(Z)+E)*((X-A*DCOS(Z))**2+3*(Y+B*DSIN(Z)+E)**2)
0006390C     3  / ((X-A*DCOS(Z))**2+(Y+B*DSIN(Z)+E)**2)**2
0006400C     4  + 2*((Y+B*DSIN(Z)+E)*(X-A*DCOS(Z))**4 + 4*(Y+B*DSIN(Z)+E)**3
0006410C     5  * (X-A*DCOS(Z))**2 - 2*Y**2*(Y+B*DSIN(Z)+E)**3
0006420C     6  - Y*(X-A*DCOS(Z))**4 + 3*(Y+B*DSIN(Z)+E)**5 + 3*Y*(Y+B*DSIN(Z)
0006430C     7  +E)**4 - 6*Y*(Y+B*DSIN(Z)+E)**2*(X-A*DCOS(Z))**2
0006432C     8  + 6*Y**2*(Y+B*DSIN(Z)+E)*(X-A*DCOS(Z))**2)
0006440C     9  / ((X-A*DCOS(Z))**2+(Y+B*DSIN(Z)+E)**2)**3)   *A*DSIN(Z)
0006450CC
0006460CC
0006610CC
0006630C       RETURN
0006800C       END
```

```
      THIS FUNCTION CALCULATES THE SHEAR STRESS AT (X,Y) DUE TO A
        POINT FORCE Y AT (A*COS(Z),B*SIN(Z) IN A SEMIINFINITE PLATE.

      REAL FUNCTION YTAU*8 (Z)
       REAL*8 Z
       REAL*8 PI,A,B,E,X,Y
       COMMON PI,X,Y /ELLIP/ A,B,E

      YTAU =-1/(4*PI)*((X-A*DCOS(Z))*((X-A*DCOS(Z))**2+3*(Y-B*DSIN(Z)
     2  -E)**2)/((X-A*DCOS(Z))**2+(Y-B*DSIN(Z)-E)**2)**2
     3  -(X-A*DCOS(Z))*((X-A*DCOS(Z))**2+3*(Y+B*DSIN(Z)+E)**2)
     4  / ((X-A*DCOS(Z))**2+(Y+B*DSIN(Z)+E)**2)**2
     5  +2*Y*(X-A*DCOS(Z))*(2*Y*(X-A*DCOS(Z))**2+2*(Y+B*DSIN(Z)+E)
     6  * (X-A*DCOS(Z))**2 + 10*(Y+B*DSIN(Z)+E)**3
     7  - 6*Y*(Y+B*DSIN(Z)+E)**2)
     8  / ((X-A*DCOS(Z))**2+(Y+B*DSIN(Z)+E)**2)**3)  *A*DSIN(Z)

       RETURN
       END
```

```
C
C     THIS FUNCTION CALCULATES THE X STRESS AT (X,Y) DUE TO A POINT
C     FORCE X AT (0,Z) ON A CRACK IN A SEMIINFINITE PLATE.
C
      REAL FUNCTION XCRAC*8 (Z)
       REAL*8 Z
       REAL*8 PI,C,H,X,Y
       COMMON PI,X,Y /CRACK/ C,H
C
      XCRAC=(1/(PI*(X**2+(Z-Y)**2)**3)
     2     *(-3*X**4 +6*X**2*(Z-Y)**2 +(Z-Y)**4)
     3     + 1/(PI*(X**2+(Z+Y)**2)**4)
     4     *(-5*X**6 +(13*(Z+Y)**2+12*(Z+Y)*Y-12*Y**2)*X**4
     5     + (Z+Y)**2*(17*(Z+Y)**2-72*(Z+Y)*Y+72*Y**2)*X**2
     6     -(Z+Y)**4*((Z+Y)**2-12*(Z+Y)*Y+12*Y**2)))
     7     * DSQRT((C)**2 -(Z-H)**2)
C
C
       RETURN
       END
```

```
      THIS FUNCTION CALCULATES THE Y STRESS AT (X,Y) DUE TO A POINT
      FORCE X AT (0,Z) ON A CRACK IN A SEMIINFINITE PLATE.

      REAL FUNCTION YCRAC*8 (Z)
       REAL*8 Z
       REAL*8 PI,C,H,X,Y
       COMMON PI,X,Y /CRACK/ C,H

      YCRAC=(1/(PI*(X**2+(Z-Y)**2)**3)
     2     *(X**4 -6*X**2*(Z-Y)**2 +(Z-Y)**4)
     3     + 1/(PI*(X**2+(Z+Y)**2)**4)
     4     *(-X**6 +(5*(Z+Y)**2-36*(Z+Y)*Y+12*Y**2)*X**4
     5     + (Z+Y)**2*(5*(Z+Y)**2+56*(Z+Y)*Y-72*Y**2)*X**2
     6     -(Z+Y)**4*((Z+Y)**2+4*(Z+Y)*Y-12*Y**2)))
     7     * DSQRT((C)**2-(Z-H)**2)


       RETURN
       END
```

```
0005920CC
0006000CC     THIS FUNCTION CALCULATES THE SHEAR STRESS AT (X,Y) DUE TO A
0006020CC     POINT FORCE X AT (0,Z) ON A CRACK IN A SEMIINFINITE PLATE.
0006100CC
0006200C      REAL FUNCTION TAUCRA*8 (Z)
0006300C       REAL*8 Z
0006310C       REAL*8 PI,C,H,X,Y
0006340C       COMMON PI,X,Y /CRACK/ C,H
0006350CC
0006360C      TAUCRA=(2/(PI*(X**2+(Z-Y)**2)**3)*X*(Z-Y)*(3*X**2-(Z-Y)**2)
0006400C     2      - 1/(PI*(X**2+(Z+Y)**2)**4)
0006410C     3      * (2*(3*(Z+Y)+2*Y)*X**5 +4*(Z+Y)*((Z+Y)**2-14*(Z+Y)*Y
0006420C     4      + 12*Y**2)*X**3 -2*(Z+Y)**3*((Z+Y)**2-18*(Z+Y)*Y+24*Y**2)*X))
0006432C     5      * DSQRT((C)**2-(Z-H)**2)
0006460CC
0006610CC
0006630C       RETURN
0006800C       END
```

4. PROGRAM FOR CRACK IN SEMI-INFINITE PLATE USING ANALYTIC INTEGRATION (BODY I)

4.1 LISTING

```
C     PROGRAM TO COMPUTE SIF FOR CRACK IN SEMIINFINITE PLATE.
C
C     METHOD OF SOLUTION IS THE BODY FORCE METHOD (H. NISITANI).
C
C     INDEX IS THE NUMBER OF INTERVALS INTO WHICH THE CRACK HAS BEEN
C      PARTITIONED.
C     DIST IS THE DISTANCE FROM THE CENTRE OF THE CRACK TO THE
C      EDGE OF THE PLATE.
C     WIDTH IS HALF THE LENGTH OF THE CRACK.
C     STRESS IS THE APPLIED STRESS AT INFINITY.
C
      INTEGER INDEX,INCREM
      REAL DIST,WIDTH,STRESS,PI,A,B,C,D,E,F,G,H,X,TEST,DET,SIF1,SIF2
      REAL ETA(101),MIDPT(101),FUNC(101,101),FORCE(100,101)
C
C
      READ(5,500) INDEX, DIST, WIDTH, STRESS
C
C
      WRITE(6,1000)
      WRITE(6,1005)
      WRITE(6,1010)
      WRITE(6,1020) INDEX
      WRITE(6,1010)
      WRITE(6,1025)
      WRITE(6,1030) DIST
      WRITE(6,1010)
      WRITE(6,1035)
      WRITE(6,1040) WIDTH
      WRITE(6,1010)
      WRITE(6,1050) STRESS
      WRITE(6,1010)
C
C     COMPUTE VALUE OF ETA(N), MIDPT(N) FOR N=1 TO INDEX+1.
C     ETA(N) AND ETA(N+1) ARE THE VALUES OF THE ENDPOINTS
C     OF THE NTH INTERVAL ON THE Y AXIS ALONG THE CRACK.
C
C     MIDPT(N) IS THE Y VALUE OF THE MIDPOINT OF THE (N-1)TH INTERVAL.
C
C
      ETA(1) = DIST - WIDTH
C
C
      PI = 3.141593
      INCREM = INDEX +1
C
      DO 100 N=2,INCREM
        ETA(N) = ETA(1) + 2*WIDTH*(N-1)/INDEX
  100   MIDPT(N) = ETA(N) - WIDTH/INDEX
C
C     CALCULATE FUNC(M,N). THIS IS THE ANALYTICALLY INTEGRATED
C       BODY FORCE FUNCTION.
C
      DO 200 M=2,INCREM
```

```
C
C        A,B,C,D,E,F,G,H,X,Y ARE INTERMEDIARY FUNCTIONS.
C
         A = MIDPT(M) + DIST
         B = MIDPT(M) - DIST
         C = SQRT(WIDTH**2 - B**2)
         D = A**2 - WIDTH**2
C
C
         DO 200 N=1,INCREM
           E = ETA(N) - DIST
           F = ETA(N) + MIDPT(M)
           G = MIDPT(M) - ETA(N)
           H = SQRT(WIDTH**2 - E**2)
           X = ARSIN((WIDTH**2 + A*E)/(WIDTH*F))
C
C
           FUNC(M,N) = 1/ PI *(H/G + B/C
     1      * ALOG(ABS((WIDTH**2 - B*E + C*H)/(WIDTH*G))))
     2      - 1/ PI *(-H/F + A/SQRT(D) * X)
     3      + 6* MIDPT(M)/ PI * (-H/F**2 *(1- A*F/D)
     4      + WIDTH**2/D**1.5 * X ) - 2*MIDPT(M)**2/ PI
     5      *(H/F**3 *(-2 + A*F/D +(2*WIDTH**2 + A**2) * F**2/D**2)
     6      + 3*WIDTH**2 *A/D**2.5 * X)
C
 200   CONTINUE
C
C      EVALUATE BODY FORCE MATRIX AS FOLLOWS:
C      (INDEX BY INDEX) MATRIX CONTAINS FORCE FUNCTION (INFLUENCE
C         COEFFICIENTS);
C      (INDEX BY ONE) MATRIX CONTAINS NEGATIVE OF STRESS.
C
       DO 300 M=1,INDEX
         FORCE(M,INDEX+1) = -STRESS
         DO 300 N=1,INDEX
           FORCE(M,N) = FUNC(M+1,N+1) - FUNC(M+1,N)
 300   CONTINUE
C
C
C      CALL SOLVE SUBROUTINE FROM FTNLIB TO SOLVE SYSTEM OF
C         LINEAR EQUATIONS.  ON RETURN FROM SOLVE, SOLUTIONS WILL
C         BE FOUND IN THE (INDEX BY ONE) MATRIX.
C
       CALL SOLVE(FORCE,100,INDEX,INDEX+1,0,DET,TEST)
       IF (TEST.NE.0) GO TO 400
C
C
       WRITE(6,1060)
       WRITE(6,1065)
       WRITE(6,1010)
C
C      CORRECTION FACTORS FOR STRESS INTENSITY FACTORS ARE NOW IN
C      THE LAST COLUMN OF THE MATRIX.  THEY ARE TO BE PRINTED,
C      AND USED TO DETERMINE THE S.I.F. AT THE TIPS OF THE CRACK.
```

```
C
C       DO 350 M=1,INDEX
          FORCE(M,INDEX+1) = FORCE(M,INDEX+1)/STRESS
C350       WRITE(6,600) M,FORCE(M,INDEX+1)
C
C       WRITE(6,1010)
        SIF1 = FORCE(1,INCREM) * SQRT(PI*WIDTH)
        SIF2 = FORCE(INDEX,INCREM) * SQRT(PI*WIDTH)
C
        WRITE(6,1070)
        WRITE(6,1075) SIF1
        WRITE(6,1010)
        WRITE(6,1080)
        WRITE(6,1085) SIF2
        WRITE(6,1010)
 400    WRITE(6,700) DET
C
CC
CC
C500    FORMAT(I4,3(1X,F10.0))
 740    FORMAT(8(1X,F6.3))
 800    FORMAT(1X,F12.6)
C600    FORMAT(I4,1X,F12.6)
 700    FORMAT(1X,'THE DETERMINANT OF THE COEFFICIENT MATRIX IS',1X,E12.6)
C1000   FORMAT(1X,'SOLUTION TO CRACK IN SEMIINFINITE PLATE')
 1005   FORMAT(3X,'BY NISITANI BODY FORCE METHOD')
 1010   FORMAT(1X)
 1020   FORMAT(1X,'THE NUMBER OF DIVISIONS IN THE CRACK IS',1X,I4)
 1025   FORMAT(1X,'THE DISTANCE FROM THE EDGE OF THE PLATE TO')
 1030   FORMAT(3X,'THE CENTRE OF THE CRACK IS',1X,F12.6)
 1035   FORMAT(1X,'THE DISTANCE FROM THE TIP OF THE CRACK TO')
 1040   FORMAT(3X,'THE CENTRE OF THE CRACK IS',1X,F12.6)
 1050   FORMAT(1X,'THE APPLIED STRESS AT INFINITY IS',1X,F12.6)
 1060   FORMAT(1X,'THE CORRECTION FACTORS FOR EACH INTERVAL,')
 1065   FORMAT(3X,'STARTING WITH THE ONE NEAREST THE EDGE, ARE')
 1070   FORMAT(1X,'THE STRESS INTENSITY FACTOR AT THE TIP OF')
 1075   FORMAT(3X,'THE CRACK NEAREST THE EDGE IS',1X,E12.6)
 1080   FORMAT(1X,'THE STRESS INTENSITY FACTOR AT THE OTHER')
 1085   FORMAT(3X,'TIP IS',1X,E12.6)
C
C
C       STOP
C       END
```

4.2 TYPICAL INPUT

INDEX (MM) = 12

DIST (e) = 1.0

WIDTH (b) = 0.8

STRESS (σ_x^∞) = 1.0

FORMAT (I4, 3 (1X, F10.0))

4.3 TYPICAL OUTPUT

```
SOLUTION TO CRACK IN SEMIINFINITE PLATE
  BY NISITANI BODY FORCE METHOD

THE NUMBER OF DIVISIONS IN THE CRACK IS    12

THE DISTANCE FROM THE EDGE OF THE PLATE TO
  THE CENTRE OF THE CRACK IS      1.000000

THE DISTANCE FROM THE TIP OF THE CRACK TO
  THE CENTRE OF THE CRACK IS      0.800000

THE APPLIED STRESS AT INFINITY IS      1.000000

THE CORRECTION FACTORS FOR EACH INTERVAL.
  STARTING WITH THE ONE NEAREST THE EDGE. ARE

   1       1.385015
   2       1.350590
   3       1.317047
   4       1.287088
   5       1.260959
   6       1.238289
   7       1.218645
   8       1.201489
   9       1.186427
  10       1.173092
  11       1.161163
  12       1.150214

THE STRESS INTENSITY FACTOR AT THE TIP OF
  THE CRACK NEAREST THE EDGE IS 0.219571E 01

THE STRESS INTENSITY FACTOR AT THE OTHER
  TIP IS 0.182347E 01

THE DETERMINANT OF THE COEFFICIENT MATRIX IS 0.197109E 09
```

5. PROGRAM FOR CRACK IN SEMI-INFINITE PLATE USING NUMERICAL INTEGRATION (BODY 5)

5.1 LISTING

```
C     PROGRAM TO COMPUTE SIF FOR CRACKIN SEMIINFINITE PLATE.
C
C     METHOD OF SOLUTION IS THE BODY FORCE METHOD (H. NISITANI).
C
C     NUMERICAL INTEGRATION IS USED INSTEAD OF ANALYTICAL
C       INTEGRATION TO DETERMINE INFLUENCE COEFFICIENTS.
C
C     INDEX IS THE NUMBER OF INTERVALS INTO WHICH THE CRACK HAS BEEN
C       PARTITIONED.
C     DIST IS THE DISTANCE FROM THE CENTRE OF THE CRACK TO THE
C       EDGE OF THE PLATE.
C     WIDTH IS THE LENGTH OF THE CRACK.
C     STRESS IS THE APPLIED STRESS AT INFINITY.
C     FRAC IS A NUMBER MUCH SMALLER THAN 1.0 USED IN THE
C       NUMERICAL INTEGRATION AROUND THE SINGULARITY.
C
      INTEGER INDEX,INCREM
      REAL*8 PI,DIST,WIDTH,STRESS,EPSIL,SI,SIM1,A,B,PREC,DET,TEST
      REAL*8 ETA(101),MIDPT(101),FORCE(100,101)
      COMMON MIDPT,DIST,WIDTH,PI,M
      EXTERNAL FUNC
C
C
      READ(5,500) INDEX, DIST, WIDTH, STRESS, FRAC
C
C
      WRITE(6,1000)
      WRITE(6,1005)
      WRITE(6,1010)
      WRITE(6,1020) INDEX
      WRITE(6,1010)
      WRITE(6,1025)
      WRITE(6,1030) DIST
      WRITE(6,1010)
      WRITE(6,1035)
      WRITE(6,1040) WIDTH
      WRITE(6,1010)
      WRITE(6,1050) STRESS
      WRITE(6,1010)
C
C
C     COMPUTE VALUE OF ETA(N), MIDPT(N) FOR N=1 TO INDEX+1.
C     ETA(N) AND ETA(N+1) ARE THE VALUES OF THE ENDPOINTS
C     OF THE NTH INTERVAL ON THE Y AXIS ALONG THE CRACK.
C
C     MIDPT(N) IS THE Y VALUE OF THE MIDPOINT OF THE (N-1)TH INTERVAL.
C
C
      ETA(1) = DIST - WIDTH
C
C
      PI = 3.141593
C
      INCREM = INDEX +1
```

```
CC
C       DO 100  N=2,INCREM
C         ETA(N) = ETA(1) + 2*WIDTH*(N-1)/INDEX
C100      MIDPT(N) = ETA(N) - WIDTH/INDEX
CC
 C
 C      EPSIL IS THE VALUE OF EPSILON FOR INTEGRATION ABOUT
 C        THE SINGULARITIES.
 C
C       EPSIL = FRAC * WIDTH / INDEX
        WRITE(6,1055) EPSIL
        WRITE(6,1010)
 C
CC
CC      EVALUATE BODY FORCE MATRIX AS FOLLOWS:
CC       (INDEX BY INDEX) MATRIX CONTAINS FORCE FUNCTION (INFLUENCE
 C        COEFFICIENTS);
CC       (INDEX BY ONE) MATRIX CONTAINS NEGATIVE OF STRESS.
CC
 C      NUMERICAL INTEGRATION IS PERFORMED OVER THE WHOLE NTH INTERVAL
 C       UNLESS (M-1.EQ.N) IN WHICH CASE THE INTEGRAL IS DIVIDED INTO
 C       THREE PARTS ABOUT THE SINGULARITY.
 C
 C
C       DO 200  M=2,INCREM
C        DO 200  N=1,INDEX
CC
C         IF (M-1.EQ.N) GO TO 175
CC
C         CALL ROMBRG( FUNC,ETA(N),ETA(N+1),1.0,1.D-7,15,SI,SIM1,INDIC,L)
C         IF (INDIC.EQ.1) GO TO 450
 C
C         FORCE(M-1,N) = SI- 1
C         GO TO 200
CC
C175      A = MIDPT(M) + EPSIL
C         B = MIDPT(M) - EPSIL
C         CALL ROMBRG( FUNC,ETA(N),B,1.0,1.D-7,14,SI,SIM1,INDIC,L)
C         IF (INDIC.EQ.1) GO TO 450
 C
C         FORCE(M-1,N) = SI -1.
C         CALL ROMBRG( FUNC,A,ETA(N+1),1.0,1.D-7,14,SI,SIM1,INDIC,L)
C         IF (INDIC.EQ.1) GO TO 450
C         FORCE(M-1,N)=FORCE(M-1,N)+SI-1. - 2*SQRT(WIDTH**2
C      2  -(MIDPT(M)-DIST)**2) / (PI * EPSIL)
CC
C200    CONTINUE
CC
C       DO 300  M=1,INDEX
C300      FORCE(M,INDEX+1) = -STRESS
CC
CC
CC      CALL SOLVD SUBROUTINE TO SOLVE SYSTEM OF LINEAR EQUATIONS.
CC
```

```
C      CALL SOLVD(FORCE,100,INDEX,INDEX+1,0.0,DET,TEST)
C      IF (TEST.NE.0) GO TO 400
CC
C      WRITE(6,1060)
       WRITE(6,1065)
       WRITE(6,1010)
 C
C      DO 350 M=1,INDEX
         FORCE(M,INCREM) = FORCE(M,INCREM)/STRESS
C350     WRITE(6,600) M,FORCE(M,INCREM)
 C
 C     CALCULATE STRESS INTENSITY FACTORS AT TIPS OF CRACK.
 C
       SIF1 = FORCE(1,INCREM) * SQRT(PI*WIDTH)
       SIF2 = FORCE(INDEX,INCREM) * SQRT(PI*WIDTH)
 C
 C
       WRITE(6,1010)
       WRITE(6,1070)
       WRITE(6,1075) SIF1
       WRITE(6,1010)
       WRITE(6,1080)
       WRITE(6,1085) SIF2
       WRITE(6,1010)
C      WRITE(6,1090) DET
 C
CC
CC
C500    FORMAT(I4,4(1X,F10.0))
C600    FORMAT(I4,1X,F12.6)
C700    FORMAT(1X,'ERROR: NUMERICAL INTEGRATION DOES NOT CONVERGE')
 800    FORMAT(1X,'ERROR: COEFFICIENT MATRIX IS SINGULAR')
C1000   FORMAT(1X,'SOLUTION TO CRACK IN SEMIINFINITE PLATE')
C1005   FORMAT(3X,'BY NISITANI BODY FORCE METHOD')
 1010   FORMAT(1X)
 1020   FORMAT(1X,'THE NUMBER OF DIVISIONS IN THE CRACK IS',1X,I4)
 1025   FORMAT(1X,'THE DISTANCE FROM THE EDGE OF THE PLATE')
 1030   FORMAT(3X,'TO THE CENTRE OF THE CRACK IS',1X,F12.6)
 1035   FORMAT(1X,'THE DISTANCE FROM THE TIP OF THE CRACK')
 1040   FORMAT(3X,'TO THE CENTRE OF THE CRACK IS',1X,F12.6)
 1050   FORMAT(1X,'THE APPLIED STRESS AT INFINITY IS',1X,F12.6)
 1055   FORMAT(1X,'THE VALUE OF EPSILON USED FOR INTEGRATION IS',1X,F12.6)
 1060   FORMAT(1X,'THE CORRECTION FACTORS FOR EACH INTERVAL,')
 1065   FORMAT(3X,'STARTING WITH THE ONE NEAREST THE EDGE, ARE')
 1070   FORMAT(1X,'THE STRESS INTENSITY FACTOR AT THE TIP OF')
 1075   FORMAT(3X,'THE CRACK NEAREST THE EDGE IS',1X,E12.6)
 1080   FORMAT(1X,'THE STRESS INTENSITY FACTOR AT THE OTHER')
 1085   FORMAT(3X,'TIP IS',1X,E12.6)
 1090   FORMAT(1X,'THE DETERMINANT OF THE COEFFICIENT MATRIX IS',1X,E12.6)
 C
CC
C      STOP
 C
 C
```

```
0006380          STOP
0006400 C
0006420C450      WRITE(6,700)
0006440          STOP
0006460 C
0006480C         END
```

5.2 TYPICAL INPUT

INDEX (MM) = 12

DIST (e) = 1.0

WIDTH (b) = 0.8

STRESS (σ_x^∞) = 1.0

FRAC = 0.01

FORMAT (I4, 4(1X, F10.0))

5.3 TYPICAL OUTPUT

```
SOLUTION TO CRACK IN SEMIINFINITE PLATE
  BY NISITANI BODY FORCE METHOD

THE NUMBER OF DIVISIONS IN THE CRACK IS    12

THE DISTANCE FROM THE EDGE OF THE PLATE
  TO THE CENTRE OF THE CRACK IS        1.000000

THE DISTANCE FROM THE TIP OF THE CRACK
  TO THE CENTRE OF THE CRACK IS        0.800000

THE APPLIED STRESS AT INFINITY IS        1.000000

THE VALUE OF EPSILON USED FOR INTEGRATION IS        0.000667

THE CORRECTION FACTORS FOR EACH INTERVAL,
  STARTING WITH THE ONE NEAREST THE EDGE, ARE

    1        1.385453
    2        1.351123
    3        1.317355
    4        1.287297
    5        1.261161
    6        1.238453
    7        1.218806
    8        1.201747
    9        1.186739
   10        1.173547
   11        1.161893
   12        1.151842

THE STRESS INTENSITY FACTOR AT THE TIP OF
  THE CRACK NEAREST THE EDGE IS 0.219803E 01

THE STRESS INTENSITY FACTOR AT THE OTHER
  TIP IS 0.182605E 01

THE DETERMINANT OF THE COEFFICIENT MATRIX IS 0.196455E 09
```

6.0 PROGRAM FOR EDGE CRACK IN SEMI-INFINITE PLATE (BODY 9)

6.1 LISTING

```
CC      PROGRAM TO COMPUTE SIF FOR CRACK AT THE EDGE OF A
CC        SEMIINFINITE PLATE.
CC      METHOD OF SOLUTION IS THE BODY FORCE METHOD (H. NISITANI).
CC
CC      NUMERICAL INTEGRATION IS USED INSTEAD OF ANALYTICAL
CC        INTEGRATION TO DETERMINE BODY FORCE COEFFICIENTS.
CC
CC      INDEX IS THE NUMBER OF INTERVALS INTO WHICH THE CRACK HAS BEEN
CC        PARTITIONED.
CC      WIDTH IS THE LENGTH OF THE CRACK.
CC      STRESS IS THE APPLIED STRESS AT INFINITY.
 C      FRAC IS A NUMBER MUCH LESS THAN 1.0 USED IN THE
 C        NUMERICAL INTEGRATION ABOUT A SINGULARITY.
CC
C       INTEGER INDEX,INCREM
C       REAL*8 PI,WIDTH,STRESS,EPSIL,SI,SIM1,A,B,PREC,DET,TEST,FRAC,SIF
C       REAL*8 ETA(101),MIDPT(101),FORCE(100,101)
C       COMMON MIDPT,WIDTH,PI,M
C       EXTERNAL FUNC2
CC
CC
C       READ(5,500) INDEX, WIDTH, STRESS, FRAC
 C
        WRITE(6,1000)
        WRITE(6,1005)
        WRITE(6,1010)
        WRITE(6,1020) INDEX
        WRITE(6,1010)
        WRITE(6,1030) WIDTH
        WRITE(6,1010)
        WRITE(6,1040) STRESS
        WRITE(6,1010)
CC
CC
CC      COMPUTE VALUE OF ETA(N) FOR N=1 TO INDEX+1.
CC      ETA(N) AND ETA(N+1) ARE THE VALUES OF THE ENDPOINTS
CC      OF THE NTH INTERVAL ON THE Y AXIS ALONG THE CRACK.
CC
CC      MIDPT(N) IS THE Y VALUE OF THE MIDPOINT OF THE (N-1)TH INTERVAL.
CC
CC
C       ETA(1) = 0.0
CC
CC
C       PI = 3.14159265
C       INCREM = INDEX +1
C       EPSIL = FRAC * WIDTH /(2 * INDEX)
        WRITE(6,1045)
        WRITE(6,1050) EPSIL
        WRITE(6,1010)
        WRITE(6,1060)
        WRITE(6,1065)
CC
C       DO 100 N=2,INCREM
```

```
0001200C          ETA(N) = WIDTH*(N-1)/INDEX
0001300C100       MIDPT(N) = ETA(N) - WIDTH/(2 * INDEX)
0001400CC
0001600CC
0001620CC       EVALUATE BODY FORCE MATRIX AS FOLLOWS:
0001630CC        (INDEX BYINDEX) MATRIX CONTAINS FORCE FUNCTION;
0001640CC        (INDEX BY ONE) MATRIX CONTAINS NEGATIVE OF STRESS.
0001650CC
0001700C        DO 200 M=2,INCREM
0002600C         DO 200 N=1,INDEX
0002700CC
0002720C          IF (M-1.EQ.N) GO TO 175
0002740 C
0002800C          CALL ROMBRG( FUNC2,ETA(N),ETA(N+1),1.0,1.D-7,16,SI,SIM1,INDIC,L)
0002900CC
0003000C          IF (INDIC.EQ.1) GO TO 400
0003005C          FORCE(M-1,N) = SI- 1
0003020C          GO TO 200
0003040CC
0003080C175       A = MIDPT(M) + EPSIL
0003100C          B = MIDPT(M) - EPSIL
0003120C          CALL ROMBRG( FUNC2,ETA(N),B,1.0,1.D-7,14,SI,SIM1,INDIC,L)
0003140C          IF (INDIC.EQ.1) GO TO 400
0003160C          FORCE(M-1,N) = SI -1.
0003180C          CALL ROMBRG( FUNC2,A,ETA(N+1),1.0,1.D-7,14,SI,SIM1,INDIC,L)
0003200C          IF (INDIC.EQ.1) GO TO 400
0003220C          FORCE(M-1,N)=FORCE(M-1,N)+SI-1. - 2*DSQRT(WIDTH**2
0003240C       2  -MIDPT(M)**2) / (PI * EPSIL)
0003400CC
0003520C200     CONTINUE
0004100CC
0004120 C
0004140 C
0004600C        DO 300 M=1,INDEX
0004700C300       FORCE(M,INDEX+1) = -STRESS
0005000CC
0005100CC
0005200CC       CALL SOLVD SUBROUTINE TO SOLVE SYSTEM OF LINEAR EQUATIONS.
0005300CC
0005400C        CALL SOLVD(FORCE,100,INDEX,INDEX+1,.5D-6,DET,TEST)
0005500C        IF (TEST.NE.0) GO TO 450
0005520CC
0005540CC
0005560C        DO 350 M=1,INDEX
0005570           FORCE(M,INCREM) = FORCE(M,INCREM)/STRESS
0005600C350       WRITE(6,600) M,FORCE(M,INDEX+1)
0005705CC
0005706 C
0005707         SIF = FORCE(INDEX,INCREM) * DSQRT(PI * WIDTH)
0005708 C
0005710         WRITE(6,1010)
0005720         WRITE(6,1070)
0005740         WRITE(6,1075) SIF
0005742         WRITE(6,1010)
```

```
0005744        WRITE(6,1090) DET
0005745 C
0005748 C
0005760CC
0005780C500    FORMAT(I4,3(1X,F10.0))
0005785 740    FORMAT(5(1X,F8.3))
0005800C600    FORMAT(I4,1X,F12.6)
0005820 1000   FORMAT(1X,'SOLUTION TO EDGE CRACK IN SEMIINFINITE PLATE')
0005840 1005   FORMAT(3X,'USING NISITANI BODY FORCE METHOD')
0005860 1010   FORMAT(1X)
0005880 1020   FORMAT(1X,'THE NUMBER OF DIVISIONS IN THE CRACK IS',1X,I4)
0005900 1030   FORMAT(1X,'THE LENGTH OF THE CRACK IS',1X,F12.6)
0005920 1040   FORMAT(1X,'THE APPLIED STRESS AT INFINITY IS',1X,F12.6)
0005940 1045   FORMAT(1X,'THE VALUE OF EPSILON IN THE NUMERICAL')
0005960 1050   FORMAT(3X,'INTEGRATION IS',1X,F12.6)
0005980 1060   FORMAT(1X,'THE CORRECTION FACTORS FOR EACH INTERVAL,STARTING')
0006000 1065   FORMAT(3X,'WITH THE ONE NEAREST THE EDGE, ARE')
0006020 1070   FORMAT(1X,'THE STRESS INTENSITY FACTOR AT THE TIP OF')
0006040 1075   FORMAT(3X,'THE CRACK IS',1X,F12.6)
0006060 1080   FORMAT(1X,'ERROR: NUMERICAL INTEGRATION DID NOT CONVERGE')
0006080 1085   FORMAT(1X,'ERROR: COEFFICIENT MATRIX IS SINGULAR')
0006100 1090   FORMAT(1X,'THE DETERMINANT OF COEFFICIENT MATRIX IS',1X,E12.6)
0006120 C
0006140CC
0006160        STOP
0006180 C
0006200 C
0006220C400    WRITE(6,1080)
0006240        STOP
0006260 C
0006280 C
0006300C450    WRITE(6,1085)
0006320C       STOP
0006340 C
0006360C       END
```

6.2 TYPICAL INPUT

INDEX (MM) = 12

DIST (e) = 1.0

STRESS (σ_x^{∞}) = 1.0

FRAC = 0.01

FORMAT (I4, 3(1X, F10.0))

6.3 TYPICAL OUTPUT

```
SOLUTION TO EDGE CRACK IN SEMIINFINITE PLATE
  USING NISITANI BODY FORCE METHOD

THE NUMBER OF DIVISIONS IN THE CRACK IS    12

THE LENGTH OF THE CRACK IS      1.000000

THE APPLIED STRESS AT INFINITY IS      1.000000

THE VALUE OF EPSILON IN THE NUMERICAL
  INTEGRATION IS      0.000417

THE CORRECTION FACTORS FOR EACH INTERVAL,STARTING
  WITH THE ONE NEAREST THE EDGE, ARE
   1      1.356457
   2      1.318769
   3      1.281655
   4      1.249164
   5      1.221302
   6      1.197477
   7      1.177049
   8      1.159454
   9      1.144221
  10      1.130968
  11      1.119419
  12      1.109599

THE STRESS INTENSITY FACTOR AT THE TIP OF
  THE CRACK IS      1.966713

THE DETERMINANT OF COEFFICIENT MATRIX IS 0.137103E 12
```

7. PROGRAM FOR ELLIPSE IN INFINITE PLATE (ELL 5)

7.1 LISTING

```
0000100CC      PROGRAM TO COMPUTE SCF FOR ELLIPSE IN AN INFINITE PLATE.
0000202CC      METHOD OF SOLUTION IS THE BODY FORCE METHOD (H. NISITANI).
0000203CC
0000204CC      NUMERICAL INTEGRATION IS USED INSTEAD OF ANALYTICAL
0000205CC        INTEGRATION TO DETERMINE INFLUENCE COEFFICIENTS.
0000210CC
0000220CC      INDEX IS THE NUMBER OF INTERVALS INTO WHICH THE ELLIPSE
0000240CC        HAS BEEN PARTITIONED.
0000245CC      WIDTH IS HALF THE WIDTH OF THE ELLIPSE ALONG
0000260CC        THE X AXIS.
0000280CC      LENGTH IS HALF THE HEIGHT OF THE ELLIPSE.
0000300CC      STRESS IS THE APPLIED STRESS AT INFINITY.
0000320CC      FRAC IS A NUMBER MUCH LESS THAN 1.0 USED IN THE
0000340CC        NUMERICAL INTEGRATION ABOUT A SINGULARITY.
0000360CC
0000380C       INTEGER INDEX,INCREM,HALF1,HALF2,QUART1,QUART2
0000400C       REAL*8 PI,WIDTH,LENGTH,X,Y,STRESS,EPSIL,SI,SIM1,C,D,PREC,DET,TEST
0000405C       REAL*8 S,T,XSUM90,THETA,HALF
0000420C       REAL*8 PHI(71),MIDPT(71),FORCE(140,210),EVAL(140,141)
0000440C       COMMON PI,WIDTH,LENGTH,X,Y
0000460C       EXTERNAL PSIGX,PSIGY,QSIGX,QSIGY,PTAU,QTAU
0000480CC
0000500CC
0000520C       READ(5,500) INDEX, WIDTH, LENGTH, STRESS, FRAC
0000540CC
0000560C       WRITE(6,1000)
0000580C       WRITE(6,1005)
0000600C       WRITE(6,1010)
0000620C       WRITE(6,1020) INDEX
0000640C       WRITE(6,1010)
0000660C       WRITE(6,1025) WIDTH
0000680C       WRITE(6,1010)
0000685C       WRITE(6,1030) LENGTH
0000690C       WRITE(6,1010)
0000700C       WRITE(6,1040) STRESS
0000720C       WRITE(6,1010)
0000740CC
0000742CC      DEFINE PI, EPSILON. EPSILON IS USED IN INTEGRATION
0000745CC        ABOUT SINGULARITIES.
0000760CC
0000780C       PI = 3.14159265359
0000800C       INCREM = INDEX +1
0000805C       T = LENGTH/WIDTH
0000820C       EPSIL = FRAC * PI/INDEX
0000840C       WRITE(6,1045)
0000860C       WRITE(6,1050) EPSIL
0000880C       WRITE(6,1010)
0000940CC
0000960CC
0000980CC      COMPUTE VALUE OF PHI(N) FOR N=1 TO INDEX+1.
0001000CC      PHI(N) AND PHI(N+1) ARE PARAMETERS FOR THE VALUE OF
0001002CC        THE ENDPOINTS OF THE NTH INTERVAL:
0001004CC      X = WIDTH * COS(PHI); Y = LENGTH * SIN(PHI)
0001006CC
```

```
C     MIDPT(M) IS THE VALUE OF THIS PARAMETER AT THE MIDPOINT
C       OF THE (N-1)TH INTERVAL.
C
C     X AND Y ARE THE X AND Y VALUES OF THE POINT ON THE ELLIPSE
C       HAVING MIDPT(M) AS ITS PARAMETER VALUE.
C
C     THETA IS THE ANGLE THE NORMAL ON THE ELLIPSE MAKES WITH THE X
C       AXIS.  IT IS ALWAYS TAKEN TO BE IN THE INTERVAL (-PI/2,PI/2).
C       THIS IS NOT THE TRUE VALUE OF THETA IN ALL QUADRANTS, BUT
C       NO CORRECTION IS NEEDED AT THIS POINT.
C
C
      PHI(1) = 0.0
C
      DO 100 N=2,INCREM
        PHI(N) = 2 * PI*(N-1)/INDEX
        MIDPT(N) = PHI(N) - PI/INDEX
  100 CONTINUE
C
      WRITE(6,1010)
C
C     PERFORM NUMERICAL INTEGRATIONS TO DETERMINE INFLUENCE
C       COEFFICIENTS.  FOR EXAMPLE, PSIGY IS THE COEFFICIENT IN
C       THE Y DIRECTION DUE TO A POINT FORCE P.
C
      DO 200 M=2,INCREM
       J=M+INDEX
       X = WIDTH * DCOS(MIDPT(M))
       Y = LENGTH * DSIN(MIDPT(M))
       S = DTAN(MIDPT(M))
C
       THETA = DATAN(S/T)
C
       C = MIDPT(M) + EPSIL
       D = MIDPT(M) - EPSIL
C
       DO 200 N=1,INDEX
        K=N+INDEX
        L=N+2*INDEX
C
        IF (M-1.EQ.N) GO TO 175
C
        CALL ROMBRG( PSIGX,PHI(N),PHI(N+1),1.0,1.D-7,10,SI,SIMI,INDIC,I)
        IF (INDIC.EQ.1) GO TO 400
        FORCE(M-1,N) = SI- 1.0
C
        CALL ROMBRG( PSIGY,PHI(N),PHI(N+1),1.0,1.D-7,10,SI,SIMI,INDIC,I)
        IF (INDIC.EQ.1) GO TO 400
        FORCE(M-1,K) = SI- 1.0
C
        CALL ROMBRG( PTAU,PHI(N),PHI(N+1),1.0,1.D-7,10,SI,SIMI,INDIC,I)
        IF (INDIC.EQ.1) GO TO 400
        FORCE(M-1,L) = SI- 1.0
C
```

```
      CALL ROMBRG( QSIGX,PHI(N),PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
      IF (INDIC.EQ.1) GO TO 400
      FORCE(J-1,N) = SI- 1.0
C
      CALL ROMBRG( QSIGY,PHI(N),PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
      IF (INDIC.EQ.1) GO TO 400
      FORCE(J-1,K) = SI- 1.0
C
      CALL ROMBRG( QTAU,PHI(N),PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
      IF (INDIC.EQ.1) GO TO 400
      FORCE(J-1,L) = SI- 1.0
C
      GO TO 200
C
C
C     INTEGRATE OVER SINGULARITIES BY TAKING PRINCIPAL VALUES.
C     THE INTEGRAL HAS FOUR PARTS:  THE FIRST TWO ARE THE NORMAL
C       INTEGRALS OVER (PHI(N),D) AND (C,PHI(N+1)) WHICH AVOID
C       THE SINGULARITY.  THE THIRD TERM INVOLVES THETA AND IS
C       EXPLAINED IN THE LITERATURE.  THE FOURTH TERM IS THE
C       ERROR IN THE INTEGRAL DUE TO THE CHOICE OF EPSILON.
C
  175 CALL ROMBRG( PSIGX,PHI(N),D,1.0,1.D-7,10,SI,SIM1,INDIC,I)
      IF (INDIC.EQ.1) GO TO 400
      FORCE(M-1,N) = SI -1.
      CALL ROMBRG( PSIGX,C,PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
      IF (INDIC.EQ.1) GO TO 400
      FORCE(M-1,N)=FORCE(M-1,N)+SI-1.0
     2       - (5+4*DCOS(2*THETA)-DCOS(4*THETA))/16
     3       + EPSIL*T/(4*PI*(S**2+T**2)**3)*(6*S**6+(3-2*T**2)*S**4
     4       -6*S**2*T**2-T**4)
C
      CALL ROMBRG( PSIGY,PHI(N),D,1.0,1.D-7,10,SI,SIM1,INDIC,I)
      IF (INDIC.EQ.1) GO TO 400
      FORCE(M-1,K) = SI -1.
      CALL ROMBRG( PSIGY,C,PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
      IF (INDIC.EQ.1) GO TO 400
      FORCE(M-1,K)=FORCE(M-1,K)+SI-1.0
     2       +(1-DCOS(4*THETA))/16
     3       + EPSIL*T/(4*PI*(S**2+T**2)**3)*(-2*S**6-(1-6*T**2)*S**4
     4       +6*S**2*T**2-T**4)
C
      CALL ROMBRG( PTAU,PHI(N),D,1.0,1.D-7,10,SI,SIM1,INDIC,I)
      IF (INDIC.EQ.1) GO TO 400
      FORCE(M-1,L) = SI -1.
      CALL ROMBRG( PTAU,C,PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
      IF (INDIC.EQ.1) GO TO 400
      FORCE(M-1,L)=FORCE(M-1,L)+SI-1.0
     2       -(2*DSIN(2*THETA)-DSIN(4*THETA))/16
     3       + EPSIL*T**2/(4*PI*(S**2+T**2)**3)*(-9*S**5-(6+2*T**2)
     4       *S**3 + T**2*S*(2-T**2))
C
      CALL ROMBRG( QSIGX,PHI(N),D,1.0,1.D-7,10,SI,SIM1,INDIC,I)
```

```
0004180C          FORCE(J-1,N) = SI -1.
0004220C          CALL ROMBRG( GSIGX,C,PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
0004240C          IF (INDIC.EQ.1) GO TO 400
0004260C          FORCE(J-1,N)=FORCE(J-1,N)+SI-1.0
0004280C       2        +(1-DCOS(4*THETA))/16
0004290C       3        + EPSIL*T/(4*PI*(S**2+T**2)**3)*(-S**6+(6-T**2)*S**2
0004300C       4        *T**2 + 6*S**4*T**2-2*T**4)
0004420CC
0004440C          CALL ROMBRG( GSIGY,PHI(N),D,1.0,1.D-7,10,SI,SIM1,INDIC,I)
0004460C          IF (INDIC.EQ.1) GO TO 400
0004480C          FORCE(J-1,K) = SI -1.
0004520C          CALL ROMBRG( GSIGY,C,PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
0004540C          IF (INDIC.EQ.1) GO TO 400
0004560C          FORCE(J-1,K)=FORCE(J-1,K)+SI-1.0
0004580C       2        -(5-4*DCOS(2*THETA)-DCOS(4*THETA))/16
0004600C       3        + EPSIL*T/(4*PI*(S**2+T**2)**3)*(-S**6+(2-T**2)*S**2
0004620C       4        *T**2 - 6*S**4*T**2+6*T**4)
0004640CC
0004660C          CALL ROMBRG( GTAU,PHI(N),D,1.0,1.D-7,10,SI,SIM1,INDIC,I)
0004680C          IF (INDIC.EQ.1) GO TO 400
0004700C          FORCE(J-1,L) = SI -1.
0004740C          CALL ROMBRG( GTAU,C,PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
0004760C          IF (INDIC.EQ.1) GO TO 400
0004780C          FORCE(J-1,L)=FORCE(J-1,L)+SI-1.0
0004800C       2        -(2*DSIN(2*THETA)+DSIN(4*THETA))/16
0004820C       3        - EPSIL/(4*PI*(S**2+T**2)**3)*((1-2*T**2)*S**5
0004840C       4        +(1+3*T**2)*2*T**2*S**3 + 9*S*T**4)
0004960CC
0004980CC
0005000C200     CONTINUE
0005020CC
0005025C        WRITE(6,1010)
0005040CC
0005180CC
0005200CC       INSERT INFLUENCE COEFFICIENTS AND STRESSES INTO EVAL
0005220CC         MATRIX ACCORDING TO HOW THEY MUST APPEAR IN THE
0005240CC         SYSTEM OF LINEAR EQUATIONS.
0005245CC
0005246 C       THETA IS AGAIN THE ANGLE BETWEEN THE NORMAL TO THE ELLIPSE
0005247 C         AND THE X AXIS BUT IS IN THE INTERVAL (-PI/2,3*PI/2).
0005248 C         THIS IS THE CORRECT VALUE OF THETA.
0005249 C
0005250C        INCREM = 2*INDEX +1
0005260CC
0005280C        DO 250 M=1,INDEX
0005300C          J=M+INDEX
0005320C          THETA = DATAN(DTAN(MIDPT(M+1))/T)
0005325           IF (MIDPT(M+1).LE.PI/2) GO TO 230
0005326           IF (MIDPT(M+1).GT.3*PI/2) GO TO 230
0005340 C
0005360           THETA = THETA + PI
0005380 C
```

```
 C
C         DO 250 N=1,INDEX
C           K = N+INDEX
C           L = N+2*INDEX
C           EVAL(M,N) = FORCE(M,N)*DCOS(THETA)
C    2                + FORCE(M,L)*DSIN(THETA)
CC
C           EVAL(J,N) = FORCE(M,K)*DSIN(THETA)
C    2                + FORCE(M,L)*DCOS(THETA)
CC
C           EVAL(M,K) = FORCE(J,N)*DCOS(THETA)
C    2                + FORCE(J,L)*DSIN(THETA)
CC
C           EVAL(J,K) = FORCE(J,K)*DSIN(THETA)
C    2                + FORCE(J,L)*DCOS(THETA)
CC
C250    CONTINUE
CC
CC
CC
C       WRITE(6,1010)
CC
CC      CALL SOLVD SUBROUTINE TO SOLVE SYSTEM OF LINEAR EQUATIONS.
CC      ON RETURN FROM SOLVD, THE PART OF THE MATRIX THAT BEFORE
CC        CONTAINED THE APPLIED STRESSES WILL NOW CONTAIN THE
CC        SOLUTIONS FOR RO-SUB-X AND RO-SUB-Y.
CC
C       CALL SOLVD(EVAL,140,INCREM-1,INCREM,.5D-6,DET,TEST)
C       IF (TEST.NE.0) GO TO 450
CC
CC
C       WRITE(6,1060)
C       WRITE(6,1065)
        WRITE(6,1010)
 C
C       DO 350 M=1,INDEX
C         J=M+INDEX
C350      WRITE(6,600) M,EVAL(M,INCREM),EVAL(J,INCREM)
CC
CC      THE STRESSES AT ARBITRARY POINTS ON THE ELLIPSE (HERE 0 AND 90
CC        DEGREES) CAN BE EVALUATED AS A LINEAR COMBINATION OF THE
CC        INFLUENCE COEFFICIENTS AND THE ROS.
CC
C       XSUM90 = STRESS
CC
 C
 C
 C
CC      PERFORM NUMERICAL INTEGRATIONS TO DETERMINE INFLUENCE
CC        COEFFICIENTS.  FOR EXAMPLE, PSIGY IS THE COEFFICIENT IN
CC        THE Y DIRECTION DUE TO A POINT FORCE P.
```

```
      X = WIDTH * DCOS(HALF)
      Y = LENGTH * DSIN(HALF)
C
      D = HALF - EPSIL
C
      DO 380 N=1,INDEX
C
       IF (N.EQ.INDNEW) GO TO 375
C
        CALL ROMBRG( PSIGX,PHI(N),PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
        IF (INDIC.EQ.1) GO TO 400
        FORCE(1,N) = SI- 1.0
C
        CALL ROMBRG( QSIGX,PHI(N),PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
        IF (INDIC.EQ.1) GO TO 400
        FORCE(2,N) = SI- 1.0
C
C
        GO TO 380
C
C
C       INTEGRATE OVER SINGULARITIES BY TAKING PRINCIPAL VALUES.
C
  375   CALL ROMBRG( PSIGX,PHI(N),D,1.0,1.D-7,10,SI,SIM1,INDIC,I)
        IF (INDIC.EQ.1) GO TO 400
        FORCE(1,N)= SI-1.0 + 3*EPSIL*T/(2*PI)
C
        CALL ROMBRG( QSIGX,PHI(N),D,1.0,1.D-7,10,SI,SIM1,INDIC,I)
        IF (INDIC.EQ.1) GO TO 400
        FORCE(2,N)= SI-1.0 - EPSIL*T/(4*PI)
C
C
C
  380 CONTINUE
C
C
      DO 370 M=1,INDEX
        J=M+INDEX
        XSUM90 = XSUM90+EVAL(M,INCREM) * FORCE(1,M)
     2       + EVAL(J,INCREM) * FORCE(2,M)
  370 CONTINUE
C
C     DETERMINE STRESS CONCENTRATION FACTOR BY DIVIDING THE EVALUATED
C       STRESS IN THE X DIRECTION AT 90 DEGREES BY THE STRESS
C       AT INFINITY.
C
C
C
      WRITE(6,1010)
C
      XSUM90 = XSUM90/STRESS
      WRITE(6,1085) XSUM90
      WRITE(6,1010)
```

```
0009140CC
0009160CC
0009180CC
0009200C500    FORMAT(I4,4(1X,F10.0))
0009220C600    FORMAT(I4,2(1X,F12.6))
0009240C1000   FORMAT(1X,'SOLUTION TO ELLIPSE IN INFINITE PLATE')
0009260C1005   FORMAT(3X,'USING NISITANI BODY FORCE METHOD')
0009280C1010   FORMAT(1X)
0009300C1020   FORMAT(1X,'THE NUMBER OF DIVISIONS IN THE ELLIPSE IS ',I4)
0009320C1025   FORMAT(1X,'HALF THE WIDTH OF THE ELLIPSE IS ',F12.6)
0009340C1030   FORMAT(1X,'HALF THE HEIGHT OF THE ELLIPSE IS ',F12.6)
0009360C1040   FORMAT(1X,'THE APPLIED STRESS AT INFINITY IS ',F12.6)
0009380C1045   FORMAT(1X,'THE VALUE OF EPSILON IN THE NUMERICAL')
0009400C1050   FORMAT(3X,'INTEGRATION IS ',F12.6)
0009420 1060   FORMAT(1X,'THE X AND Y BODY FORCE DENSITIES, STARTING AT')
0009440 1065   FORMAT(3X,'THE SIDE OF THE ELLIPSE (PHI = 0), ARE:')
0009500C1085   FORMAT(1X,'THE STRESS CONCENTRATION FACTOR IS ',2(F12.6,1X))
0009520C1090   FORMAT(1X,'THE DETERMINANT OF COEFFICIENT MATRIX IS ',E12.6)
0009540C1100   FORMAT(1X,'ERROR: NUMERICAL INTEGRATION DID NOT CONVERGE.')
0009560C1110   FORMAT(1X,'ERROR: COEFFICIENT MATRIX IS SINGULAR.')
0009580CC
0009600CC
0009620C       STOP
0009640CC
0009660CC
0009680C400    WRITE(6,1100)
0009700C       STOP
0009720CC
0009740CC
0009760C450    WRITE(6,1110)
0009780C       STOP
0009800CC
0009820C       END
```

7.2 TYPICAL INPUT

INDEX (MM) = 12

WIDTH (a) = 2.0

LENGTH (b) = 1.0

STRESS (σ_x^∞) = 1.0

FRAC = 0.05

FORMAT (I4, 4(1X, F10.0))

7.3 TYPICAL OUTPUT

```
SOLUTION TO ELLIPSE IN INFINITE PLATE
  USING NISITANI BODY FORCE METHOD

THE NUMBER OF DIVISIONS IN THE ELLIPSE IS    12

HALF THE WIDTH OF THE ELLIPSE IS       2.000000

HALF THE HEIGHT OF THE ELLIPSE IS       1.000000

THE APPLIED STRESS AT INFINITY IS       1.000000

THE VALUE OF EPSILON IN THE NUMERICAL
  INTEGRATION IS       0.013090


THE X AND Y BODY FORCE DENSITIES, STARTING AT
  THE SIDE OF THE ELLIPSE (PHI = 0), ARE:

  1      2.000474     -1.002336
  2      1.999895     -1.000864
  3      1.999652     -1.000324
  4      1.999652     -1.000324
  5      1.999895     -1.000864
  6      2.000474     -1.002336
  7      2.000474     -1.002336
  8      1.999895     -1.000864
  9      1.999652     -1.000324
 10      1.999652     -1.000324
 11      1.999895     -1.000864
 12      2.000474     -1.002336

THE STRESS CONCENTRATION FACTOR IS       2.000041

THE DETERMINANT OF COEFFICIENT MATRIX IS 0.122099E-13
```

8.0 PROGRAM FOR ELLIPSE OR SEMI-ELLIPSE IN SEMI-INFINITE PLATE (ELL 8)

8.1 LISTING

```
C     PROGRAM TO COMPUTE SCF FOR ELLIPSE IN A SEMIINFINITE PLATE.
C     METHOD OF SOLUTION IS THE BODY FORCE METHOD (H. NISITANI).
C
C     NUMERICAL INTEGRATION IS USED INSTEAD OF ANALYTICAL
C       INTEGRATION TO DETERMINE INFLUENCE COEFFICIENTS.
C
C     INDEX IS THE NUMBER OF INTERVALS INTO WHICH THE ELLIPSE
C       HAS BEEN PARTITIONED.
C     WIDTH IS HALF THE WIDTH OF THE ELLIPSE ALONG
C       THE X AXIS.
C     LENGTH IS HALF THE HEIGHT OF THE ELLIPSE.
C     DIST IS THE DISTANCE FROM THE CENTRE OF THE ELLIPSE TO THE
C       EDGE OF THE PLATE.  IF DIST = 0. THEN WE HAVE A SEMIELLIPSE
C       AT THE EDGE OF A SEMIINFINITE PLATE.
C     STRESS IS THE APPLIED STRESS AT INFINITY.
C     FRAC IS A NUMBER MUCH LESS THAN 1.0 USED IN THE
C       NUMERICAL INTEGRATION ABOUT A SINGULARITY.
C
      INTEGER INDEX,INCREM,NEW
      REAL*8 PI,WIDTH,LENGTH,X,Y,STRESS,EPSIL,SI,SIMI,C,D,PREC,DET,TEST
      REAL*8 S,T,XSUM90,THETA,DIST,FRAC
      REAL*8 PHI(101),MIDPT(101),FORCE(200,300),EVAL(200,201)
      COMMON PI,X,Y /ELLIP/ WIDTH,LENGTH,DIST
      EXTERNAL XSIGX,XSIGY,YSIGX,YSIGY,XTAU,YTAU
C
C
      READ(5,500) INDEX, WIDTH, LENGTH, DIST, STRESS, FRAC
C
      WRITE(6,1000)
      WRITE(6,1005)
      WRITE(6,1010)
      WRITE(6,1020) INDEX
      WRITE(6,1010)
      WRITE(6,1025) WIDTH
      WRITE(6,1010)
      WRITE(6,1030) LENGTH
      WRITE(6,1010)
      WRITE(6,1035)
      WRITE(6,1037) DIST
      WRITE(6,1010)
      WRITE(6,1040) STRESS
      WRITE(6,1010)
C
C     DEFINE PI, EPSILON. EPSILON IS USED IN INTEGRATION
C       ABOUT SINGULARITIES.
C
      PI = 3.14159265359
      INCREM = INDEX +1
      T = LENGTH/WIDTH
      EPSIL = FRAC * PI/INDEX
      WRITE(6,1045)
      WRITE(6,1050) EPSIL
      WRITE(6,1010)
C
```

```
C
C      COMPUTE VALUE OF PHI(N) FOR N=1 TO INDEX+1.
C      PHI(N) AND PHI(N+1) ARE PARAMETERS FOR THE VALUE OF
C        THE ENDPOINTS OF THE NTH INTERVAL ALONG THE ELLIPSE.
C        X = WIDTH * COS(PHI); Y = LENGTH * SIN(PHI)
C
C      MIDPT(M) IS THE VALUE OF THIS PARAMETER AT THE MIDPOINT
C        OF THE (N-1)TH INTERVAL.
C
C      X AND Y ARE THE X AND Y VALUES OF THE POINT ON THE ELLIPSE
C        HAVING MIDPT(M) AS ITS PARAMETER VALUE.
C
C      THETA IS THE ANGLE THE NORMAL ON THE ELLIPSE MAKES WITH THE X
C        AXIS.  IT IS ALWAYS TAKEN TO BE IN THE INTERVAL (-PI/2,PI/2).
C        THIS IS NOT THE TRUE VALUE OF THETA IN ALL QUADRANTS, BUT
C        NO CORRECTION IS NEEDED AT THIS POINT.
C
C      WE SPLIT INTO TWO CASES FOR THE PARTITION: IF DIST=0, A
C        SEMIELLIPSE IS DONE.  OTHERWISE AN ELLIPSE IS DONE.
C
       IF (DIST.NE.0.0) GO TO 80
C
       PHI(1) = 0.0
       DO 50 N=2,INCREM
         PHI(N) = PI*(N-1)/INDEX
         MIDPT(N) = PHI(N) - PI/(2*INDEX)
50     CONTINUE
       GO TO 150
C
80     PHI(1) = -PI/2.0
       DO 100 N=2,INCREM
         PHI(N) = -PI/2.0 + 2*PI*(N-1)/INDEX
         MIDPT(N) = PHI(N) - PI/INDEX
100    CONTINUE
C
C
C      PERFORM NUMERICAL INTEGRATIONS TO DETERMINE INFLUENCE
C        COEFFICIENTS.  FOR EXAMPLE, XSIGY IS THE COEFFICIENT IN
C        THE Y DIRECTION DUE TO A POINT FORCE X.
C
C      I.E. XSIGY IS THE STRESS AT THE (M-1)TH INTERVAL IN THE
C        Y DIRECTION DUE TO A POINT FORCE AT THE NTH INTERVAL
C        IN THE X DIRECTION.
C      DUE TO SYMMETRY THE (M-1,N) COEFFICIENTS AND THE
C        (INDEX-M+2,INDEX-N+1) COEFFICIENTS ARE EQUAL, SO THEY
C        ARE EVALUATED SIMULTANEOUSLY.
C
C
150    INDNEW=INT(INDEX/2.0)
C
       DO 200 M=2,INCREM
        J=M+INDEX
        MM = INDEX -M +3
```

```
      X = WIDTH * DCOS(MIDPT(M))
      Y = LENGTH * DSIN(MIDPT(M)) + DIST
      S = DTAN(MIDPT(M))
C
      THETA = DATAN(S/T)
C
      C = MIDPT(M) + EPSIL
      D = MIDPT(M) - EPSIL
C
      DO 200 N=1,INDNEW
       K=N+INDEX
       L=N+2*INDEX
       NN = INDEX -N +1
       KK = NN + INDEX
       LL = NN + 2*INDEX
C
       IF (M-1.EQ.N) GO TO 175
C
       CALL ROMBRG( XSIGX,PHI(N),PHI(N+1),1.0,1.D-7,15,SI,SIM1,INDIC,I)
       IF (INDIC.EQ.1) GO TO 400
       FORCE(M-1,N) = SI- 1.0
       FORCE(MM-1,NN) = SI- 1.0
C
       CALL ROMBRG( XSIGY,PHI(N),PHI(N+1),1.0,1.D-7,15,SI,SIM1,INDIC,I)
       IF (INDIC.EQ.1) GO TO 400
       FORCE(M-1,K) = SI- 1.0
       FORCE(MM-1,KK) = SI- 1.0
C
       CALL ROMBRG( XTAU,PHI(N),PHI(N+1),1.0,1.D-7,15,SI,SIM1,INDIC,I)
       IF (INDIC.EQ.1) GO TO 400
       FORCE(M-1,L) = SI- 1.0
       FORCE(MM-1,LL) = 1.0 - SI
C
       CALL ROMBRG( YSIGX,PHI(N),PHI(N+1),1.0,1.D-7,15,SI,SIM1,INDIC,I)
       IF (INDIC.EQ.1) GO TO 400
       FORCE(J-1,N) = SI- 1.0
       FORCE(JJ-1,NN) = SI- 1.0
C
       CALL ROMBRG( YSIGY,PHI(N),PHI(N+1),1.0,1.D-7,15,SI,SIM1,INDIC,I)
       IF (INDIC.EQ.1) GO TO 400
       FORCE(J-1,K) = SI- 1.0
       FORCE(JJ-1,KK) = SI- 1.0
C
       CALL ROMBRG( YTAU,PHI(N),PHI(N+1),1.0,1.D-7,15,SI,SIM1,INDIC,I)
       IF (INDIC.EQ.1) GO TO 400
       FORCE(J-1,L) = SI- 1.0
       FORCE(JJ-1,LL) = 1.0 - SI
C
       GO TO 200
C
C
C      INTEGRATE OVER SINGULARITIES BY TAKING PRINCIPAL VALUES.
C      THE INTEGRAL HAS FOUR PARTS:  THE FIRST TWO ARE THE NORMAL
C        INTEGRALS OVER (PHI(N),D) AND (C,PHI(N+1)) WHICH AVOID
```

```
0003406CC       THE SINGULARITY.  THE THIRD TERM INVOLVES THETA AND IS
0003408CC       EXPLAINED IN THE LITERATURE.  THE FOURTH TERM IS THE
0003410CC       ERROR IN THE INTEGRAL DUE TO THE CHOICE OF EPSILON.
0003420CC
0003500C175    CALL ROMBRG( XSIGX,PHI(N),D,1.0,1.D-7,15,SI,SIM1,INDIC,I)
0003520C       IF (INDIC.EQ.1) GO TO 400
0003540C       FORCE(M-1,N) = SI -1.
0003560C       CALL ROMBRG( XSIGX,C,PHI(N+1),1.0,1.D-7,15,SI,SIM1,INDIC,I)
0003580C       IF (INDIC.EQ.1) GO TO 400
0003600C       FORCE(M-1,N)=FORCE(M-1,N)+SI-1.0
0003601C     2       - (5+4*DCOS(2*THETA)-DCOS(4*THETA))/16
0003620C     3       + EPSIL*T/(4*PI*(S**2+T**2)**3)*(6*S**6+(3-2*T**2)*S**4
0003640C     4       -6*S**2*T**2-T**4)
0003660C       FORCE(MM-1,NN)=FORCE(M-1,N)
0003720CC
0003740C       CALL ROMBRG( XSIGY,PHI(N),D,1.0,1.D-7,15,SI,SIM1,INDIC,I)
0003760C       IF (INDIC.EQ.1) GO TO 400
0003780C       FORCE(M-1,K) = SI -1.
0003820C       CALL ROMBRG( XSIGY,C,PHI(N+1),1.0,1.D-7,15,SI,SIM1,INDIC,I)
0003840C       IF (INDIC.EQ.1) GO TO 400
0003860C       FORCE(M-1,K)=FORCE(M-1,K)+SI-1.0
0003880C     2       +(1-DCOS(4*THETA))/16
0003890C     3       + EPSIL*T/(4*PI*(S**2+T**2)**3)*(-2*S**6-(1-6*T**2)*S**4
0003900C     4       +6*S**2*T**2-T**4)
0003905C        FORCE(MM-1,KK)=FORCE(M-1,K)
0003920CC
0003940C       CALL ROMBRG( XTAU,PHI(N),D,1.0,1.D-7,15,SI,SIM1,INDIC,I)
0003960C       IF (INDIC.EQ.1) GO TO 400
0003980C       FORCE(M-1,L) = SI -1.
0004020C       CALL ROMBRG( XTAU,C,PHI(N+1),1.0,1.D-7,15,SI,SIM1,INDIC,I)
0004040C       IF (INDIC.EQ.1) GO TO 400
0004060C       FORCE(M-1,L)=FORCE(M-1,L)+SI-1.0
0004080C     2       -(2*DSIN(2*THETA)-DSIN(4*THETA))/16
0004090C     3       + EPSIL*T**2/(4*PI*(S**2+T**2)**3)*(-9*S**5-(6+2*T**2)
0004100C     4       *S**3 + T**2*S*(2-T**2))
0004105C       FORCE(MM-1,LL)= -FORCE(M-1,L)
0004120CC
0004140C       CALL ROMBRG( YSIGX,PHI(N),D,1.0,1.D-7,15,SI,SIM1,INDIC,I)
0004160C       IF (INDIC.EQ.1) GO TO 400
0004180C       FORCE(J-1,N) = SI -1.
0004220C       CALL ROMBRG( YSIGX,C,PHI(N+1),1.0,1.D-7,15,SI,SIM1,INDIC,I)
0004240C       IF (INDIC.EQ.1) GO TO 400
0004260C       FORCE(J-1,N)=FORCE(J-1,N)+SI-1.0
0004280C     2       +(1-DCOS(4*THETA))/16
0004290C     3       + EPSIL*T/(4*PI*(S**2+T**2)**3)*(-S**6+(6-T**2)*S**2
0004300C     4       *T**2 + 6*S**4*T**2-2*T**4)
0004320C       FORCE(JJ-1,NN)=FORCE(J-1,N)
0004420CC
0004440C       CALL ROMBRG( YSIGY,PHI(N),D,1.0,1.D-7,15,SI,SIM1,INDIC,I)
0004460C       IF (INDIC.EQ.1) GO TO 400
0004480C       FORCE(J-1,K) = SI -1.
0004520C       CALL ROMBRG( YSIGY,C,PHI(N+1),1.0,1.D-7,15,SI,SIM1,INDIC,I)
0004540C       IF (INDIC.EQ.1) GO TO 400
0004560C       FORCE(J-1,K)=FORCE(J-1,K)+SI-1.0
```

```
      2         -(5-4*DCOS(2*THETA)-DCOS(4*THETA))/16
      3         + EPSIL*T/(4*PI*(S**2+T**2)**3)*(-S**6+(2-T**2)*S**2
      4         *T**2 - 6*S**4*T**2+6*T**4)
          FORCE( JJ-1,KK)=FORCE( J-1,K)
C
          CALL ROMBRG( YTAU,PHI(N),D,1.0,1.D-7,15,SI,SIM1,INDIC,I)
          IF (INDIC.EQ.1) GO TO 400
          FORCE(J-1,L) = SI -1.
          CALL ROMBRG( YTAU,C,PHI(N+1),1.0,1.D-7,15,SI,SIM1,INDIC,I)
          IF (INDIC.EQ.1) GO TO 400
          FORCE( J-1,L)=FORCE(J-1,L)+SI-1.0
      2         -(2*DSIN(2*THETA)+DSIN(4*THETA))/16
      3         - EPSIL/(4*PI*(S**2+T**2)**3)*((1-2*T**2)*S**5
      4         +(1+3*T**2)*2*T**2*S**3 + 9*S*T**4)
          FORCE(JJ-1,LL)= -FORCE(J-1,L)
C
C
  200   CONTINUE
C
C
C       INSERT INFLUENCE COEFFICIENTS AND STRESSES AT INFINITY
C         INTO EVAL MATRIX ACCORDING TO HOW THEY MUST APPEAR
C         IN THE SYSTEM OF LINEAR EQUATIONS.
C
C       THETA IS AGAIN THE ANGLE BETWEEN THE NORMAL TO THE ELLIPSE
C         AND THE X AXIS BUT IS IN THE INTERVAL (-PI/2,3*PI/2).
C         THIS IS THE CORRECT VALUE OF THETA.
C
        INCREM = 2*INDEX +1
C
        DO 250 M=1,INDEX
          J=M+INDEX
          THETA = DATAN(DTAN(MIDPT(M+1))/T)
          IF (MIDPT(M+1).LE.PI/2) GO TO 230
          IF (MIDPT(M+1).GT.3*PI/2) GO TO 230
C
          THETA = THETA + PI
C
  230     EVAL(M,INCREM) = -STRESS*DCOS(THETA)
          EVAL(J,INCREM) = 0.0
C
          DO 250 N=1,INDEX
            K = N+INDEX
            L = N+2*INDEX
            EVAL(M,N) = FORCE(M,N)*DCOS(THETA)
      2                 + FORCE(M,L)*DSIN(THETA)
C
            EVAL(J,N) = FORCE(M,K)*DSIN(THETA)
      2                 + FORCE(M,L)*DCOS(THETA)
C
            EVAL(M,K) = FORCE(J,N)*DCOS(THETA)
      2                 + FORCE(J,L)*DSIN(THETA)
C
            EVAL(J,K) = FORCE(J,K)*DSIN(THETA)
```

```
     2                 + FORCE(J,L)*DCOS(THETA)
C
  250 CONTINUE
C
C
C
C     CALL SOLVD SUBROUTINE TO SOLVE SYSTEM OF LINEAR EQUATIONS.
C     ON RETURN FROM SOLVD, THE PART OF THE MATRIX THAT BEFORE
C       CONTAINED THE APPLIED STRESSES WILL NOW CONTAIN THE
C       SOLUTIONS FOR RO-SUB-X AND RO-SUB-Y, THE DENSITIES OF
C       THE BODY FORCES.
C
      CALL SOLVD(EVAL,200,INCREM-1,INCREM,.5D-6,DET,TEST)
      IF (TEST.NE.0) GO TO 450
C
C
      WRITE(6,1055)
      IF (DIST.EQ.0.0) GO TO 340
      WRITE(6,1060)
      GO TO 345
  340 WRITE(6,1065)
  345 WRITE(6,1010)
C
      DO 350 M=1,INDEX
        J=M+INDEX
  350   WRITE(6,600) M,EVAL(M,INCREM),EVAL(J,INCREM)
C
C
C     PERFORM NUMERICAL INTEGRATIONS TO DETERMINE INFLUENCE
C       COEFFICIENTS.  FOR EXAMPLE, PSIGY IS THE COEFFICIENT IN
C       THE Y DIRECTION DUE TO A POINT FORCE P.
C
      INDNEW = INT(INDEX/2.0)
      X = 0.0
      IF (DIST.EQ.0.0) GO TO 360
      Y = DIST - LENGTH
      NEW = 1
      C = -PI/2.0 + EPSIL
      GO TO 365
  360 Y = LENGTH
      NEW = INDNEW
      D = PI/2.0 - EPSIL
C
  365 DO 380 N=1,INDNEW
C
        IF (N.EQ.NEW) GO TO 375
C
        CALL ROMBRG( XSIGX,PHI(N),PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
        IF (INDIC.EQ.1) GO TO 400
        FORCE(1,N) = SI- 1.0
C
        CALL ROMBRG( YSIGX,PHI(N),PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
        IF (INDIC.EQ.1) GO TO 400
        FORCE(2,N) = SI- 1.0
```

```
0007800CC
0007820CC
0007840C        GO TO 380
0007860CC
0007880CC
0007900CC       INTEGRATE OVER SINGULARITIES BY TAKING PRINCIPAL VALUES.
0007920CC
0007925C375    IF (DIST.EQ.0.0) GO TO 376
0007926C        CALL ROMBRG( XSIGX,C,PHI(N+1),1.0,1.D-7,15,SI,SIM1,INDIC,I)
0007928C        GO TO 377
0007940C376     CALL ROMBRG( XSIGX,PHI(N),D,1.0,1.D-7,15,SI,SIM1,INDIC,I)
0007960C377     IF (INDIC.EQ.1) GO TO 400
0008040C        FORCE(1,N)= SI-1.0 + 3*EPSIL*T/(2*PI)
0008060CC
0008080CC
0008100CC
0008102C       IF (DIST.EQ.0.0) GO TO 378
0008104C        CALL ROMBRG( YSIGX,C,PHI(N+1),1.0,1.D-7,15,SI,SIM1,INDIC,I)
0008106C        GO TO 379
0008120C378     CALL ROMBRG( YSIGX,PHI(N),D,1.0,1.D-7,15,SI,SIM1,INDIC,I)
0008140C379     IF (INDIC.EQ.1) GO TO 400
0008220C        FORCE(2,N) = SI-1.0 - EPSIL*T/(4*PI)
0008240CC
0008280CC
0008300C380   CONTINUE
0008320CC
0008340CC
0008400CC
0008420C      XSUM90 = STRESS
0008440CC
0008460C      DO 370 M=1,INDNEW
0008480C        J=M+INDEX
0008500C        XSUM90 = XSUM90 + 2.0 * EVAL(M,INCREM) * FORCE(1,M)
0008520C     2           + 2.0 * EVAL(J,INCREM) + FORCE(2,M)
0008560C370   CONTINUE
0008580CC
0008600CC     DETERMINE STRESS CONCENTRATION FACTOR BY DIVIDING THE EVALUATED
0008620CC       STRESS IN THE X DIRECTION AT 90 DEGREES BY THE STRESS
0008640CC       AT INFINITY.
0008660CC
0008680CC
0008700C      WRITE(6,1010)
0008720CC
0008740C      XSUM90 = XSUM90/STRESS
0008760C      WRITE(6,1085) XSUM90
0008780C      WRITE(6,1010)
0008800C      WRITE(6,1090) DET
0008820CC
0008840CC
0008860CC
0008880C500   FORMAT(I4,5(1X,F10.0))
0008885 740   FORMAT(8(1X,F6.3))
0008900C555   FORMAT(I4)
0008920C600   FORMAT(I4,2(1X,F12.6))
```

```
0008940C866     FORMAT(2(1X,F12.6))
0008960C800     FORMAT(1X,F12.6)
0008980C1000    FORMAT(1X,'SOLUTION TO ELLIPSE IN SEMIINFINITE PLATE')
0009000C1005    FORMAT(3X,'USING NISITANI BODY FORCE METHOD')
0009020C1010    FORMAT(1X)
0009040C1020    FORMAT(1X,'THE NUMBER OF DIVISIONS IN THE ELLIPSE IS ', I4)
0009060C1025    FORMAT(1X,'HALF THE WIDTH OF THE ELLIPSE IS ',F12.6)
0009080C1030    FORMAT(1X,'HALF THE HEIGHT OF THE ELLIPSE IS ',F12.6)
0009100C1035    FORMAT(1X,'THE DISTANCE FROM THE CENTRE OF ELLIPSE TO')
0009120C1037    FORMAT(3X,'THE EDGE OF THE PLATE IS ',F12.6)
0009140C1040    FORMAT(1X,'THE APPLIED STRESS AT INFINITY IS ',F12.6)
0009160C1045    FORMAT(1X,'THE VALUE OF EPSILON IN THE NUMERICAL')
0009180C1050    FORMAT(3X,'INTEGRATION IS ',F12.6)
0009200C1055    FORMAT(1X,'THE X AND Y BODY FORCE DENSITIES, STARTING')
0009220C1060    FORMAT(3X,'WITH THE BOTTOM OF THE ELLIPSE, ARE')
0009240C1065    FORMAT(3X,'WITH THE SIDE OF THE ELLIPSE, ARE')
0009260C1085    FORMAT(1X,'THE STRESS CONCENTRATION FACTOR IS ',2(F12.6,1X))
0009280C1090    FORMAT(1X,'THE DETERMINANT OF COEFFICIENT MATRIX IS ',E12.6)
0009300C1100    FORMAT(1X,'ERROR: NUMERICAL INTEGRATION DID NOT CONVERGE.')
0009320C1110    FORMAT(1X,'ERROR: COEFFICIENT MATRIX IS SINGULAR.')
0009340CC
0009360CC
0009380C        STOP
0009400CC
0009420CC
0009440C400     WRITE(6,1100)
0009460C        STOP
0009480CC
0009500CC
0009520C450     WRITE(6,1110)
0009540C        STOP
0009560CC
0009580C        END
```

8.2 TYPICAL INPUT

INDEX (MM) = 12

WIDTH (a) = 1.0

LENGTH (b) = 4.0

DIST (e) = 0.0

STRESS (σ_x^∞) = 1.0

FRAC = 0.01

FORMAT (I4, 5(1X, F10.0))

8.3 TYPICAL OUTPUT

SOLUTION TO ELLIPSE IN SEMIINFINITE PLATE
USING NISITANI BODY FORCE METHOD

THE NUMBER OF DIVISIONS IN THE ELLIPSE IS 12

HALF THE WIDTH OF THE ELLIPSE IS 1.000000

HALF THE HEIGHT OF THE ELLIPSE IS 4.000000

THE DISTANCE FROM THE CENTRE OF ELLIPSE TO
THE EDGE OF THE PLATE IS 0.000000

THE APPLIED STRESS AT INFINITY IS 1.000000

THE VALUE OF EPSILON IN THE NUMERICAL
INTEGRATION IS 0.002618

THE X AND Y BODY FORCE DENSITIES, STARTING
WITH THE SIDE OF THE ELLIPSE, ARE

1	10.479455	6.235629
2	10.021310	1.518940
3	9.752178	0.286842
4	9.599312	-0.247939
5	9.515675	-0.492201
6	9.478575	-0.589385
7	9.478575	-0.589385
8	9.515675	-0.492201
9	9.599312	-0.247939
10	9.752178	0.286842
11	10.021310	1.518940
12	10.479455	6.235629

THE STRESS CONCENTRATION FACTOR IS 9.499096

THE DETERMINANT OF COEFFICIENT MATRIX IS 0.139862E-16

9. PROGRAM FOR SEMI-ELLIPSE AND CRACK IN A SEMI-INFINITE PLATE (ELCRA I)

9.1 LISTING

```
C     PROGRAM TO COMPUTE SIF FOR CRACK IN A SEMIINFINITE PLATE
C       HAVING A SEMIELLIPTICAL NOTCH.
C
C     METHOD OF SOLUTION IS THE BODY FORCE METHOD (H. NISITANI).
C
C     NUMERICAL INTEGRATION IS USED INSTEAD OF ANALYTICAL
C       INTEGRATION TO DETERMINE INFLUENCE COEFFICIENTS.
C
C     INDEX1 IS THE NUMBER OF INTERVALS INTO WHICH THE SEMIELLIPSE
C       HAS BEEN PARTITIONED.
C     INDEX2 IS THE NUMBER OF INTERVALS INTO WHICH THE CRACK
C       HAS BEEN PARTITIONED.
C     WIDTH IS HALF THE WIDTH OF THE SEMIELLIPSE ALONG
C       THE X AXIS.
C     LENGTH IS THE HEIGHT OF THE SEMIELLIPSE.
C     CRAC IS HALF THE LENGTH OF THE CRACK.
C     HEIGHT IS THE DISTANCE FROM THE CENTRE OF THE CRACK TO THE EDGE
C       OF THE PLATE.
C     STRESS IS THE APPLIED STRESS AT INFINITY.
C     FRAC IS A NUMBER MUCH LESS THAN 1.0 USED IN THE
C       NUMERICAL INTEGRATION ABOUT A SINGULARITY.
C
      INTEGER INDEX1,INDEX2,INCREM,HALF1,HALF2
      REAL*8 PI,WIDTH,LENGTH,X,Y,STRESS,EPSIL,SI,SIM1,C,D,PREC,DET,TEST
      REAL*8 S,T,THETA,DIST,CRAC,A1,A2,A3,A4,A5,A6,A7,A8,A9
      REAL*8 FRAC,HEIGHT,SIF1,SIF2
      REAL*8 PHI(101),MIDPT(101),FORCE(200,300),EVAL(300,301)
      REAL*8 ETA(101),CENTRE(101),FUNC(100,101),FORCE2(100,300)
      COMMON PI,X,Y /ELLIP/ WIDTH,LENGTH,DIST /CRACK/ CRAC,HEIGHT
      EXTERNAL XSIGX,XSIGY,YSIGX,YSIGY,XTAU,YTAU,XCRAC,YCRAC,TAUCRA
C
C
      READ(5,600) INDEX1,INDEX2,WIDTH,LENGTH,CRAC,HEIGHT
      READ(5,650) STRESS,FRAC
C
      WRITE(6,1000)
      WRITE(6,1005)
      WRITE(6,1010)
      WRITE(6,1015) INDEX1
      WRITE(6,1010)
      WRITE(6,1020) INDEX2
      WRITE(6,1010)
      WRITE(6,1025) WIDTH
      WRITE(6,1010)
      WRITE(6,1030) LENGTH
      WRITE(6,1010)
      WRITE(6,1035) CRAC
      WRITE(6,1010)
      WRITE(6,1037) HEIGHT
      WRITE(6,1010)
      WRITE(6,1040) STRESS
      WRITE(6,1010)
C
C     DEFINE PI, EPSILON. EPSILON IS USED IN INTEGRATION
```

```
C        ABOUT SINGULARITIES ON THE SEMIELLIPSE.
C
       PI = 3.14159265359
       EPSIL = FRAC * PI/INDEX1
       WRITE(6,1045)
       WRITE(6,1050) EPSIL
       WRITE(6,1010)
C
       DIST = 0.0
       T = LENGTH/WIDTH
C
C      COMPUTE VALUE OF PHI(N) FOR N=1 TO INDEX1 + 1.
C      PHI(N) AND PHI(N+1) ARE PARAMETERS FOR THE VALUE OF
C        THE ENDPOINTS OF THE NTH INTERVAL ON THE ELLIPSE:
C        X = WIDTH * COS(PHI); Y = LENGTH * SIN(PHI)
C
C      COMPUTE ETA(N) FOR N=1 TO INDEX2 + 1.
C      ETA(N) AND ETA(N+1) ARE THE Y-VALUES OF THE ENDPOINTS OF
C        THE NTH INTERVAL ON THE CRACK.
C
C
C      MIDPT(N) IS THE VALUE OF PHI AT THE MIDPOINT
C        OF THE (N-1)TH INTERVAL ON THE ELLIPSE.
C
C      CENTRE(N) IS THE Y-VALUE OF THE MIDPOINT OF THE (N-1)TH
C        INTERVAL ON THE CRACK.
C
C      X AND Y ARE THE X AND Y VALUES OF THE POINT ON THE ELLIPSE
C        HAVING MIDPT(M) AS ITS PARAMETER VALUE.
C
C      THETA IS THE ANGLE THE NORMAL ON THE ELLIPSE MAKES WITH THE X
C        AXIS.  IT IS ALWAYS TAKEN TO BE IN THE INTERVAL (-PI/2,PI/2).
C        THIS IS NOT THE TRUE VALUE OF THETA IN ALL QUADRANTS, BUT
C        NO CORRECTION IS NEEDED AT THIS POINT.
C
C
       INCREM=INDEX1+1
       PHI(1) = 0.0
C
       DO 100 N=2,INCREM
         PHI(N) = PI*(N-1)/INDEX1
         MIDPT(N) = PHI(N) - PI/(2*INDEX1)
100    CONTINUE
C
C
       INCREM=INDEX2+1
       ETA(1) = HEIGHT - CRAC
C
C
       DO 150 N=2,INCREM
         ETA(N) = ETA(1) + 2.0*CRAC*(N-1)/INDEX2
         CENTRE(N) = ETA(N) - CRAC/INDEX2
150    CONTINUE
C
```

```
C
C     PERFORM NUMERICAL INTEGRATIONS TO DETERMINE INFLUENCE
C       COEFFICIENTS ON THE BOUNDARY OF THE SEMIELLIPSE DUE TO
        POINT FORCES IN THE X AND Y DIRECTION ON THE ELLIPSE.
        FOR EXAMPLE, XSIGY IS THE FORCE IN THE Y DIRECTION
C       DUE TO A POINT FORCE X IN THE X DIRECTION.
C
      INCREM=INDEX1+1
      INDNEW=INT(INDEX1/2.0)
C
      DO 200 M=2,INCREM
       J=M+INDEX1
       MM = INDEX1 -M +3
       JJ = MM + INDEX1
       X = WIDTH * DCOS(MIDPT(M))
       Y = LENGTH * DSIN(MIDPT(M))
       S = DTAN(MIDPT(M))
C
       THETA = DATAN(S/T)
C
       C = MIDPT(M) + EPSIL
       D = MIDPT(M) - EPSIL
C
       DO 200 N=1,INDNEW
        K=N+INDEX1
        L=N+2*INDEX1
        NN = INDEX1 -N +1
        KK = NN + INDEX1
        LL = NN + 2*INDEX1
C
        IF (M-1.EQ.N) GO TO 175
C
        CALL ROMBRG( XSIGX,PHI(N),PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
        IF (INDIC.EQ.1) GO TO 400
        FORCE(M-1,N) = SI- 1.0
        FORCE(MM-1,NN) = SI- 1.0
C
        CALL ROMBRG( XSIGY,PHI(N),PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
        IF (INDIC.EQ.1) GO TO 400
        FORCE(M-1,K) = SI- 1.0
        FORCE(MM-1,KK) = SI- 1.0
C
        CALL ROMBRG( XTAU,PHI(N),PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
        IF (INDIC.EQ.1) GO TO 400
        FORCE(M-1,L) = SI- 1.0
        FORCE(MM-1,LL) = 1.0 - SI
C
        CALL ROMBRG( YSIGX,PHI(N),PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
        IF (INDIC.EQ.1) GO TO 400
        FORCE(J-1,N) = SI- 1.0
        FORCE(JJ-1,NN) = SI- 1.0
C
        CALL ROMBRG( YSIGY,PHI(N),PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
        IF (INDIC.EQ.1) GO TO 400
```

```
      FORCE(J-1,K) = SI- 1.0
      FORCE(JJ-1,KK) = SI- 1.0
C
      CALL ROMBRG( YTAU,PHI(N),PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
      IF (INDIC.EQ.1) GO TO 400
      FORCE(J-1,L) = SI- 1.0
      FORCE(JJ-1,LL) = 1.0 - SI
C
C
      GO TO 200
C
C
C     INTEGRATE OVER SINGULARITIES BY TAKING PRINCIPAL VALUES.
C     THE INTEGRAL HAS FOUR PARTS:  THE FIRST TWO ARE THE NORMAL
C       INTEGRALS OVER (PHI(N),D) AND (C,PHI(N+1)) WHICH AVOID
C       THE SINGULARITY.  THE THIRD TERM INVOLVES THETA AND IS
C       EXPLAINED IN THE LITERATURE.  THE FOURTH TERM IS THE
C       ERROR IN THE INTEGRAL DUE TO THE CHOICE OF EPSILON.
C
  175 CALL ROMBRG( XSIGX,PHI(N),D,1.0,1.D-7,10,SI,SIM1,INDIC,I)
      IF (INDIC.EQ.1) GO TO 400
      FORCE(M-1,N) = SI -1.
      CALL ROMBRG( XSIGX,C,PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
      IF (INDIC.EQ.1) GO TO 400
      FORCE(M-1,N)=FORCE(M-1,N)+SI-1.0
     2      - (5+4*DCOS(2*THETA)-DCOS(4*THETA))/16
     3      + EPSIL*T/(4*PI*(S**2+T**2)**3)*(6*S**6+(3-2*T**2)*S**4
     4      -6*S**2*T**2-T**4)
      FORCE(MM-1,NN)=FORCE(M-1,N)
C
      CALL ROMBRG( XSIGY,PHI(N),D,1.0,1.D-7,10,SI,SIM1,INDIC,I)
      IF (INDIC.EQ.1) GO TO 400
      FORCE(M-1,K) = SI -1.
      CALL ROMBRG( XSIGY,C,PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
      IF (INDIC.EQ.1) GO TO 400
      FORCE(M-1,K)=FORCE(M-1,K)+SI-1.0
     2      +(1-DCOS(4*THETA))/16
     3      + EPSIL*T/(4*PI*(S**2+T**2)**3)*(-2*S**6-(1-6*T**2)*S**4
     4      +6*S**2*T**2-T**4)
       FORCE(MM-1,KK)=FORCE(M-1,K)
C
      CALL ROMBRG( XTAU,PHI(N),D,1.0,1.D-7,10,SI,SIM1,INDIC,I)
      IF (INDIC.EQ.1) GO TO 400
      FORCE(M-1,L) = SI -1.
      CALL ROMBRG( XTAU,C,PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
      IF (INDIC.EQ.1) GO TO 400
      FORCE(M-1,L)=FORCE(M-1,L)+SI-1.0
     2      -(2*DSIN(2*THETA)-DSIN(4*THETA))/16
     3      + EPSIL*T**2/(4*PI*(S**2+T**2)**3)*(-9*S**5-(6+2*T**2)
     4      *S**3 + T**2*S*(2-T**2))
      FORCE(MM-1,LL)= -FORCE(M-1,L)
C
      CALL ROMBRG( YSIGX,PHI(N),D,1.0,1.D-7,10,SI,SIM1,INDIC,I)
      IF (INDIC.EQ.1) GO TO 400
```

```
      FORCE(J-1,N) = SI -1.
      CALL ROMBRG( YSIGX,C,PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
      IF (INDIC.EQ.1) GO TO 400
      FORCE(J-1,N)=FORCE(J-1,N)+SI-1.0
     2      +(1-DCOS(4*THETA))/16
     3      + EPSIL*T/(4*PI*(S**2+T**2)**3)*(-S**6+(6-T**2)*S**2
     4      *T**2 + 6*S**4*T**2-2*T**4)
      FORCE(JJ-1,NN)=FORCE(J-1,N)
C
      CALL ROMBRG( YSIGY,PHI(N),D,1.0,1.D-7,10,SI,SIM1,INDIC,I)
      IF (INDIC.EQ.1) GO TO 400
      FORCE(J-1,K) = SI -1.
      CALL ROMBRG( YSIGY,C,PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
      IF (INDIC.EQ.1) GO TO 400
      FORCE(J-1,K)=FORCE(J-1,K)+SI-1.0
     2      -(5-4*DCOS(2*THETA)-DCOS(4*THETA))/16
     3      + EPSIL*T/(4*PI*(S**2+T**2)**3)*(-S**6+(2-T**2)*S**2
     4      *T**2 - 6*S**4*T**2+6*T**4)
      FORCE(JJ-1,KK)=FORCE(J-1,K)
C
      CALL ROMBRG( YTAU,PHI(N),D,1.0,1.D-7,10,SI,SIM1,INDIC,I)
      IF (INDIC.EQ.1) GO TO 400
      FORCE(J-1,L) = SI -1.
      CALL ROMBRG( YTAU,C,PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
      IF (INDIC.EQ.1) GO TO 400
      FORCE(J-1,L)=FORCE(J-1,L)+SI-1.0
     2      -(2*DSIN(2*THETA)+DSIN(4*THETA))/16
     3      - EPSIL/(4*PI*(S**2+T**2)**3)*((1-2*T**2)*S**5
     4      +(1+3*T**2)*2*T**2*S**3 + 9*S*T**4)
      FORCE(JJ-1,LL)= -FORCE(J-1,L)
C
C200  CONTINUE
C
C
C
C
C     EVALUATE THE STRESSES ON THE ELLIPTIC INTERVALS DUE TO
C       POINT FORCES ON THE CRACK INTERVALS.
C
C
      INDNEW = INT(INDEX1/2.0) + 1
C
      DO 210 M=2,INDNEW
        MM = INDEX1 -M +3
        X = WIDTH * DCOS(MIDPT(M))
        Y = LENGTH * DSIN(MIDPT(M))
        DO 210 N=1,INDEX2
          K = N+INDEX2
          L = N+2*INDEX2
C
        CALL ROMBRG(XCRAC,ETA(N),ETA(N+1),1.0,1.D-7,20,SI,SIM1,INDIC,I)
        IF (INDIC.EQ.1) GO TO 400
        FORCE2(M-1,N) = SI- 1.0
        FORCE2(MM-1,N) = SI- 1.0
C
```

```
         CALL ROMBRG(YCRAC,ETA(N),ETA(N+1),1.0,1.D-7,20,SI,SIM1,INDIC,I)
         IF (INDIC.EQ.1) GO TO 400
         FORCE2(M-1,K) = SI- 1.0
         FORCE2(MM-1,K) = SI- 1.0
C
         CALL ROMBRG(TAUCRA,ETA(N),ETA(N+1),1.0,1.D-7,20,SI,SIM1,INDIC,I)
         IF (INDIC.EQ.1) GO TO 400
         FORCE2(M-1,L) = SI- 1.0
         FORCE2(MM-1,L) = 1.0 - SI
C
  210 CONTINUE
C
C
C
C     INSERT INFLUENCE COEFFICIENTS AND STRESSES AT INFINITY
C       INTO EVAL MATRIX ACCORDING TO HOW THEY MUST APPEAR
C       IN THE SYSTEM OF LINEAR EQUATIONS.
C
C     THETA IS AGAIN THE ANGLE BETWEEN THE NORMAL TO THE ELLIPSE
C       AND THE X AXIS BUT IS IN THE INTERVAL (-PI/2,PI/2).
C       THIS IS THE CORRECT VALUE OF THETA.
C
      INCREM = 2*INDEX1 + INDEX2 +1
C
      DO 250 M=1,INDEX1
         J=M+INDEX1
         THETA = DATAN(DTAN(MIDPT(M+1))/T)
         IF (MIDPT(M+1).LE.PI/2) GO TO 230
         IF (MIDPT(M+1).GT.3*PI/2) GO TO 230
C
         THETA = THETA + PI
C
  230    EVAL(M,INCREM) = -STRESS*DCOS(THETA)
         EVAL(J,INCREM) = 0.0
C
         DO 240 N=1,INDEX1
            K = N+INDEX1
            L = N+2*INDEX1
            EVAL(M,N) = FORCE(M,N)*DCOS(THETA)
     2                + FORCE(M,L)*DSIN(THETA)
C
            EVAL(J,N) = FORCE(M,K)*DSIN(THETA)
     2                + FORCE(M,L)*DCOS(THETA)
C
            EVAL(M,K) = FORCE(J,N)*DCOS(THETA)
     2                + FORCE(J,L)*DSIN(THETA)
C
            EVAL(J,K) = FORCE(J,K)*DSIN(THETA)
     2                + FORCE(J,L)*DCOS(THETA)
C
  240    CONTINUE
         DO 250 N=1,INDEX2
           K = N + INDEX2
           L = N + 2*INDEX2
```

```
            I = N + 2*INDEX1
            EVAL(M,I) = FORCE2(M,N)*DCOS(THETA)
     2                + FORCE2(M,L)*DSIN(THETA)
C
            EVAL(J,I) = FORCE2(M,K)*DSIN(THETA)
     2                + FORCE2(M,L)*DCOS(THETA)
C
  250   CONTINUE
C
C
C
        INCREM = INDEX2 + 1
C
C       CALCULATE FUNC(M,N). THIS IS THE ANALYTICALLY INTEGRATED
C         FUNCTION FOR THE EVALUATION OF THE CRACK INFLUENCE
C         COEFFICIENTS.
C       FUNC(M,N+1) - FUNC(M,N) WILL BE THE STRESS IN THE X
C         DIRECTION AT THE (M-1)TH CRACK INTERVAL DUE TO A POINT
C         FORCE X AT THE NTH CRACK INTERVAL.
C
C
        DO 300 M=2,INCREM
C
C         A1,A2,A3,A4,A5,A6,A7,A8,A9 ARE INTERMEDIARY FUNCTIONS.
C
C
          A1 = CENTRE(M) + HEIGHT
          A2 = CENTRE(M) - HEIGHT
          A3 = DSQRT(CRAC**2 - A2**2)
          A4 = A1**2 - CRAC**2
C
C
          DO 300 N=1,INCREM
            A5 = ETA(N) - HEIGHT
            A6 = ETA(N) + CENTRE(M)
            A7 = CENTRE(M) - ETA(N)
            A8 = DSQRT(CRAC**2 - A5**2)
            A9 = DARSIN((CRAC**2 + A1*A5)/(CRAC*A6))
C
C
            FUNC(M,N) = 1/ PI *(A8/A7 + A2/A3
     1       * DLOG(DABS((CRAC**2 - A2*A5 + A3*A8)/(CRAC*A7))))
     2       - 1/ PI *(-A8/A6 + A1/DSQRT(A4) * A9)
     3       + 6* CENTRE(M)/ PI * (-A8/A6**2 *(1- A1*A6/A4)
     4       + CRAC**2/A4**1.5 * A9 ) - 2*CENTRE(M)**2/ PI
     5       *(A8/A6**3 *(-2 + A1*A6/A4+(2*CRAC**2 + A1**2)*A6**2/A4**2)
     6       + 3*CRAC**2 *A1/A4**2.5 * A9)
C
  300   CONTINUE
C
C
C
C       THE STRESSES ON THE CRACK DUE TO THE CRACK ARE NOW INSERTED
C         IN EVAL MATRIX.  ALSO EVALUATED ARE THE STRESSES ON THE
```

```
C     CRACK IN THE X DIRECTION DUE TO POINT FORCES ON THE ELLIPSE
C     IN THE X AND Y DIRECTIONS.  THESE STRESSES ARE
C     XSIGX AND YSIGX WITH X=0 AND Y=CENTRE(M)
C
      INCREM = 2*INDEX1 + INDEX2 + 1
      INDNEW = INDEX1/2
      X = 0.0
C
C
      DO 500 M=1,INDEX2
        Y = CENTRE(M+1)
        J = M + 2*INDEX1
        EVAL(J,INCREM) = -STRESS
C
        DO 475 N=1,INDNEW
          K  = N + INDEX1
          NN = INDEX1 -N +1
          KK = NN + INDEX1
         CALL ROMBRG(XSIGX,PHI(N),PHI(N+1),1.0,1.D-7,15,SI,SIM1,INDIC,I)
          IF (INDIC.EQ.1) GO TO 400
          EVAL(J,N) = SI - 1.0
          EVAL(J,NN) = SI- 1.0
C
         CALL ROMBRG(YSIGX,PHI(N),PHI(N+1),1.0,1.D-7,15,SI,SIM1,INDIC,I)
          IF (INDIC.EQ.1) GO TO 400
          EVAL(J,K) = SI - 1.0
          EVAL(J,KK) = SI- 1.0
  475   CONTINUE
C
        DO 500 N=1,INDEX2
          L = N + 2*INDEX1
          EVAL(J,L) = FUNC(M+1,N+1) - FUNC(M+1,N)
  500 CONTINUE
C
C
C
C
C
C     CALL SOLVD SUBROUTINE TO SOLVE SYSTEM OF LINEAR EQUATIONS.
C     ON RETURN FROM SOLVD, THE PART OF THE MATRIX THAT BEFORE
C       CONTAINED THE APPLIED STRESSES WILL NOW CONTAIN THE
C       SOLUTIONS FOR RO-SUB-X, RO-SUB-Y (THE DENSITIES OF THE
C       BODY FORCES ALONG THE ELLIPSE), AND THE CRACK CORRECTION
C       FACTORS.
C
C
      CALL SOLVD(EVAL,300,INCREM-1,INCREM,.5D-6,DET,TEST)
      IF (TEST.NE.0) GO TO 450
C
C
      WRITE(6,1055)
      WRITE(6,1065)
```

```
      DO 530 M=1,INDEX1
        J=M+INDEX1
530     WRITE(6,700) M,EVAL(M,INCREM),EVAL(J,INCREM)
C
      WRITE(6,1010)
      WRITE(6,1070)
      WRITE(6,1075)
C
      DO 550 M=1,INDEX2
        K = M + 2*INDEX1
        EVAL(K,INCREM) = EVAL(K,INCREM)/STRESS
        WRITE(6,750) M,EVAL(K,INCREM)
550   CONTINUE
C
C
C     CALCULATE THE STRESS INTENSITY FACTORS SIF1,SIF2 AT BOTH
C        TIPS OF THE CRACK.
C
      II=2*INDEX1
C
      SIF1 = EVAL(II+1,INCREM) * DSQRT(PI*CRAC) * STRESS
      SIF2 = EVAL(INCREM-1,INCREM) * DSQRT(PI*CRAC) * STRESS
C
C
      WRITE(6,1010)
      WRITE(6,1080)
      WRITE(6,1085) SIF1
      WRITE(6,1080)
      WRITE(6,1088) SIF2
      WRITE(6,1010)
      WRITE(6,1090) DET
C
C
C
600   FORMAT(I4,1X,I4,4(1X,F10.0))
650   FORMAT(F10.0,1X,F10.0)
700   FORMAT(I4,2(1X,F12.6))
740   FORMAT(8(1X,F6.3))
750   FORMAT(I4,1X,F12.6)
800   FORMAT(1X,E24.18)
1000  FORMAT(1X,'SOLUTION TO CRACK IN SEMIINFINITE PLATE HAVING A ')
1005  FORMAT(3X,'SEMIELLIPTICAL NOTCH USING NISITANI BODY FORCE METHOD')
1010  FORMAT(1X)
1015  FORMAT(1X,'THE NUMBER OF DIVISIONS IN THE SEMIELLIPSE IS ',I4)
1020  FORMAT(1X,'THE NUMBER OF DIVISIONS IN THE CRACK IS ',I4)
1025  FORMAT(1X,'HALF THE WIDTH OF THE SEMIELLIPSE IS ',F12.6)
1030  FORMAT(1X,'THE HEIGHT OF THE SEMIELLIPSE IS ',F12.6)
1035  FORMAT(1X,'THE LENGTH OF THE CRACK IS ',F12.6)
1037  FORMAT(1X,'THE DISTANCE FROM THE EDGE TO CRACK CENTRE IS ',F12.6)
1040  FORMAT(1X,'THE APPLIED STRESS AT INFINITY IS ',F12.6)
1045  FORMAT(1X,'THE VALUE OF EPSILON IN THE NUMERICAL')
1050  FORMAT(3X,'INTEGRATION IS ',F12.6)
1055  FORMAT(1X,'THE X AND Y BODY FORCE DENSITIES, STARTING')
1065  FORMAT(3X,'WITH THE SIDE OF THE ELLIPSE, ARE')
```

```
 1070 FORMAT(1X,'THE CORRECTION FACTORS FOR EACH CRACK INTERVAL,')
 1075 FORMAT(3X,'STARTING WITH THE ONE NEAREST THE ELLIPSE, ARE')
 1080 FORMAT(1X,'THE STRESS INTENSITY FACTOR AT THE TIP OF')
 1085 FORMAT(3X,'THE CRACK NEAREST THE EDGE IS ',F12.6)
 1088 FORMAT(3X,'THE CRACK FURTHEST FROM THE EDGE IS ',F12.6)
 1090 FORMAT(1X,'THE DETERMINANT OF COEFFICIENT MATRIX IS ',E12.6)
 1100 FORMAT(1X,'ERROR: NUMERICAL INTEGRATION DID NOT CONVERGE.')
 1110 FORMAT(1X,'ERROR: COEFFICIENT MATRIX IS SINGULAR.')
C
C
      STOP
C
C
  400 WRITE(6,1100)
      STOP
C
C
  450 WRITE(6,1110)
      STOP
C
      END
```

9.2 TYPICAL INPUT

INDEX 1 (M1) = 12

INDEX 2 (M2) = 12

WIDTH (a) = 1000.0*

LENGTH (b) = 1000.0*

CRAC (c) = 100.0*

HEIGHT (e) = 1500.0*

STRESS (σ_x^∞) = 1.0

FRAC = 0.05

FORMAT (I4, 1X, I4, 4(1X, F10.0)

FORMAT (F10.0, 1X, F10.0)

* Program stops due to roundoff errors if smaller values of a,b,c and e are chosen.

9.3 TYPICAL OUTPUT

SOLUTION TO CRACK IN SEMIINFINITE PLATE HAVING A
SEMIELLIPTICAL NOTCH USING NISITANI BODY FORCE METHOD

THE NUMBER OF DIVISIONS IN THE SEMIELLIPSE IS 12

THE NUMBER OF DIVISIONS IN THE CRACK IS 12

HALF THE WIDTH OF THE SEMIELLIPSE IS 1000.000000

THE HEIGHT OF THE SEMIELLIPSE IS 1000.000000

THE LENGTH OF THE CRACK IS 100.000000

THE DISTANCE FROM THE EDGE TO CRACK CENTRE IS 1500.000000

THE APPLIED STRESS AT INFINITY IS 1.000000

THE VALUE OF EPSILON IN THE NUMERICAL
INTEGRATION IS 0.013090

THE X AND Y BODY FORCE DENSITIES, STARTING
WITH THE SIDE OF THE ELLIPSE, ARE

1	3.036725	1.454622
2	3.172604	0.424099
3	3.265192	0.089852
4	3.335564	-0.079543
5	3.368478	-0.178077
6	3.175544	-0.148367
7	3.175544	-0.148367
8	3.368478	-0.178077
9	3.335564	-0.079543
10	3.265192	0.089852
11	3.172604	0.424099
12	3.036725	1.454622

THE CORRECTION FACTORS FOR EACH CRACK INTERVAL,
STARTING WITH THE ONE NEAREST THE ELLIPSE, ARE

1	1.637645
2	1.625330
3	1.613797
4	1.602758
5	1.592126
6	1.581854
7	1.571910
8	1.562267
9	1.552898
10	1.543769
11	1.534818
12	1.525850

THE STRESS INTENSITY FACTOR AT THE TIP OF
THE CRACK NEAREST THE EDGE IS 29.026504
THE STRESS INTENSITY FACTOR AT THE TIP OF

```
  THE CRACK FURTHEST FROM THE EDGE IS     27.044994
THE DETERMINANT OF COEFFICIENT MATRIX IS 0.597485E-05
```

10. REFERENCES

1. Nisitani, H., 'Solutions of Notch Problems by Body Force Method', Mechanics of Fracture Vol. 5, ed. G.C. Sih, Sijthoff and Noordhoff, The Netherlands, 1978.

2. Fraga, W.E. and Hewitt, R.L., 'Implementation of Nisitani's Body Force Method for the Solution of Notch Problems', Aeronautical Note AN-17, National Research Council of Canada, October 1983.