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### Computer programs for solving notch problems using Nisitani's body force method

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LABORATORY TECHNICAL REPORT

LTR - ST - 1479

COMPUTER PROGRAMS FOR SOLVING NOTCH PROBLEMS  
USING NISITANI'S BODY FORCE METHOD

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RAPPORT TECHNIQUE DE LABORATOIRE

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USING NISITANI'S BODY FORCE METHOD

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## 1. INTRODUCTION

A numerical method for the determination of stress intensity factors and stress concentration factors for cracks and notches has been presented by Nisitani (1). The method, known as the body force method, approximates a plate with a hole (or crack) by a plate without a hole but having body forces distributed along the boundary of the hole.

This method has been implemented on the NRC-IBM 360 system and has been described in a previous report (2). This report describes and lists the computer programs that were used to determine the stress intensity factors or stress concentration factors for the five types of notch and crack problems described in that report.

## 2. DEFINITIONS OF MAIN VARIABLES

- DIST -  $e$  - the distance from the edge of the semi-infinite plate to the centre of the crack or ellipse.
- EPSIL -  $\epsilon$  - the numerical value calculated from FRAC used to define the endpoints of the integral over the singularity.
- ETA -  $\eta$  - the  $y$  values of the endpoints of the crack intervals.
- EVAL  
FORCE  
FORCE 2 - matrices containing influence coefficients or linear combinations of influence coefficients
- FRAC - the fraction of an interval which is to be integrated analytically over a singularity.
- INDEX - MM - the number of intervals to be used.
- LENGTH -  $a$  - half the length of an ellipse along the  $y$ -axis.
- MIDPT - the value of  $y$  for the midpoint of a crack interval, or the value of  $\phi$  for the midpoint of an elliptic interval.
- PHI -  $\phi$  - the value of the elliptic parameter at the endpoints of the elliptic intervals.
- STRESS -  $\sigma_x^\infty$  - the applied stress at infinity.
- THETA -  $\theta$  - the angle that the normal to the ellipse makes with the  $x$ -axis.
- WIDTH -  $b$  - half the width of the ellipse along the  $x$ -axis or the half length of a crack.
- X,Y -  $x, y$  - the  $x$  and  $y$  values of the midpoint of the crack or elliptic interval where stresses are being evaluated.

### 3. CALLING HIERARCHY AND FUNCTION SUBROUTINES

In the calculation of the influence coefficients, most programs use function subroutines to perform numerical integrations using a library routine ROMBRG. The calling hierarchy for these programs is given below.

Program Name	Description	Subroutines Called	
BODY 1	crack in semi-infinite plate (analytic integration)	None	
BODY 5	crack in semi-infinite plate (numerical integration)	FUNC	$(\gamma_{xM}^{XN})$
BODY 9	edge crack in semi-infinite plate	FUNC 2	$(\gamma_{xM}^{XN})$
ELL 5	ellipse in infinite plate	PSIGX	$(\sigma_{xM}^{XN})$
		PSIGY	$(\sigma_{yM}^{XN})$
		PTAU	$(\tau_{xyM}^{XN})$
		QSIGX	$(\sigma_{xM}^{YN})$
		QSIGY	$(\sigma_{yM}^{YN})$
		QTAU	$(\tau_{xyM}^{YN})$
ELL8	ellipse or semi-ellipse in semi-infinite plate	XSIGX	$(\sigma_{xM}^{XN})$
		XSIGY	$(\sigma_{yM}^{XN})$
		XTAU	$(\tau_{xyM}^{XN})$
		YSIGX	$(\sigma_{xM}^{YN})$
		YSIGY	$(\sigma_{yM}^{YN})$
		YTAU	$(\tau_{xyM}^{YN})$
ELCRA 1	semi-ellipse and crack in semi-infinite plate	XSIGX	$(\sigma_{xM}^{XN})$
		XSIGY	$(\sigma_{yM}^{XN})$
		XTAU	$(\tau_{xyM}^{XN})$

YSIGX	$(\sigma_{xM}^{YN})$
YSIGY	$(\sigma_{yM}^{YN})$
YTAU	$(\tau_{xyM}^{YN})$
XCRAC	$(\sigma_{xMe}^{YN})$
YCRAC	$(\sigma_{yMe}^{YN})$
TAUCRA	$(\tau_{xyMe}^{YN})$

### 3.1 LISTINGS OF FUNCTION SUBROUTINES

200 = S

```
0005920CC
0006000CC      THIS FUNCTION CALCULATES THE INTEGRAND FOR NUMERICAL INTEGRATION
0006020CC      FOR CRACK IN SEMIINFINITE PLATE.
0006040CC      IT IS CALLED BY THE MAIN PROGRAM: SOURCE.BODY5
0006100CC
0006200C      REAL FUNCTION FUNC*8 (X)
0006300C      REAL*8 X
0006310C      REAL*8 MIDPT(101),DIST,WIDTH,PI
0006340C      COMMON MIDPT,DIST,WIDTH,PI,M
0006360CC
0006400C      FUNC = 1/PI *(1/(MIDPT(M) - X)**2 - 1/(MIDPT(M) + X)**2
0006500C      2 + 12*MIDPT(M)/(MIDPT(M)+X)**3 - 12*MIDPT(M)**2/(MIDPT(M)+X)**4)
0006600C      3 * SQRT(WIDTH**2 - (X - DIST)**2)
0006610CC
0006630C      RETURN
0006800C      END
```

```
0006920CC
0006000CC      THIS FUNCTION CALCULATES THE INTEGRAND FOR NUMERICAL INTEGRATION
0006020CC      FOR CRACK AT EDGE OF SEMIINFINITE PLATE.
0006040CC
0006060CC      IT IS CALLED BY THE MAIN PROGRAM: SOURCE.BODY9
0006100CC
0006200C      REAL FUNCTION FUNC2*8 (X)
0006300C          REAL*8 X
0006310C          REAL*8 MIDPT(101),WIDTH,PI
0006340C          COMMON MIDPT,WIDTH,PI,M
0006360CC
0006400C          FUNC2 = 1/PI *(1/(MIDPT(M) - X)**2 - 1/(MIDPT(M) + X)**2
0006500C          2 + 12*MIDPT(M)/(MIDPT(M)+X)**3 - 12*MIDPT(M)**2/(MIDPT(M)+X)**4)
0006602C          3 * SQRT(WIDTH**2 - X**2)
0006610CC
0006630C          RETURN
0006800C          END
```

```

0005920CC
0006000CC      THIS FUNCTION CALCULATES THE X STRESS AT (X,Y) DUE TO A POINT
0006020CC      FORCE P IN THE X DIRECTION AT (A*COS(Z),B*SIN(Z)) IN AN
0006100CC      INFINITE PLATE.
0006120CC
0006200C      REAL FUNCTION PSIGX*8 (Z)
0006300C      REAL*8 Z
0006310C      REAL*8 PI,A,B,X,Y
0006340C      COMMON PI,A,B,X,Y
0006350CC
0006360C      PSIGX =-1/(4*PI)*(X-A*DCOS(Z))*(3*(X-A*DCOS(Z))**2+(Y-B*DSIN(Z))
0006370C      2 **2)/((X-A*DCOS(Z))**2+(Y-B*DSIN(Z))**2)**2 *B*DCOS(Z)
0006610CC
0006630C      RETURN
0006800C      END

```

```
0005920CC
0006000CC      THIS FUNCTION CALCULATES THE Y STRESS AT (X,Y) DUE TO A POINT
0006020CC      FORCE P IN THE X DIRECTION AT (A*DCOS(Z),B*SIN(Z)) IN AN
0006100CC      INFINITE PLATE.
0006120CC
0006200C      REAL FUNCTION PSIGY*8 (Z)
0006300C      REAL*8 Z
0006310C      REAL*8 PI,A,B,X,Y
0006340C      COMMON PI,A,B,X,Y
0006350CC
0006360C      PSIGY =1.0/(4*PI)*((X-A*DCOS(Z))*((X-A*DCOS(Z))**2-(Y-B*DSIN(Z))
0006370C      2 **2)/((X-A*DCOS(Z))**2+(Y-B*DSIN(Z))**2)**2 *B*DCOS(Z)
0006610CC
0006630C      RETURN
0006800C      END
```

ELF 4 3

```
0005920CC
0006000CC      THIS FUNCTION CALCULATES THE SHEAR STRESS AT (X,Y) DUE TO A
0006020CC      POINT FORCE P IN THE X DIRECTION AT (A*COS(Z),B*SIN(Z)) IN
0006100CC      AN INFINITE PLATE.
0006120CC
0006200C      REAL FUNCTION PTAU*8 (Z)
0006300C      REAL*8 Z
0006310C      REAL*8 PI,A,B,X,Y
0006340C      COMMON PI,A,B,X,Y
0006350CC
0006360C      PTAU =-1.0/(4*PI)*((Y-B*D SIN(Z))*3*(X-A*D COS(Z))**2+(Y-B*D SIN(Z))
0006370C      2 **2)/((X-A*D COS(Z))**2+(Y-B*D SIN(Z))**2)**2 *B*D COS(Z)
0006610CC
0006630C      RETURN
0006800C      END
```

FILE # 3

```
0005920CC
0006000CC      THIS FUNCTION CALCULATES THE SHEAR STRESS AT (X,Y) DUE TO A
0006020CC      POINT FORCE P IN THE X DIRECTION AT (A*DCOS(Z),B*DSIN(Z)) IN
0006100CC      AN INFINITE PLATE.
0006120CC
0006200C      REAL FUNCTION PTAU*8 (Z)
0006300C      REAL*8 Z
0006310C      REAL*8 PI,A,B,X,Y
0006340C      COMMON PI,A,B,X,Y
0006350CC
0006360C      PTAU=-1.0/(4*PI)*(Y-B*DSIN(Z))*(3*(X-A*DCOS(Z))**2+(Y-B*DSIN(Z))
0006370C      2 **2)/((X-A*DCOS(Z))**2+(Y-B*DSIN(Z))**2)**2 *B*DCOS(Z)
0006610CC
0006630C      RETURN
0006800C      END
```

```

00060200C
00060000C
00060200C THIS FUNCTION CALCULATES THE X STRESS AT (X,Y) DUE TO A POINT
00061000C FORCE Q IN THE Y DIRECTION AT (A*COS(Z),B*SIN(Z)) IN AN
00061200C INFINITE PLATE.
0006200C
0006300C REAL FUNCTION QSIGX*8 (Z)
0006300C REAL*8 Z
0006310C REAL*8 P1,A,B,X,Y
0006340C COMMON P1,A,B,X,Y
0006350C
0006360C QSIGX =-1.0/(4*PI)*{(Y-B*DSIN(Z))*{(X-A*DCOS(Z))**2-(Y-B*DSIN(Z))
0006370C 2 **2)/( (X-A*DCOS(Z))**2+(Y-B*DSIN(Z))**2)**2 *A*DSIN(Z)
00066100C
0006630C RETURN
0006800C END

```

E11#1

```
0005920CC
0006000CC      THIS FUNCTION CALCULATES THE Y STRESS AT (X,Y) DUE TO A POINT
0006020CC      FORCE Q IN THE Y DIRECTION AT (A*COS(Z),B*SIN(Z)) IN AN
0006100CC      INFINITE PLATE.
0006120CC
0006200C      REAL FUNCTION QSIGY*8 (Z)
0006300C      REAL*8 Z
0006310C      REAL*8 PI,A,B,X,Y
0006340C      COMMON PI,A,B,X,Y
0006350CC
0006360C      QSIGY =-1./(4*PI)*{(Y-B*DSIN(Z))*((X-A*DCOS(Z))**2+3*(Y-B*DSIN(Z))
0006370C      2 **2)/((X-A*DCOS(Z))**2+(Y-B*DSIN(Z))**2)**2 *A*DSIN(Z)
0006610CC
0006630C      RETURN
0006800C      END
```

0006920CC  
0006000CC  
0006020CC  
0006100CC  
0006120CC  
0006200C  
0006300C  
0006310C  
0006340C  
0006350CC  
0006360C  
0006370C  
0006610CC  
0006630C  
00066600C

THIS FUNCTION CALCULATES THE SHEAR STRESS AT (X,Y) DUE TO A  
POINT FORCE Q IN THE Y DIRECTION AT (A#COS(Z),B#SIN(Z)) IN  
AN INFINITE PLATE.

REAL FUNCTION QTAU#8 (Z)

REAL#8 Z  
REAL#8 P1,A,B,X,Y  
COMMON P1,A,B,X,Y

$$QTAU = -1.6 / (4 * PI) * (X - A * DCOS(Z)) * ((X - A * DCOS(Z)) ** 2 + 3 * (Y - B * DSIN(Z)) ** 2) / ((X - A * DCOS(Z)) ** 2 + (Y - B * DSIN(Z)) ** 2) ** 2 * A * DSIN(Z)$$

RETURN  
END

```

0005920CC
0006000CC      THIS FUNCTION CALCULATES THE X STRESS AT (X,Y) DUE TO A POINT
0006020CC      FORCE X AT (A*COS(Z),B*SIN(Z)) IN A SEMIINFINITE PLATE.
0006100CC
0006200C      REAL FUNCTION XSIGX*B (Z)
0006300C      REAL*B Z
0006310C      REAL*B PI,A,B,E,X,Y
0006340C      COMMON PI,X,Y /ELLIP/ A,B,E
0006350CC
0006360C      XSIGX=-1/(4*PI)*((X-A*DCOS(Z))*((3*(X-A*DCOS(Z))**2+(Y-B*DSIN(Z)
0006370C 2  -E)**2)/((X-A*DCOS(Z))**2+(Y-B*DSIN(Z)-E)**2)**2
0006380C 3  + (3*(X-A*DCOS(Z))**2+(Y+B*DSIN(Z)+E)**2)
0006390C 4  / (((X-A*DCOS(Z))**2+(Y+B*DSIN(Z)+E)**2)**2
0006400C 5  + 2*((X-A*DCOS(Z))**4 - 2*Y*(Y+B*DSIN(Z)+E)
0006410C 6  * (X-A*DCOS(Z))**2 + 6*Y*(Y+B*DSIN(Z)+E)**3
0006420C 7  + 2*Y**2*(X-A*DCOS(Z))**2 - (Y+B*DSIN(Z)+E)**4
0006430C 8  - 6*Y**2*(Y+B*DSIN(Z)+E)**2)
0006440C 9  / (((X-A*DCOS(Z))**2+(Y+B*DSIN(Z)+E)**2)**3) *B*DCOS(Z)
0006450CC
0006460CC
0006610CC
0006630C      RETURN
0006680CC      END

```

```

0005920CC
0006000CC      THIS FUNCTION CALCULATES THE Y STRESS AT (X,Y) DUE TO A POINT
0006020CC      FORCE X AT (A*COS(Z),B*SIN(Z)) IN A SEMIINFINITE PLATE.
0006100CC
0006200C      REAL FUNCTION XSIGY*8 (Z)
0006300C      REAL*8 Z
0006310C      REAL*8 PI,A,B,E,X,Y
0006340C      COMMON PI,X,Y /ELLIP/ A,B,E
0006350CC
0006360C      XSIGY =1/{4*PI}*(X-A*DCOS(Z))*{((X-A*DCOS(Z))**2-(Y-B*DSIN(Z)-E)
0006370C 2  **2)/((X-A*DCOS(Z))**2+(Y-B*DSIN(Z)-E)**2)**2
0006380C 3  +((X-A*DCOS(Z))**2-(Y+B*DSIN(Z)+E)**2)
0006390C 4  / ((X-A*DCOS(Z))**2+(Y+B*DSIN(Z)+E)**2)**2
0006400C 5  - 2*{(X-A*DCOS(Z))**4 + 6*Y*(Y+B*DSIN(Z)+E)
0006410C 6  * (X-A*DCOS(Z))**2 - 2*Y*(Y+B*DSIN(Z)+E)**3
0006420C 7  - 2*Y**2*(X-A*DCOS(Z))**2 - (Y+B*DSIN(Z)+E)**4
0006430C 8  + 6*Y**2*(Y+B*DSIN(Z)+E)**2)
0006440C 9  / ((X-A*DCOS(Z))**2+(Y+B*DSIN(Z)+E)**2)**3) *B*DCOS(Z)
0006450CC
0006460CC
0006610CC
0006630C      RETURN
0006800C      END

```

10053

```
0005920CC      THIS FUNCTION CALCULATES THE SHEAR STRESS AT (X,Y) DUE TO A
0006000CC      POINT FORCE X AT (A*COS(Z),B*SIN(Z)) IN A SEMIINFINITE PLATE.
0006020CC
0006100CC      REAL FUNCTION XTAU*B (Z)
0006200C      REAL*8 Z
0006300C      REAL*8 PI,A,B,E,X,Y
0006310C      COMMON PI,X,Y /ELLIP/ A,B,E
0006340C
0006350CC      XTAU =-1/(4*PI)*((Y-B*DSIN(Z)-E)*(3*(X-A*DCOS(Z))**2+(Y-B*DSIN(Z)
0006360C      2  -E)**2)/((X-A*DCOS(Z))**2+(Y-B*DSIN(Z)-E)**2)**2
0006370C      3  +(Y+B*DSIN(Z)+E)*(3*(X-A*DCOS(Z))**2+(Y+B*DSIN(Z)+E)**2)
0006380C      4  / ((X-A*DCOS(Z))**2+(Y+B*DSIN(Z)+E)**2)**2
0006390C      5  - 2*Y*(-(X-A*DCOS(Z))**4 - 6*Y*(Y+B*DSIN(Z)+E)
0006400C      6  * (X-A*DCOS(Z))**2 + 2*Y*(Y+B*DSIN(Z)+E)**3
0006410C      7  - (Y+B*DSIN(Z)+E)**4
0006420C      8  + 6*(Y+B*DSIN(Z)+E)**2*(X-A*DCOS(Z))**2)
0006430C      9  / ((X-A*DCOS(Z))**2+(Y+B*DSIN(Z)+E)**2)**3) *B*DCOS(Z)
0006440C
0006450CC
0006460CC
0006610CC      RETURN
0006630C      END
0006800C
```

00060000C  
00060001C  
00060002C  
00060003C  
00060004C  
00060005C  
00060006C  
00060007C  
00060008C  
00060009C  
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00060092C  
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00060096C  
00060097C  
00060098C  
00060099C  
00060100C

THIS FUNCTION CALCULATES THE X STRESS AT (X,Y) DUE TO A POINT  
FORCE Y AT (A\*COS(Z),B\*SIN(Z)) IN A SEMIINFINITE PLATE.

REAL FUNCTION YSIGX\*8 (Z)

REAL\*8 Z

REAL\*8 PI,A,B,E,X,Y

COMMON PI,X,Y /ELLIP/ A,B,E

YSIGX = -1/(4\*PI)\*((Y-B\*DSIN(Z)-E)\*((X-A\*DCOS(Z))\*\*2-(Y-B\*DSIN(Z)  
1 -E)\*\*2)/((X-A\*DCOS(Z))\*\*2+(Y-B\*DSIN(Z)-E)\*\*2)\*\*2  
2 - (Y+B\*DSIN(Z)+E)\*((X-A\*DCOS(Z))\*\*2-(Y+B\*DSIN(Z)+E)\*\*2)  
3 / ((X-A\*DCOS(Z))\*\*2+(Y+B\*DSIN(Z)+E)\*\*2)\*\*2  
4 + 2\*(3\*Y\*(X-A\*DCOS(Z))\*\*4 + (Y+B\*DSIN(Z)+E)  
5 \* (X-A\*DCOS(Z))\*\*4 - 5\*Y\*(Y+B\*DSIN(Z)+E)\*\*4  
6 - 6\*Y\*\*2\*(Y+B\*DSIN(Z)+E)\*(X-A\*DCOS(Z))\*\*2 + 4\*(Y+B\*DSIN(Z)+E)\*\*3  
7 \* (X-A\*DCOS(Z))\*\*2 + 6\*Y\*(Y+B\*DSIN(Z)+E)\*\*2\*(X-A\*DCOS(Z))\*\*2  
8 + 3\*(Y+B\*DSIN(Z)+E)\*\*5 + 2\*Y\*\*2\*(Y+B\*DSIN(Z)+E)\*\*3)  
9 / ((X-A\*DCOS(Z))\*\*2+(Y+B\*DSIN(Z)+E)\*\*2)\*\*3 \*A\*DSIN(Z)

RETURN

END

2005

```
0005920CC
0006000CC      THIS FUNCTION CALCULATES THE Y STRESS AT (X,Y) DUE TO A POINT
0006020CC      FORCE Y AT (A*COS(Z),B*SIN(Z)) IN A SEMIINFINITE PLATE.
0006100CC
0006200C      REAL FUNCTION YSIGY*8 (Z)
0006300C      REAL*8 Z
0006310C      REAL*8 PI,A,B,E,X,Y
0006340C      COMMON PI,X,Y /ELLIP/ A,B,E
0006350CC
0006360C      YSIGY =-1/(4*PI)*((Y-B*DSIN(Z)-E)*((X-A*DCOS(Z))**2+3*(Y-B*DSIN(Z)
1  )-E)**2)/(((X-A*DCOS(Z))**2+(Y-B*DSIN(Z)-E)**2)**2
0006370C      2  - (Y+B*DSIN(Z)+E)*((X-A*DCOS(Z))**2+3*(Y+B*DSIN(Z)+E)**2)
0006380C      3  / (((X-A*DCOS(Z))**2+(Y+B*DSIN(Z)+E)**2)**2
0006390C      4  + 2*((Y+B*DSIN(Z)+E)*(X-A*DCOS(Z))**4 + 4*(Y+B*DSIN(Z)+E)**3
0006400C      5  * (X-A*DCOS(Z))**2 - 2*Y**2*(Y+B*DSIN(Z)+E)**3
0006410C      6  - Y*(X-A*DCOS(Z))**4 + 3*(Y+B*DSIN(Z)+E)**5 + 3*Y*(Y+B*DSIN(Z)
0006420C      7  +E)**4 - 6*Y*(Y+B*DSIN(Z)+E)**2*(X-A*DCOS(Z))**2
0006432C      8  + 6*Y**2*(Y+B*DSIN(Z)+E)*(X-A*DCOS(Z))**2)
0006440C      9  / (((X-A*DCOS(Z))**2+(Y+B*DSIN(Z)+E)**2)**3) *A*DSIN(Z)
0006450CC
0006460CC
0006610CC
0006630C      RETURN
0006800C      END
```

0005920CC  
0006000CC  
0006020CC  
0006100CC  
0006200C  
0006300C  
0006310C  
0006340C  
0006350CC  
0006360C  
0006370C  
0006380C  
0006390C  
0006400C  
0006410C  
0006430C  
0006440C  
0006460CC  
0006610CC  
0006630C  
0006800C

THIS FUNCTION CALCULATES THE SHEAR STRESS AT (X,Y) DUE TO A  
POINT FORCE Y AT (A\*DCOS(Z),B\*DSIN(Z)) IN A SEMIINFINITE PLATE.

REAL FUNCTION YTAU\*8 (Z)

REAL\*8 Z

REAL\*8 PI,A,B,E,X,Y

COMMON PI,X,Y /ELLIP/ A,B,E

YTAU = -1/(4\*PI)\*((X-A\*DCOS(Z))\*((X-A\*DCOS(Z))\*\*2+3\*(Y-B\*DSIN(Z)  
2 -E)\*\*2)/((X-A\*DCOS(Z))\*\*2+(Y-B\*DSIN(Z)-E)\*\*2)\*\*2  
3 -(X-A\*DCOS(Z))\*((X-A\*DCOS(Z))\*\*2+3\*(Y+B\*DSIN(Z)+E)\*\*2)  
4 / ((X-A\*DCOS(Z))\*\*2+(Y+B\*DSIN(Z)+E)\*\*2)\*\*2  
5 +2\*Y\*(X-A\*DCOS(Z))\*(2\*Y\*(X-A\*DCOS(Z))\*\*2+2\*(Y+B\*DSIN(Z)+E)  
6 \* (X-A\*DCOS(Z))\*\*2 + 10\*(Y+B\*DSIN(Z)+E)\*\*3  
7 - 6\*Y\*(Y+B\*DSIN(Z)+E)\*\*2)  
8 / ((X-A\*DCOS(Z))\*\*2+(Y+B\*DSIN(Z)+E)\*\*2)\*\*3) \*A\*DSIN(Z)

RETURN

END

CL-27

```
0005920CC
0006000CC THIS FUNCTION CALCULATES THE X STRESS AT (X,Y) DUE TO A POINT
0006020CC FORCE X AT (0,Z) ON A CRACK IN A SEMIINFINITE PLATE.
0006100CC
0006200C REAL FUNCTION XCRAC*8 (Z)
0006300C REAL*8 Z
0006310C REAL*8 PI,C,H,X,Y
0006340C COMMON PI,X,Y /CRACK/ C,H
0006350CC
0006360C XCRAC=(1/(PI*(X**2+(Z-Y)**2)**3)
0006380C 2 *(-3*X**4 +6*X**2*(Z-Y)**2 +(Z-Y)**4)
0006400C 3 + 1/(PI*(X**2+(Z+Y)**2)**4)
0006420C 4 *(-5*X**6 +(13*(Z+Y)**2+12*(Z+Y)*Y-12*Y**2)*X**4
0006430C 5 + (Z+Y)**2*(17*(Z+Y)**2-72*(Z+Y)*Y+72*Y**2)*X**2
0006432C 6 -(Z+Y)**4*((Z+Y)**2-12*(Z+Y)*Y+12*Y**2))
0006440C 7 * DSQRT((C)**2 -(Z-H)**2)
0006460CC
0006610CC
0006630C RETURN
0006800C END
```

00059800C  
00060000C  
00060200C  
00061000C  
00062000C  
00063000C  
00063100C  
00063400C  
00063500C  
00063600C  
00063800C  
00064000C  
00064200C  
00064300C  
00064320C  
00064400C  
00064600C  
00066100C  
00066300C  
00068000C

THIS FUNCTION CALCULATES THE Y STRESS AT (X,Y) DUE TO A POINT  
FORCE X AT (0,Z) ON A CRACK IN A SEMIINFINITE PLATE.

REAL FUNCTION YCRAC\*8 (Z)

REAL\*8 Z

REAL\*8 PI,C,H,X,Y

COMMON PI,X,Y /CRACK/ C,H

YCRAC=(1/(PI\*(X\*\*2+(Z-Y)\*\*2)\*\*3)

2 \*(X\*\*4 -6\*X\*\*2\*(Z-Y)\*\*2 +(Z-Y)\*\*4)

3 + 1/(PI\*(X\*\*2+(Z+Y)\*\*2)\*\*4)

4 \*(-X\*\*6 +(5\*(Z+Y)\*\*2-36\*(Z+Y)\*Y+12\*Y\*\*2)\*X\*\*4

5 + (Z+Y)\*\*2\*(5\*(Z+Y)\*\*2+56\*(Z+Y)\*Y-72\*Y\*\*2)\*X\*\*2

6 -(Z+Y)\*\*4\*((Z+Y)\*\*2+4\*(Z+Y)\*Y-12\*Y\*\*2))

7 \* DSQRT((C)\*\*2-(Z-H)\*\*2)

RETURN

END

```

0005920CC
0006000CC
0006020CC
0006100CC
0006200C
0006300C
0006310C
0006340C
0006350CC
0006360C
0006400C
0006410C
0006420C
0006432C
0006460CC
0006610CC
0006630C
0006800C

THIS FUNCTION CALCULATES THE SHEAR STRESS AT (X,Y) DUE TO A
POINT FORCE X AT (0,Z) ON A CRACK IN A SEMIINFINITE PLATE.

REAL FUNCTION TAUCRA*8 (Z)
REAL*8 Z
REAL*8 PI,C,H,X,Y
COMMON PI,X,Y /CRACK/ C,H

TAUCRA=(2/(PI*(X**2+(Z-Y)**2)**3)*X*(Z-Y)*(3*X**2-(Z-Y)**2)
2  - 1/(PI*(X**2+(Z+Y)**2)**4)
3  * (2*(3*(Z+Y)+2*Y)*X**5 +4*(Z+Y)*((Z+Y)**2-14*(Z+Y)*Y
4  + 12*Y**2)*X**3 -2*(Z+Y)**3*((Z+Y)**2-18*(Z+Y)*Y+24*Y**2)*X))
5  * DSQRT((C)**2-(Z-H)**2)

RETURN
END

```

#### 4. PROGRAM FOR CRACK IN SEMI-INFINITE PLATE USING ANALYTIC INTEGRATION (BODY I)

##### 4.1 LISTING

```

0000100CC PROGRAM TO COMPUTE SIF FOR CRACK IN SEMIINFINITE PLATE.
0000200CC
0000202CC METHOD OF SOLUTION IS THE BODY FORCE METHOD (H. NISITANI).
0000210CC
0000220CC INDEX IS THE NUMBER OF INTERVALS INTO WHICH THE CRACK HAS BEEN
0000240CC PARTITIONED.
0000242CC DIST IS THE DISTANCE FROM THE CENTRE OF THE CRACK TO THE
0000244CC EDGE OF THE PLATE.
0000245CC WIDTH IS HALF THE LENGTH OF THE CRACK.
0000243CC STRESS IS THE APPLIED STRESS AT INFINITY.
0000260CC
0000280C INTEGER INDEX,INCREM
0000300C REAL DIST,WIDTH,STRESS,PI,A,B,C,D,E,F,G,H,X,TEST,DET,SIF1,SIF2
0000320C REAL ETA(101),MIDPT(101),FLNC(101,101),FORCE(100,101)
0000340CC
0000360CC
0000400C READ(5,500) INDEX, DIST, WIDTH, STRESS
0000420CC
0000421 C
0000422 WRITE(6,1000)
0000440 WRITE(6,1005)
0000460 WRITE(6,1010)
0000480 WRITE(6,1020) INDEX
0000500 WRITE(6,1010)
0000505 WRITE(6,1025)
0000520 WRITE(6,1030) DIST
0000540 WRITE(6,1010)
0000545 WRITE(6,1035)
0000560 WRITE(6,1040) WIDTH
0000580 WRITE(6,1010)
0000600 WRITE(6,1050) STRESS
0000620 WRITE(6,1010)
0000720CC
0000740CC COMPUTE VALUE OF ETA(N), MIDPT(N) FOR N=1 TO INDEX+1.
0000760CC ETA(N) AND ETA(N+1) ARE THE VALUES OF THE ENDPOINTS
0000780CC OF THE NTH INTERVAL ON THE Y AXIS ALONG THE CRACK.
0000800CC
0000820CC MIDPT(N) IS THE Y VALUE OF THE MIDPOINT OF THE (N-1)TH INTERVAL.
0000840CC
0000860CC
0000900C ETA(1) = DIST - WIDTH
0001000CC
0001020CC
0001021 PI = 3.141593
0001025C INCREM = INDEX +1
0001040CC
0001100C DO 100 N=2,INCREM
0001200C ETA(N) = ETA(1) + 2*WIDTH*(N-1)/INDEX
0001300C100 MIDPT(N) = ETA(N) - WIDTH/INDEX
0001400CC
0001500CC CALCULATE FUNC(M,N). THIS IS THE ANALYTICALLY INTEGRATED
0001600CC BODY FORCE FUNCTION.
0001620 C
0001700C DO 200 M=2,INCREM

```

```

0001800CC
0001900CC      A,B,C,D,E,F,G,H,X,Y ARE INTERMEDIARY FUNCTIONS.
0002000CC
0002100C      A = MIDPT(M) + DIST
0002200C      B = MIDPT(M) - DIST
0002300C      C = SQRT(WIDTH**2 - E**2)
0002400C      D = A**2 - WIDTH**2
0002420 C
0002500CC
0002600C      DO 200 N=1,INCREM
0002700C          E = ETA(N) - DIST
0002800C          F = ETA(N) + MIDPT(M)
0002900C          G = MIDPT(M) - ETA(N)
0003000C          H = SQRT(WIDTH**2 - E**2)
0003100C          X = ARSIN((WIDTH**2 + A**2)/(WIDTH*F))
0003200CC
0003300CC
0003400C      FUNC(M,N) = 1/ PI *(H/G + B/C
0003500C      1      * LOG(ABS((WIDTH**2 - B**2 + C**2)/(WIDTH*G))))
0003600C      2      - 1/ PI *(-H/F + A/SQRT(D) * X)
0003700C      3      + 6* MIDPT(M)/ PI * (-H/F**2 *(1- A*F/D)
0003800C      4      + WIDTH**2/D**1.5 * X ) - 2*MIDPT(M)**2/ PI
0003900C      5      *(H/F**3 *(1-2 + A*F/D *(2*WIDTH**2 + A**2) * F**2/D**2)
0004000C      6      + 3*WIDTH**2 *A/D**2.5 * X)
0004100CC
0004120C200    CONTINUE
0004140 C
0004200CC      EVALUATE BODY FORCE MATRIX AS FOLLOWS:
0004300CC      (INDEX BY INDEX) MATRIX CONTAINS FORCE FUNCTION (INFLUENCE
0004320 C      COEFFICIENTS);
0004400CC      (INDEX BY ONE) MATRIX CONTAINS NEGATIVE OF STRESS.
0004500CC
0004600C      DO 300 M=1,INDEX
0004700C          FORCE(M,INDEX+1) = -STRESS
0004800C          DO 300 N=1,INDEX
0004900C              FORCE(M,N) = FUNC(M+1,N+1) - FUNC(M+1,N)
0004940 300    CONTINUE
0005000CC
0005100CC
0005200CC      CALL SOLVE SUBROUTINE FROM FTNLID TO SOLVE SYSTEM OF
0005300CC      LINEAR EQUATIONS. ON RETURN FROM SOLVE, SOLUTIONS WILL
0005320 C      BE FOUND IN THE (INDEX BY ONE) MATRIX.
0005340 C
0005400C      CALL SOLVE(FORCE,100,INDEX,INDEX+1,0,DET,TEST)
0005500C      IF (TEST.NE.0) GO TO 400
0005520CC
0005540CC
0005545      WRITE(6,1060)
0005550      WRITE(6,1065)
0005551      WRITE(6,1010)
0005560 C
0005580 C      CORRECTION FACTORS FOR STRESS INTENSITY FACTORS ARE NOW IN
0005600 C      THE LAST COLUMN OF THE MATRIX. THEY ARE TO BE PRINTED,
0005620 C      AND USED TO DETERMINE THE S.I.F. AT THE TIPS OF THE CRACK.

```

```

0005625 C
0005640C DO 350 M=1,INDEX
0005660 FORCE(M,INDEX+1) = FORCE(M,INDEX+1)/STRESS
0005680C350 WRITE(6,600) M,FORCE(M,INDEX+1)
0005700 C
0005720C WRITE(6,1010)
0005740 SIF1 = FORCE(1,INCREM) * SQRT(PI*WIDTH)
0005760 SIF2 = FORCE(INDEX,INCREM) * SQRT(PI*WIDTH)
0005780 C
0005800 WRITE(6,1070)
0005820 WRITE(6,1075) SIF1
0005840 WRITE(6,1010)
0005860 WRITE(6,1080)
0005880 WRITE(6,1085) SIF2
0005900 WRITE(6,1010)
0005920 400 WRITE(6,700) DET
0005940 C
0005960CC
0005980CC
0006000C500 FORMAT(14,3(1X,F10.0))
0006001 740 FORMAT(8(1X,F6.3))
0006005 800 FORMAT(1X,F12.6)
0006020C600 FORMAT(14,1X,F12.6)
0006040 700 FORMAT(1X,'THE DETERMINANT OF THE COEFFICIENT MATRIX IS',1X,E12.6)
0006060C1000 FORMAT(1X,'SOLUTION TO CRACK IN SEMIINFINITE PLATE')
0006080 1005 FORMAT(3X,'BY NISITANI BODY FORCE METHOD')
0006100 1010 FORMAT(1X)
0006120 1020 FORMAT(1X,'THE NUMBER OF DIVISIONS IN THE CRACK IS',1X,I4)
0006140 1025 FORMAT(1X,'THE DISTANCE FROM THE EDGE OF THE PLATE TO')
0006160 1030 FORMAT(3X,'THE CENTRE OF THE CRACK IS',1X,F12.6)
0006180 1035 FORMAT(1X,'THE DISTANCE FROM THE TIP OF THE CRACK TO')
0006200 1040 FORMAT(3X,'THE CENTRE OF THE CRACK IS',1X,F12.6)
0006220 1050 FORMAT(1X,'THE APPLIED STRESS AT INFINITY IS',1X,F12.6)
0006240 1060 FORMAT(1X,'THE CORRECTION FACTORS FOR EACH INTERVAL,')
0006260 1065 FORMAT(3X,'STARTING WITH THE ONE NEAREST THE EDGE, ARE')
0006280 1070 FORMAT(1X,'THE STRESS INTENSITY FACTOR AT THE TIP OF')
0006300 1075 FORMAT(3X,'THE CRACK NEAREST THE EDGE IS',1X,E12.6)
0006320 1080 FORMAT(1X,'THE STRESS INTENSITY FACTOR AT THE OTHER')
0006340 1085 FORMAT(3X,'TIP IS',1X,E12.6)
0006360 C
0006380 C
0006400C STOP
0006420C END

```

#### 4.2 TYPICAL INPUT

INDEX (MM) = 12

DIST (e) = 1.0

WIDTH (b) = 0.8

STRESS ( $\sigma_{\infty}^{\infty}$ ) = 1.0

FORMAT (I4, 3 (IX, F10.0))

#### 4.3 TYPICAL OUTPUT

SOLUTION TO CRACK IN SEMIINFINITE PLATE  
BY NISITANI BODY FORCE METHOD

THE NUMBER OF DIVISIONS IN THE CRACK IS 12

THE DISTANCE FROM THE EDGE OF THE PLATE TO  
THE CENTRE OF THE CRACK IS 1.500000

THE DISTANCE FROM THE TIP OF THE CRACK TO  
THE CENTRE OF THE CRACK IS 0.800000

THE APPLIED STRESS AT INFINITY IS 1.000000

THE CORRECTION FACTORS FOR EACH INTERVAL,  
STARTING WITH THE ONE NEAREST THE EDGE, ARE

1	1.385015
2	1.350590
3	1.317047
4	1.287088
5	1.260959
6	1.238289
7	1.218545
8	1.201489
9	1.185427
10	1.173092
11	1.151163
12	1.150214

THE STRESS INTENSITY FACTOR AT THE TIP OF  
THE CRACK NEAREST THE EDGE IS 0.219571E 01

THE STRESS INTENSITY FACTOR AT THE OTHER  
TIP IS 0.182347E 01

THE DETERMINANT OF THE COEFFICIENT MATRIX IS 0.197109E 09

5. PROGRAM FOR CRACK IN SEMI-INFINITE PLATE USING NUMERICAL INTEGRATION (BODY 5)

5.1 LISTING

```

0000100CC PROGRAM TO COMPUTE SIF FOR CRACK IN SEMI INFINITE PLATE.
0000200CC METHOD OF SOLUTION IS THE BODY FORCE METHOD (H. NISITANI).
0000202CC
0000203 C
0000204 C NUMERICAL INTEGRATION IS USED INSTEAD OF ANALYTICAL
0000205 C INTEGRATION TO DETERMINE INFLUENCE COEFFICIENTS.
0000210CC
0000220CC INDEX IS THE NUMBER OF INTERVALS INTO WHICH THE CRACK HAS BEEN
0000240CC PARTITIONED.
0000242CC DIST IS THE DISTANCE FROM THE CENTRE OF THE CRACK TO THE
0000244CC EDGE OF THE PLATE.
0000246CC WIDTH IS THE LENGTH OF THE CRACK.
0000248CC STRESS IS THE APPLIED STRESS AT INFINITY.
0000250 C FRAC IS A NUMBER MUCH SMALLER THAN 1.0 USED IN THE
0000252 C NUMERICAL INTEGRATION AROUND THE SINGULARITY.
0000260CC
0000280C INTEGER INDEX, INCREM
0000321C REAL*8 PI, DIST, WIDTH, STRESS, EPSIL, SI, SIM1, A, B, PREC, DET, TEST
0000322C REAL*8 ETA(101), MIDPT(101), FORCE(100, 101)
0000340C COMMON MIDPT, DIST, WIDTH, PI, M
0000360C EXTERNAL FUNC
0000380CC
0000400CC
0000420C READ(5, 500) INDEX, DIST, WIDTH, STRESS, FRAC
0000421 C
0000422 C
0000423 WRITE(6, 1000)
0000424 WRITE(6, 1005)
0000425 WRITE(6, 1010)
0000426 WRITE(6, 1020) INDEX
0000427 WRITE(6, 1010)
0000428 WRITE(6, 1025)
0000429 WRITE(6, 1030) DIST
0000440 WRITE(6, 1010)
0000460 WRITE(6, 1035)
0000480 WRITE(6, 1040) WIDTH
0000500 WRITE(6, 1010)
0000520 WRITE(6, 1050) STRESS
0000525 WRITE(6, 1010)
0000540CC
0000560CC COMPUTE VALUE OF ETA(N), MIDPT(N) FOR N=1 TO INDEX+1.
0000580CC ETA(N) AND ETA(N+1) ARE THE VALUES OF THE ENDPOINTS
0000600CC OF THE NTH INTERVAL ON THE Y AXIS ALONG THE CRACK.
0000700CC
0000800CC MIDPT(N) IS THE Y VALUE OF THE MIDPOINT OF THE (N-1)TH INTERVAL.
0000820CC
0000840CC
0000860CC
0000900C ETA(1) = DIST - WIDTH
0001000CC
0001020CC
0001021C PI = 3.141593
0001022 C
0001025C INCREM = INDEX + 1

```

```

0001040CC
0001100C      DO 100 N=2,INCREM
0001200C      ETA(N) = ETA(1) + 2*WIDTH*(N-1)/INDEX
0001300C100   MIDPT(N) = ETA(N) - WIDTH/INDEX
0001400CC
0001402 C
0001404 C      EPSIL IS THE VALUE OF EPSILON FOR INTEGRATION ABOUT
0001406 C      THE SINGULARITIES.
0001410 C
0001420C      EPSIL = FRAC * WIDTH / INDEX
0001460      WRITE(6,1055) EPSIL
0001465      WRITE(6,1010)
0001480 C
0001600CC
0001620CC      EVALUATE BDDY FORCE MATRIX AS FOLLOWS:
0001630CC      (INDEX BY INDEX) MATRIX CONTAINS FORCE FUNCTION (INFLUENCE
0001632 C      COEFFICIENTS);
0001640CC      (INDEX BY ONE) MATRIX CONTAINS NEGATIVE OF STRESS.
0001650CC
0001655 C      NUMERICAL INTEGRATION IS PERFORMED OVER THE WHOLE NTH INTERVAL
0001660 C      UNLESS (M-1.EQ.N) IN WHICH CASE THE INTEGRAL IS DIVIDED INTO
0001665 C      THREE PARTS ABOUT THE SINGULARITY.
0001670 C
0001675 C
0001700C      DO 200 M=2,INCREM
0002600C      DO 200 N=1,INDEX
0002700CC
0002720C      IF (M-1.EQ.N) GO TO 175
0002740CC
0002800C      CALL ROMBRG( FUNC,ETA(N),ETA(N+1),1.0,1.D-7,15,SI,SIM1,INDIC,L)
0003000C      IF (INDIC.EQ.1) GO TO 450
0003001 C
0003005C      FORCE(M-1,N) = SI - 1
0003020C      GO TO 200
0003040CC
0003080C175   A = MIDPT(M) + EPSIL
0003100C      B = MIDPT(M) - EPSIL
0003120C      CALL ROMBRG( FUNC,ETA(N),B,1.0,1.D-7,14,SI,SIM1,INDIC,L)
0003140C      IF (INDIC.EQ.1) GO TO 450
0003145 C
0003160C      FORCE(M-1,N) = SI - 1.
0003180C      CALL ROMBRG( FUNC,A,ETA(N+1),1.0,1.D-7,14,SI,SIM1,INDIC,L)
0003200C      IF (INDIC.EQ.1) GO TO 450
0003220C      FORCE(M-1,N)=FORCE(M-1,N)+SI-1. - 2*SQRT(WIDTH**2
0003240C 2      -(MIDPT(M)-DIST)**2) / (PI * EPSIL)
0003400CC
0003520C200   CONTINUE
0004100CC
0004600C      DO 300 M=1,INDEX
0004700C300   FORCE(M,INDEX+1) = -STRESS
0005000CC
0005100CC
0005200CC      CALL SOLVD SUBROUTINE TO SOLVE SYSTEM OF LINEAR EQUATIONS.
0005300CC

```

```

0005400C CALL SOLVD(FORCE,100,INDEX,INDEX+1,0.0,DET,TEST)
0005500C IF (TEST.NE.0) GO TO 400
0005520CC
0005540C WRITE(6,1060)
0005545 WRITE(6,1065)
0005550 WRITE(6,1010)
0005552 C
0005560C DO 350 M=1,INDEX
0005570 FORCE(M,INCREM) = FORCE(M,INCREM)/STRESS
0005600C350 WRITE(6,600) M,FORCE(M,INCREM)
0005602 C
0005604 C CALCULATE STRESS INTENSITY FACTORS AT TIPS OF CRACK.
0005606 C
0005608 SIF1 = FORCE(1,INCREM) * SQRT(PI*WIDTH)
0005610 SIF2 = FORCE(INDEX,INCREM) * SQRT(PI*WIDTH)
0005611 C
0005612 C
0005620 WRITE(6,1010)
0005640 WRITE(6,1070)
0005660 WRITE(6,1075) SIF1
0005680 WRITE(6,1010)
0005700 WRITE(6,1080)
0005720 WRITE(6,1085) SIF2
0005740 WRITE(6,1010)
0005760C WRITE(6,1090) DET
0005780 C
0005800CC
0005820CC
0005840C500 FORMAT(I4,4(1X,F10.0))
0005860C600 FORMAT(I4,1X,F12.6)
0005880C700 FORMAT(1X,'ERROR: NUMERICAL INTEGRATION DOES NOT CONVERGE')
0005900 800 FORMAT(1X,'ERROR: COEFFICIENT MATRIX IS SINGULAR')
0005920C1000 FORMAT(1X,'SOLUTION TO CRACK IN SEMI-INFINITE PLATE')
0005940C1005 FORMAT(3X,'BY NISITANI BODY FORCE METHOD')
0005960 1010 FORMAT(1X)
0005980 1020 FORMAT(1X,'THE NUMBER OF DIVISIONS IN THE CRACK IS',1X,I4)
0006000 1025 FORMAT(1X,'THE DISTANCE FROM THE EDGE OF THE PLATE')
0006020 1030 FORMAT(3X,'TO THE CENTRE OF THE CRACK IS',1X,F12.6)
0006040 1035 FORMAT(1X,'THE DISTANCE FROM THE TIP OF THE CRACK')
0006060 1040 FORMAT(3X,'TO THE CENTRE OF THE CRACK IS',1X,F12.6)
0006080 1050 FORMAT(1X,'THE APPLIED STRESS AT INFINITY IS',1X,F12.6)
0006100 1055 FORMAT(1X,'THE VALUE OF EPSILON USED FOR INTEGRATION IS',1X,F12.6)
0006120 1060 FORMAT(1X,'THE CORRECTION FACTORS FOR EACH INTERVAL,')
0006140 1065 FORMAT(3X,'STARTING WITH THE ONE NEAREST THE EDGE, ARE')
0006160 1070 FORMAT(1X,'THE STRESS INTENSITY FACTOR AT THE TIP OF')
0006180 1075 FORMAT(3X,'THE CRACK NEAREST THE EDGE IS',1X,E12.6)
0006200 1080 FORMAT(1X,'THE STRESS INTENSITY FACTOR AT THE OTHER')
0006220 1085 FORMAT(3X,'TIP IS',1X,E12.6)
0006240 1090 FORMAT(1X,'THE DETERMINANT OF THE COEFFICIENT MATRIX IS',1X,E12.6)
0006260 C
0006280CC
0006300C STOP
0006320 C
0006340 C

```

0006380  
0006400 C  
0006420C450  
0006440  
0006460 C  
0006480C  
STOP  
WRITE(6,700)  
STOP  
END

## 5.2 TYPICAL INPUT

INDEX (MM) = 12

DIST (e) = 1.0

WIDTH (b) = 0.8

STRESS ( $\sigma_{\infty}$ ) = 1.0

FRAC = 0.01

FORMAT (I4, 4(IX, F10.0))

## 5.3 TYPICAL OUTPUT

SOLUTION TO CRACK IN SEMIINFINITE PLATE  
BY NISITANI BODY FORCE METHOD

THE NUMBER OF DIVISIONS IN THE CRACK IS 12

THE DISTANCE FROM THE EDGE OF THE PLATE  
TO THE CENTRE OF THE CRACK IS 1.000000

THE DISTANCE FROM THE TIP OF THE CRACK  
TO THE CENTRE OF THE CRACK IS 0.800000

THE APPLIED STRESS AT INFINITY IS 1.000000

THE VALUE OF EPSILON USED FOR INTEGRATION IS 0.000667

THE CORRECTION FACTORS FOR EACH INTERVAL,  
STARTING WITH THE ONE NEAREST THE EDGE, ARE

1	1.386453
2	1.351123
3	1.317365
4	1.287297
5	1.261161
6	1.238453
7	1.218806
8	1.201747
9	1.186739
10	1.173547
11	1.161893
12	1.151842

THE STRESS INTENSITY FACTOR AT THE TIP OF  
THE CRACK NEAREST THE EDGE IS 0.219803E 01

THE STRESS INTENSITY FACTOR AT THE OTHER  
TIP IS 0.182605E 01

THE DETERMINANT OF THE COEFFICIENT MATRIX IS 0.196455E 09

6.0 PROGRAM FOR EDGE CRACK IN SEMI-INFINITE PLATE (BODY 9)

6.1 LISTING

```

0000100CC PROGRAM TO COMPUTE SIF FOR CRACK AT THE EDGE OF A
0000200CC SEMIINFINITE PLATE.
0000202CC METHOD OF SOLUTION IS THE BODY FORCE METHOD (H. NISITANI).
0000203CC
0000204CC NUMERICAL INTEGRATION IS USED INSTEAD OF ANALYTICAL
0000205CC INTEGRATION TO DETERMINE BODY FORCE COEFFICIENTS.
0000210CC
0000220CC INDEX IS THE NUMBER OF INTERVALS INTO WHICH THE CRACK HAS BEEN
0000240CC PARTITIONED.
0000246CC WIDTH IS THE LENGTH OF THE CRACK.
0000248CC STRESS IS THE APPLIED STRESS AT INFINITY.
0000250 C FRAC IS A NUMBER MUCH LESS THAN 1.0 USED IN THE
0000252 C NUMERICAL INTEGRATION ABOUT A SINGULARITY.
0000260CC
0000280C INTEGER INDEX,INCREM
0000321C REAL*8 PI,WIDTH,STRESS,EPSIL,SI,SIM1,A,B,PREC,DET,TEST,FRAC,SIF
0000322C REAL*8 ETA(101),MIDPT(101),FORCE(100,101)
0000340C COMMON MIDPT,WIDTH,PI,M
0000360C EXTERNAL FUNC2
0000380CC
0000400CC
0000420C READ(5,500) INDEX, WIDTH, STRESS, FRAC
0000421 C
0000422 WRITE(6,1000)
0000423 WRITE(6,1005)
0000424 WRITE(6,1010)
0000425 WRITE(6,1020) INDEX
0000440 WRITE(6,1010)
0000480 WRITE(6,1030) WIDTH
0000485 WRITE(6,1010)
0000500 WRITE(6,1040) STRESS
0000580 WRITE(6,1010)
0000680CC
0000700CC
0000720CC COMPUTE VALUE OF ETA(N) FOR N=1 TO INDEX+1.
0000740CC ETA(N) AND ETA(N+1) ARE THE VALUES OF THE ENDPOINTS
0000760CC OF THE NTH INTERVAL ON THE Y AXIS ALONG THE CRACK.
0000800CC
0000820CC MIDPT(N) IS THE Y VALUE OF THE MIDPOINT OF THE (N-1)TH INTERVAL.
0000840CC
0000860CC
0000900C ETA(1) = 0.0
0001000CC
0001020CC
0001021C PI = 3.14159265
0001025C INCREM = INDEX + 1
0001030C EPSIL = FRAC * WIDTH / (2 * INDEX)
0001031 WRITE(6,1045)
0001032 WRITE(6,1050) EPSIL
0001033 WRITE(6,1010)
0001034 WRITE(6,1060)
0001035 WRITE(6,1065)
0001040CC
0001100C DO 100 N=2,INCREM

```

```

0001200C      ETA(N) = WIDTH*(N-1)/INDEX
0001300C100   MIDPT(N) = ETA(N) - WIDTH/(2 * INDEX)
0001400CC
0001600CC
00016200C    EVALUATE BODY FORCE MATRIX AS FOLLOWS:
00016300C      (INDEX BY INDEX) MATRIX CONTAINS FORCE FUNCTION;
00016400C      (INDEX BY ONE) MATRIX CONTAINS NEGATIVE OF STRESS.
00016500C
0001700C      DO 200 M=2,INCREM
0002600C      DO 200 N=1,INDEX
0002700CC
0002720C      IF (M-1.EQ.N) GO TO 175
0002740 C
0002800C      CALL ROMBRG( FUNC2,ETA(N),ETA(N+1),1.0,1.0-7,16,SI,SIM1,INDIC,L)
0002900CC
0003000C      IF (INDIC.EQ.1) GO TO 400
0003005C      FORCE(M-1,N) = SI - 1
0003020C      GO TO 200
00030400C
0003080C175   A = MIDPT(M) + EPSIL
0003100C      B = MIDPT(M) - EPSIL
0003120C      CALL ROMBRG( FUNC2,ETA(N),B,1.0,1.0-7,14,SI,SIM1,INDIC,L)
0003140C      IF (INDIC.EQ.1) GO TO 400
0003160C      FORCE(M-1,N) = SI - 1.
0003180C      CALL ROMBRG( FUNC2,A,ETA(N+1),1.0,1.0-7,14,SI,SIM1,INDIC,L)
0003200C      IF (INDIC.EQ.1) GO TO 400
0003220C      FORCE(M-1,N)=FORCE(M-1,N)+SI-1. - 2*DSQRT(WIDTH**2
0003240C      2 -MIDPT(M)**2) / (PI * EPSIL)
0003400CC
0003520C200   CONTINUE
0004100CC
0004120 C
0004140 C
0004600C      DO 300 M=1,INDEX
0004700C300   FORCE(M,INDEX+1) = -STRESS
0005000CC
0005100CC
0005200CC      CALL SOLVD SUBROUTINE TO SOLVE SYSTEM OF LINEAR EQUATIONS.
0005300CC
0005400C      CALL SOLVD(FORCE,100,INDEX,INDEX+1,.5D-6,DET,TEST)
0005500C      IF (TEST.NE.0) GO TO 450
0005520CC
00055400C
0005560C      DO 350 M=1,INDEX
0005570      FORCE(M,INCREM) = FORCE(M,INCREM)/STRESS
0005600C350   WRITE(6,600) M,FORCE(M,INDEX+1)
0005705CC
0005706 C
0005707      SIF = FORCE(INDEX,INCREM) * DSQRT(PI * WIDTH)
0005708 C
0005710      WRITE(6,1010)
0005720      WRITE(6,1070)
0005740      WRITE(6,1075) SIF
0005742      WRITE(6,1010)

```

```

0005744      WRITE(6,1090) DET
0005745 C
0005748 C
0005760CC
0005780C500  FORMAT(I4,3(IX,F10.0))
0005785 740  FORMAT(S(IX,F5.3))
0005800C600  FORMAT(I4,IX,F12.6)
0005820 1000  FORMAT(1X,'SOLUTION TO EDGE CRACK IN SEMIINFINITE PLATE')
0005840 1000  FORMAT(3X,'USING NISITANI EDDY FORCE METHOD')
0005860 1010  FORMAT(1X)
0005880 1020  FORMAT(1X,'THE NUMBER OF DIVISIONS IN THE CRACK IS',1X,I4)
0005900 1030  FORMAT(1X,'THE LENGTH OF THE CRACK IS',1X,F12.6)
0005920 1040  FORMAT(1X,'THE APPLIED STRESS AT INFINITY IS',1X,F12.6)
0005940 1040  FORMAT(1X,'THE VALUE OF EPSILON IN THE NUMERICAL')
0005960 1050  FORMAT(3X,'INTEGRATION IS',1X,F12.6)
0005980 1060  FORMAT(1X,'THE CORRECTION FACTORS FOR EACH INTERVAL, STARTING')
0006000 1065  FORMAT(3X,'WITH THE ONE NEAREST THE EDGE, ARE')
0006020 1070  FORMAT(1X,'THE STRESS INTENSITY FACTOR AT THE TIP OF')
0006040 1075  FORMAT(3X,'THE CRACK IS',1X,F12.6)
0006060 1080  FORMAT(1X,'ERROR: NUMERICAL INTEGRATION DID NOT CONVERGE')
0006080 1085  FORMAT(1X,'ERROR: COEFFICIENT MATRIX IS SINGULAR')
0006100 1090  FORMAT(1X,'THE DETERMINANT OF COEFFICIENT MATRIX IS',1X,E12.6)
0006120 C
0006140CC
0006160      STOP
0006180 C
0006200 C
0006220C400  WRITE(6,1080)
0006240      STOP
0006260 C
0006280 C
0006300C450  WRITE(6,1085)
0006320C      STOP
0006340 C
0006360C      END

```

## 6.2 TYPICAL INPUT

INDEX (MM) = 12

DIST (e) = 1.0

STRESS ( $\sigma_x^0$ ) = 1.0

FRAC = 0.01

FORMAT (14, 3(IX, F10.0))

## 6.3 TYPICAL OUTPUT

SOLUTION TO EDGE CRACK IN SEMIINFINITE PLATE  
USING NISITANI BODY FORCE METHOD

THE NUMBER OF DIVISIONS IN THE CRACK IS 12

THE LENGTH OF THE CRACK IS 1.000000

THE APPLIED STRESS AT INFINITY IS 1.000000

THE VALUE OF EPSILON IN THE NUMERICAL  
INTEGRATION IS 0.000417

THE CORRECTION FACTORS FOR EACH INTERVAL, STARTING  
WITH THE ONE NEAREST THE EDGE, ARE

1	1.356457
2	1.318769
3	1.281655
4	1.249164
5	1.221302
6	1.197477
7	1.177049
8	1.159454
9	1.144221
10	1.130968
11	1.119419
12	1.109599

THE STRESS INTENSITY FACTOR AT THE TIP OF  
THE CRACK IS 1.966713

THE DETERMINANT OF COEFFICIENT MATRIX IS 0.137103E 12

## 7. PROGRAM FOR ELLIPSE IN INFINITE PLATE (ELL 5)

### 7.1 LISTING

```

0000100CC PROGRAM TO COMPUTE SCF FOR ELLIPSE IN AN INFINITE PLATE.
0000202CC METHOD OF SOLUTION IS THE BODY FORCE METHOD (H. NISITANI).
0000203CC
0000204CC NUMERICAL INTEGRATION IS USED INSTEAD OF ANALYTICAL
0000205CC INTEGRATION TO DETERMINE INFLUENCE COEFFICIENTS.
0000210CC
0000220CC INDEX IS THE NUMBER OF INTERVALS INTO WHICH THE ELLIPSE
0000240CC HAS BEEN PARTITIONED.
0000246CC WIDTH IS HALF THE WIDTH OF THE ELLIPSE ALONG
0000260CC THE X AXIS.
0000280CC LENGTH IS HALF THE HEIGHT OF THE ELLIPSE.
0000300CC STRESS IS THE APPLIED STRESS AT INFINITY.
0000320CC FRAC IS A NUMBER MUCH LESS THAN 1.0 USED IN THE
0000340CC NUMERICAL INTEGRATION ABOUT A SINGULARITY.
0000360CC
0000380C INTEGER INDEX, INCREM, HALF1, HALF2, QUART1, QUART2
0000400C REAL*8 PI, WIDTH, LENGTH, X, Y, STRESS, EPSIL, SI, SIM1, C, D, PREC, DET, TEST
0000405C REAL*8 S, T, XSUM90, THETA, HALF
0000420C REAL*8 PHI(71), MIDPT(71), FORCE(140,210), EVAL(140,141)
0000440C COMMON PI, WIDTH, LENGTH, X, Y
0000460C EXTERNAL PSIGX, PSIGY, QSIGX, QSIGY, PTAU, QTAU
0000480CC
0000500CC
0000520C READ(5,500) INDEX, WIDTH, LENGTH, STRESS, FRAC
0000540CC
0000560C WRITE(6,1000)
0000580C WRITE(6,1005)
0000600C WRITE(6,1010)
0000620C WRITE(6,1020) INDEX
0000640C WRITE(6,1010)
0000660C WRITE(6,1025) WIDTH
0000680C WRITE(6,1010)
0000685C WRITE(6,1030) LENGTH
0000690C WRITE(6,1010)
0000700C WRITE(6,1040) STRESS
0000720C WRITE(6,1010)
0000740CC
0000742CC DEFINE PI, EPSILON. EPSILON IS USED IN INTEGRATION
0000745CC ABOUT SINGULARITIES.
0000760CC
0000780C PI = 3.14159265359
0000800C INCREM = INDEX + 1
0000805C T = LENGTH/WIDTH
0000820C EPSIL = FRAC * PI/INDEX
0000840C WRITE(6,1045)
0000860C WRITE(6,1050) EPSIL
0000880C WRITE(6,1010)
0000940CC
0000960CC
0000980CC COMPUTE VALUE OF PHI(N) FOR N=1 TO INDEX+1.
0001000CC PHI(N) AND PHI(N+1) ARE PARAMETERS FOR THE VALUE OF
0001002CC THE ENDPOINTS OF THE NTH INTERVAL:
0001004CC X = WIDTH * COS(PHI); Y = LENGTH * SIN(PHI)
0001006CC

```

```

0001020CC      MIDPT(M) IS THE VALUE OF THIS PARAMETER AT THE MIDPOINT
0001040CC      OF THE (N-1)TH INTERVAL.
0001060CC
0001080CC      X AND Y ARE THE X AND Y VALUES OF THE POINT ON THE ELLIPSE
0001100CC      HAVING MIDPT(M) AS ITS PARAMETER VALUE.
0001120CC
0001122 C      THETA IS THE ANGLE THE NORMAL ON THE ELLIPSE MAKES WITH THE X
0001124 C      AXIS. IT IS ALWAYS TAKEN TO BE IN THE INTERVAL (-PI/2,PI/2).
0001126 C      THIS IS NOT THE TRUE VALUE OF THETA IN ALL QUADRANTS, BUT
0001128 C      NO CORRECTION IS NEEDED AT THIS POINT.
0001130 C
0001132 C
0001140C      PHI(1) = 0.0
0001160CC
0001180C      DO 100 N=2,INCREM
0001200C          PHI(N) = 2 * PI*(N-1)/INDEX
0001300C          MIDPT(N) = PHI(N) - PI/INDEX
0001340C100      CONTINUE
0001400CC
0001420C      WRITE(6,1010)
0001600CC
0001620CC      PERFORM NUMERICAL INTEGRATIONS TO DETERMINE INFLUENCE
0001630CC      COEFFICIENTS. FOR EXAMPLE, PSIGY IS THE COEFFICIENT IN
0001640CC      THE Y DIRECTION DUE TO A POINT FORCE P.
0001650CC
0001700C      DO 200 M=2,INCREM
0001720C          J=M+INDEX
0001740C          X = WIDTH * DCOS(MIDPT(M))
0001760C          Y = LENGTH * DSIN(MIDPT(M))
0001762C          S = DTAN(MIDPT(M))
0001764 C
0001772C          THETA = DATAN(S/T)
0001777 C
0001780C          C = MIDPT(M) + EPSIL
0001800C          D = MIDPT(M) - EPSIL
0001820CC
0002600C          DO 200 N=1,INDEX
0002630C              K=N+INDEX
0002640C              L=N+2*INDEX
0002700CC
0002720C              IF (M-1.EQ.N) GO TO 175
0002740CC
0002800C              CALL ROMBRG( PSIGX,PHI(N),PHI(N+1),1.0,1.0D-7,10,SI,SIMI,INDIC,1)
0003000C              IF (INDIC.EQ.1) GO TO 400
0003005C              FORCE(M-1,N) = SI- 1.0
0003010CC
0003011C              CALL ROMBRG( PSIGY,PHI(N),PHI(N+1),1.0,1.0D-7,10,SI,SIMI,INDIC,1)
0003013C              IF (INDIC.EQ.1) GO TO 400
0003014C              FORCE(M-1,K) = SI- 1.0
0003040CC
0003060C              CALL ROMBRG( PTAU,PHI(N),PHI(N+1),1.0,1.0D-7,10,SI,SIMI,INDIC,1)
0003080C              IF (INDIC.EQ.1) GO TO 400
0003100C              FORCE(M-1,L) = SI- 1.0
0003120CC

```

```

0003140C CALL ROMBRG( OSIGX,PHI(N),PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
0003160C IF (INDIC.EQ.1) GO TO 400
0003180C FORCE(J-1,N) = SI- 1.0
0003200CC

```

```

0003201C CALL ROMBRG( OSIGY,PHI(N),PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
0003203C IF (INDIC.EQ.1) GO TO 400
0003204C FORCE(J-1,K) = SI- 1.0
0003240CC

```

```

0003260C CALL ROMBRG( QTAU,PHI(N),PHI(N+1),1.0,1.L-7,10,SI,SIM1,INDIC,I)
0003280C IF (INDIC.EQ.1) GO TO 400
0003300C FORCE(J-1,L) = SI- 1.0
0003320CC
0003340C GO TO 200
0003360CC
0003380CC

```

```

0003400CC INTEGRATE OVER SINGULARITIES BY TAKING PRINCIPAL VALUES.
0003402 C THE INTEGRAL HAS FOUR PARTS: THE FIRST TWO ARE THE NORMAL
0003404 C INTEGRALS OVER (PHI(N),D) AND (C,PHI(N+1)) WHICH AVOID
0003406 C THE SINGULARITY. THE THIRD TERM INVOLVES THETA AND IS
0003408 C EXPLAINED IN THE LITERATURE. THE FOURTH TERM IS THE
0003410 C ERROR IN THE INTEGRAL DUE TO THE CHOICE OF EPSILON.
0003420CC

```

```

0003500C175 CALL ROMBRG( PSIGX,PHI(N),D,1.0,1.D-7,10,SI,SIM1,INDIC,I)
0003520C IF (INDIC.EQ.1) GO TO 400
0003540C FORCE(M-1,N) = SI -1.
0003560C CALL ROMBRG( PSIGX,C,PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
0003580C IF (INDIC.EQ.1) GO TO 400
0003600C FORCE(M-1,N)=FORCE(M-1,N)+SI-1.0
0003601C 2 - (5+4*DCOS(2*THETA)-DCOS(4*THETA))/16
0003620C 3 + EPSIL*T/(4*PI*(S**2+T**2)**3)*(6*S**6+(3-2*T**2)*S**4
0003640C 4 -6*S**2*T**2-T**4)
0003720CC

```

```

0003740C CALL ROMBRG( PSIGY,PHI(N),D,1.0,1.D-7,10,SI,SIM1,INDIC,I)
0003760C IF (INDIC.EQ.1) GO TO 400
0003780C FORCE(M-1,K) = SI -1.
0003820C CALL ROMBRG( PSIGY,C,PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
0003840C IF (INDIC.EQ.1) GO TO 400
0003860C FORCE(M-1,K)=FORCE(M-1,K)+SI-1.0
0003880C 2 +(1-DCOS(4*THETA))/16
0003890C 3 + EPSIL*T/(4*PI*(S**2+T**2)**3)*(-2*S**6+(1-6*T**2)*S**4
0003900C 4 +6*S**2*T**2-T**4)
0003920CC

```

```

0003940C CALL ROMBRG( PTAU,PHI(N),D,1.0,1.D-7,10,SI,SIM1,INDIC,I)
0003960C IF (INDIC.EQ.1) GO TO 400
0003980C FORCE(M-1,L) = SI -1.
0004020C CALL ROMBRG( PTAU,C,PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
0004040C IF (INDIC.EQ.1) GO TO 400
0004060C FORCE(M-1,L)=FORCE(M-1,L)+SI-1.0
0004080C 2 -(2*DSIN(2*THETA)-DSIN(4*THETA))/16
0004090C 3 + EPSIL*T**2/(4*PI*(S**2+T**2)**3)*(-9*S**5-(6+2*T**2)
0004100C 4 *S**3 + T**2*S*(2-T**2))
0004120CC

```

```

0004140C CALL ROMBRG( OSIGX,PHI(N),D,1.0,1.D-7,10,SI,SIM1,INDIC,I)

```

```

0004180C      FORCE(J-1,N) = SI -1.
0004220C      CALL ROMBRG( GSIGX,C,PHI(N+1),1.0,1.0-D-7,10,SI,SIM1,INDIC,I)
0004240C      IF (INDIC.EQ.1) GO TO 400
0004260C      FORCE(J-1,N)=FORCE(J-1,N)+SI-1.0
0004280C      2      + (1-DCOS(4*THETA))/16
0004290C      3      + EPSIL*T/(4*PI*(S**2+T**2)**3)*(-S**6+(6-T**2)*S**2
0004300C      4      *T**2 + 6*S**4*T**2-2*T**4)
0004420CC
0004440C      CALL ROMBRG( GSIGY,PHI(N),D,1.0,1.0-D-7,10,SI,SIM1,INDIC,I)
0004460C      IF (INDIC.EQ.1) GO TO 400
0004480C      FORCE(J-1,K) = SI -1.
0004520C      CALL ROMBRG( GSIGY,C,PHI(N+1),1.0,1.0-D-7,10,SI,SIM1,INDIC,I)
0004540C      IF (INDIC.EQ.1) GO TO 400
0004560C      FORCE(J-1,K)=FORCE(J-1,K)+SI-1.0
0004580C      2      -(5-4*DCOS(2*THETA)-DCOS(4*THETA))/16
0004600C      3      + EPSIL*T/(4*PI*(S**2+T**2)**3)*(-S**6+(2-T**2)*S**2
0004620C      4      *T**2 - 6*S**4*T**2+6*T**4)
0004640CC
0004660C      CALL ROMBRG( QTAU,PHI(N),D,1.0,1.0-D-7,10,SI,SIM1,INDIC,I)
0004680C      IF (INDIC.EQ.1) GO TO 400
0004700C      FORCE(J-1,L) = SI -1.
0004740C      CALL ROMBRG( QTAU,C,PHI(N+1),1.0,1.0-D-7,10,SI,SIM1,INDIC,I)
0004760C      IF (INDIC.EQ.1) GO TO 400
0004780C      FORCE(J-1,L)=FORCE(J-1,L)+SI-1.0
0004800C      2      -(2*DSIN(2*THETA)+CSIN(4*THETA))/16
0004820C      3      - EPSIL/(4*PI*(S**2+T**2)**3)*((1-2*T**2)*S**5
0004840C      4      *(1+3*T**2)*2*T**2*S**3 + 9*S*T**4)
0004960CC
0004980CC
0005000C200  CONTINUE
0005020CC
0005025C      WRITE(6,1010)
0005040CC
0005180CC
0005200CC      INSERT INFLUENCE COEFFICIENTS AND STRESSES INTO EVAL
0005220CC      MATRIX ACCORDING TO HOW THEY MUST APPEAR IN THE
0005240CC      SYSTEM OF LINEAR EQUATIONS.
0005245CC
0005245 C      THETA IS AGAIN THE ANGLE BETWEEN THE NORMAL TO THE ELLIPSE
0005247 C      AND THE X AXIS BUT IS IN THE INTERVAL (-PI/2,3*PI/2).
0005248 C      THIS IS THE CORRECT VALUE OF THETA.
0005249 C
0005250C      INCREM = 2*INDEX +1
0005260CC
0005280C      DO 250 M=1,INDEX
0005300C      J=M+INDEX
0005320C      THETA = DATAN(OTAN(MIDPT(M+1))/T)
0005325 C      IF (MIDPT(M+1).LE.PI/2) GO TO 230
0005325 C      IF (MIDPT(M+1).GT.3*PI/2) GO TO 230
0005340 C
0005360      THETA = THETA + PI
0005380 C

```

```

0005425 C
0005440C      DO 250 N=1,INDEX
0005460C          K = N+INDEX
0005480C          L = N+2*INDEX
0005500C          EVAL(M,N) = FORCE(M,N)*DCOS(THETA)
0005520C      2          + FORCE(M,L)*DSIN(THETA)
0005540CC
0005560C          EVAL(J,N) = FORCE(M,K)*DSIN(THETA)
0005580C      2          + FORCE(M,L)*DCOS(THETA)
0005600CC
0005620C          EVAL(M,K) = FORCE(J,N)*DCOS(THETA)
0005640C      2          + FORCE(J,L)*DSIN(THETA)
0005660CC
0005680C          EVAL(J,K) = FORCE(J,K)*DSIN(THETA)
0005700C      2          + FORCE(J,L)*DCOS(THETA)
0005720CC
0005740C250  CONTINUE
0005760CC
0005780CC
0005840CC
0005845C      WRITE(6,1010)
0005860CC
0005880CC      CALL SOLVD SUBROUTINE TO SOLVE SYSTEM OF LINEAR EQUATIONS.
0005900CC      ON RETURN FROM SOLVD, THE PART OF THE MATRIX THAT BEFORE
0005920CC      CONTAINED THE APPLIED STRESSES WILL NOW CONTAIN THE
0005940CC      SOLUTIONS FOR RO-SUB-X AND RO-SUB-Y.
0005960CC
0005980C      CALL SOLVD(EVAL,140,INCREM-1,INCREM,.5D-6,DET,TEST)
0006000C      IF (TEST.NE.0) GO TO 450
0006020CC
0006040CC
0006045C      WRITE(6,1060)
0006046C      WRITE(6,1065)
0006050      WRITE(6,1010)
0006052 C
0006060C      DO 350 M=1,INDEX
0006080C          J=M+INDEX
0006100C350      WRITE(6,600) M,EVAL(M,INCREM),EVAL(J,INCREM)
0006120CC
0006140CC      THE STRESSES AT ARBITRARY POINTS ON THE ELLIPSE (HERE 0 AND 90
0006160CC      DEGREES) CAN BE EVALUATED AS A LINEAR COMBINATION OF THE
0006180CC      INFLUENCE COEFFICIENTS AND THE ROS.
0006200CC
0006220C      XSUM90 = STRESS
0006340CC
0006345 C
0006350 C
0006352 C
0006360CC      PERFORM NUMERICAL INTEGRATIONS TO DETERMINE INFLUENCE
0006380CC      COEFFICIENTS. FOR EXAMPLE, PSIGY IS THE COEFFICIENT IN
0006400CC      THE Y DIRECTION DUE TO A POINT FORCE P.

```

```

0006480C      X = WIDTH * DCOS(HALF)
0006500C      Y = LENGTH * DSIN(HALF)
0006580 C
0006620C      D = HALF - EPSIL
0006640CC
0006660C      DO 380 N=1,INDEX
0006720CC
0006740C      IF (N.EQ.INDNEW) GO TO 375
0006760CC
0006780C      CALL ROMBRG( PSIGX,PHI(N),PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
0006800C      IF (INDIC.EQ.1) GO TO 400
0006820C      FORCE(1,N) = SI - 1.0
0006840CC
0007020C      CALL ROMBRG( QSIGX,PHI(N),PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
0007040C      IF (INDIC.EQ.1) GO TO 400
0007060C      FORCE(2,N) = SI - 1.0
0007080CC
0007240CC
0007260C      GO TO 380
0007280CC
0007300CC
0007320CC      INTEGRATE OVER SINGULARITIES BY TAKING PRINCIPAL VALUES.
0007440CC
0007460C375   CALL ROMBRG( PSIGX,PHI(N),D,1.0,1.D-7,10,SI,SIM1,INDIC,I)
0007480C      IF (INDIC.EQ.1) GO TO 400
0007560C      FORCE(1,N) = SI - 1.0 + 3*EPSIL*T/(2*PI)
0007640CC
0008060C      CALL ROMBRG( QSIGX,PHI(N),D,1.0,1.D-7,10,SI,SIM1,INDIC,I)
0008080C      IF (INDIC.EQ.1) GO TO 400
0008160C      FORCE(2,N) = SI - 1.0 - EPSIL*T/(4*PI)
0008240CC
0008640CC
0008660CC
0008680C380   CONTINUE
0008685 C
0008690 C
0008700C      DO 370 M=1,INDEX
0008720C      J=M+INDEX
0008740C      XSUM90 = XSUM90+EVAL(M,INCREM) * FORCE(1,M)
0008760C      2      + EVAL(J,INCREM) * FORCE(2,M)
0008780C370   CONTINUE
0008800CC
0008820CC      DETERMINE STRESS CONCENTRATION FACTOR BY DIVIDING THE EVALUATED
0008840CC      STRESS IN THE X DIRECTION AT 90 DEGREES BY THE STRESS
0008860CC      AT INFINITY.
0008880 C
0008900CC
0008920CC
0009020C      WRITE(6,1010)
0009040CC
0009060C      XSUM90 = XSUM90/STRESS
0009080C      WRITE(6,1085) XSUM90
0009100C      WRITE(6,1010)

```

```

0009140CC
0009160CC
0009180CC
0009200C500  FORMAT(I4,4(1X,F10.0))
0009220C600  FORMAT(I4,2(1X,F12.6))
0009240C1000  FORMAT(1X,'SOLUTION TO ELLIPSE IN INFINITE PLATE')
0009260C1005  FORMAT(3X,'USING NISITANT BODY FORCE METHOD')
0009280C1010  FORMAT(1X)
0009300C1020  FORMAT(1X,'THE NUMBER OF DIVISIONS IN THE ELLIPSE IS ',I4)
0009320C1025  FORMAT(1X,'HALF THE WIDTH OF THE ELLIPSE IS ',F12.6)
0009340C1030  FORMAT(1X,'HALF THE HEIGHT OF THE ELLIPSE IS ',F12.6)
0009360C1040  FORMAT(1X,'THE APPLIED STRESS AT INFINITY IS ',F12.6)
0009380C1045  FORMAT(1X,'THE VALUE OF EPSILON IN THE NUMERICAL')
0009400C1050  FORMAT(3X,'INTEGRATION IS ',F12.6)
0009420 1055  FORMAT(1X,'THE X AND Y BODY FORCE DENSITIES, STARTING AT')
0009440 1060  FORMAT(3X,'THE SIDE OF THE ELLIPSE (PHI = 0), ARE:')
0009500C1085  FORMAT(1X,'THE STRESS CONCENTRATION FACTOR IS ',2(F12.6,1X))
0009520C1090  FORMAT(1X,'THE DETERMINANT OF COEFFICIENT MATRIX IS ',E12.6)
0009540C1100  FORMAT(1X,'ERROR: NUMERICAL INTEGRATION DID NOT CONVERGE.')
0009560C1110  FORMAT(1X,'ERROR: COEFFICIENT MATRIX IS SINGULAR.')
0009580CC
0009600CC
0009620C  STOP
0009640CC
0009660CC
0009680C400  WRITE(6,1100)
0009700C  STOP
0009720CC
0009740CC
0009760C450  WRITE(6,1110)
0009780C  STOP
0009800CC
0009820C  END

```

## 7.2 TYPICAL INPUT

INDEX (MM) = 12

WIDTH (a) = 2.0

LENGTH (b) = 1.0

STRESS ( $\sigma_{\infty}^{\infty}$ ) = 1.0

FRAC = 0.05

FORMAT (I4, 4(IX, F10.0))

## 7.3 TYPICAL OUTPUT

SOLUTION TO ELLIPSE IN INFINITE PLATE  
USING NISITANI BODY FORCE METHOD

THE NUMBER OF DIVISIONS IN THE ELLIPSE IS 12  
HALF THE WIDTH OF THE ELLIPSE IS 2.000000  
HALF THE HEIGHT OF THE ELLIPSE IS 1.000000  
THE APPLIED STRESS AT INFINITY IS 1.000000  
THE VALUE OF EPSILON IN THE NUMERICAL  
INTEGRATION IS 0.013090

THE X AND Y BODY FORCE DENSITIES, STARTING AT  
THE SIDE OF THE ELLIPSE ( $\phi = 0$ ), ARE:

1	2.000474	-1.002336
2	1.999895	-1.000864
3	1.999652	-1.000324
4	1.999652	-1.000324
5	1.999895	-1.000864
6	2.000474	-1.002336
7	2.000474	-1.002336
8	1.999895	-1.000864
9	1.999652	-1.000324
10	1.999652	-1.000324
11	1.999895	-1.000864
12	2.000474	-1.002336

THE STRESS CONCENTRATION FACTOR IS 2.00001

THE DETERMINANT OF COEFFICIENT MATRIX IS 0.122099E-13

## 8.0 PROGRAM FOR ELLIPSE OR SEMI-ELLIPSE IN SEMI-INFINITE PLATE (ELL 8)

### 8.1 LISTING

```

0000100CC PROGRAM TO COMPUTE SCF FOR ELLIPSE IN A SEMIINFINITE PLATE.
0000202CC METHOD OF SOLUTION IS THE BODY FORCE METHOD (H. NISITANI).
0000203CC
0000204CC NUMERICAL INTEGRATION IS USED INSTEAD OF ANALYTICAL
0000205CC INTEGRATION TO DETERMINE INFLUENCE COEFFICIENTS.
0000210CC
0000220CC INDEX IS THE NUMBER OF INTERVALS INTO WHICH THE ELLIPSE
0000240CC HAS BEEN PARTITIONED.
0000246CC WIDTH IS HALF THE WIDTH OF THE ELLIPSE ALONG
0000260CC THE X AXIS.
0000280CC LENGTH IS HALF THE HEIGHT OF THE ELLIPSE.
0000305CC DIST IS THE DISTANCE FROM THE CENTRE OF THE ELLIPSE TO THE
0000310CC EDGE OF THE PLATE. IF DIST = 0, THEN WE HAVE A SEMIELLIPSE
0000311CC AT THE EDGE OF A SEMIINFINITE PLATE.
0000312CC STRESS IS THE APPLIED STRESS AT INFINITY.
0000320CC FRAC IS A NUMBER MUCH LESS THAN 1.0 USED IN THE
0000340CC NUMERICAL INTEGRATION ABOUT A SINGULARITY.
0000360CC
0000380C INTEGER INDEX,INCREM,NEW
0000400C REAL*8 PI,WIDTH,LENGTH,X,Y,STRESS,EPSIL,SI,SIMI,C,D,PREC,DET,TEST
0000405C REAL*8 S,T,XSUM90,THETA,DIST,FRAC
0000420C REAL*8 PHI(101),MIDPT(101),FORCE(200,300),EVAL(200,201)
0000440C COMMON PI,X,Y /ELLIP/ WIDTH,LENGTH,DIST
0000460C EXTERNAL XSIGX,XSIGY,YSIGX,YSIGY,XTAU,YTAU
0000480CC
0000500CC
0000520C READ(5,500) INDEX, WIDTH, LENGTH, DIST, STRESS, FRAC
0000540CC
0000560C WRITE(6,1000)
0000580C WRITE(6,1005)
0000600C WRITE(6,1010)
0000620C WRITE(6,1020) INDEX
0000640C WRITE(6,1010)
0000660C WRITE(6,1025) WIDTH
0000680C WRITE(6,1010)
0000685C WRITE(6,1030) LENGTH
0000686C WRITE(6,1010)
0000700C WRITE(6,1035)
0000720C WRITE(6,1037) DIST
0000740C WRITE(6,1010)
0000760C WRITE(6,1040) STRESS
0000780C WRITE(6,1010)
0000800CC
0000820CC DEFINE PI, EPSILON. EPSILON IS USED IN INTEGRATION
0000840CC ABOUT SINGULARITIES.
0000860CC
0000880C PI = 3.14159265359
0000900C INCREM = INDEX +1
0000920C T = LENGTH/WIDTH
0000940C EPSIL = FRAC * PI/INDEX
0000960C WRITE(6,1045)
0000980C WRITE(6,1050) EPSIL
0001000C WRITE(6,1010)
0001020CC

```

```

0001040CC
0001060CC COMPUTE VALUE OF PHI(N) FOR N=1 TO INDEX+1.
0001080CC PHI(N) AND PHI(N+1) ARE PARAMETERS FOR THE VALUE OF
0001100CC THE ENDPOINTS OF THE NTH INTERVAL ALONG THE ELLIPSE.
0001120CC X = WIDTH * COS(PHI); Y = LENGTH * SIN(PHI)
0001140CC
0001160CC MIDPT(M) IS THE VALUE OF THIS PARAMETER AT THE MIDPOINT
0001180CC OF THE (N-1)TH INTERVAL.
0001200CC
0001220CC X AND Y ARE THE X AND Y VALUES OF THE POINT ON THE ELLIPSE
0001240CC HAVING MIDPT(M) AS ITS PARAMETER VALUE.
0001260CC
0001280CC THETA IS THE ANGLE THE NORMAL ON THE ELLIPSE MAKES WITH THE X
0001300CC AXIS. IT IS ALWAYS TAKEN TO BE IN THE INTERVAL (-PI/2,PI/2).
0001320CC THIS IS NOT THE TRUE VALUE OF THETA IN ALL QUADRANTS, BUT
0001340CC NO CORRECTION IS NEEDED AT THIS POINT.
0001380CC
0001420CC WE SPLIT INTO TWO CASES FOR THE PARTITION: IF DIST=0, A
0001440CC SEMIELLIPSE IS DONE, OTHERWISE AN ELLIPSE IS DONE.
0001445CC
0001460C IF (DIST.NE.0.0) GO TO 80
0001480CC
0001485C PHI(1) = 0.0
0001500C DO 50 N=2,INCREM
0001520C PHI(N) = PI*(N-1)/INDEX
0001540C MIDPT(N) = PHI(N) - PI/(2*INDEX)
0001560C50 CONTINUE
0001565C GO TO 150
0001580CC
0001585C80 PHI(1) = -PI/2.0
0001600C DO 100 N=2,INCREM
0001620C PHI(N) = -PI/2.0 + 2*PI*(N-1)/INDEX
0001640C MIDPT(N) = PHI(N) - PI/INDEX
0001660C100 CONTINUE
0001680CC
0001700CC
0001720CC PERFORM NUMERICAL INTEGRATIONS TO DETERMINE INFLUENCE
0001740CC COEFFICIENTS. FOR EXAMPLE, XSIGY IS THE COEFFICIENT IN
0001760CC THE Y DIRECTION DUE TO A POINT FORCE X.
0001780CC
0001800CC I.E. XSIGY IS THE STRESS AT THE (M-1)TH INTERVAL IN THE
0001820CC Y DIRECTION DUE TO A POINT FORCE AT THE NTH INTERVAL
0001860CC IN THE X DIRECTION.
0001865CC DUE TO SYMMETRY THE (M-1,N) COEFFICIENTS AND THE
0001870CC (INDEX-M+2,INDEX-N+1) COEFFICIENTS ARE EQUAL, SO THEY
0001872CC ARE EVALUATED SIMULTANEOUSLY.
0001874CC
0001880CC
0001900C150 INDNEW=INT(INDEX/2.0)
0001920CC
0001940C DO 200 M=2,INCREM
0001960C J=M+INDEX
0001980C MM = INDEX -M +3

```

2

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0002020C      X = WIDTH * DCOS(MIDPT(M))
0002040C      Y = LENGTH * DSIN(MIDPT(M)) + DIST
0002060C      S = DTAN(MIDPT(M))
0002080CC
0002100C      THETA = DATAN(S/T)
0002120CC
0002140C      C = MIDPT(M) + EPSIL
0002160C      D = MIDPT(M) - EPSIL
0002180CC
0002600C      DO 200 N=1,INDNEW
0002630C          K=N+INDEX
0002640C          L=N+2*INDEX
0002650C          NN = INDEX -N +1
0002660C          KK = NN + INDEX
0002670C          LL = NN + 2*INDEX
0002700CC
0002720C          IF (M-1.EQ.N) GO TO 175
0002740CC
0002800C          CALL ROMBRG( XSIGX,PHI(N),PHI(N+1),1.0,1.D-7,15,SI,SIM1,INDIC,I)
0003000C          IF (INDIC.EQ.1) GO TO 400
0003005C          FORCE(M-1,N) = SI- 1.0
0003006C          FORCE(MM-1,NN) = SI- 1.0
0003010CC
0003011C          CALL ROMBRG( XSIGY,PHI(N),PHI(N+1),1.0,1.D-7,15,SI,SIM1,INDIC,I)
0003013C          IF (INDIC.EQ.1) GO TO 400
0003014C          FORCE(M-1,K) = SI- 1.0
0003020C          FORCE(MM-1,KK) = SI- 1.0
0003040CC
0003060C          CALL ROMBRG( XTAU,PHI(N),PHI(N+1),1.0,1.D-7,15,SI,SIM1,INDIC,I)
0003080C          IF (INDIC.EQ.1) GO TO 400
0003100C          FORCE(M-1,L) = SI- 1.0
0003105C          FORCE(MM-1,LL) = 1.0 - SI
0003120CC
0003140C          CALL ROMBRG( YSIGX,PHI(N),PHI(N+1),1.0,1.D-7,15,SI,SIM1,INDIC,I)
0003160C          IF (INDIC.EQ.1) GO TO 400
0003180C          FORCE(J-1,N) = SI- 1.0
0003185C          FORCE(JJ-1,NN) = SI- 1.0
0003200CC
0003201C          CALL ROMBRG( YSIGY,PHI(N),PHI(N+1),1.0,1.D-7,15,SI,SIM1,INDIC,I)
0003203C          IF (INDIC.EQ.1) GO TO 400
0003204C          FORCE(J-1,K) = SI- 1.0
0003210C          FORCE(JJ-1,KK) = SI- 1.0
0003240CC
0003260C          CALL ROMBRG( YTAU,PHI(N),PHI(N+1),1.0,1.D-7,15,SI,SIM1,INDIC,I)
0003280C          IF (INDIC.EQ.1) GO TO 400
0003300C          FORCE(J-1,L) = SI- 1.0
0003305C          FORCE(JJ-1,LL) = 1.0 - SI
0003320CC
0003340C          GO TO 200
0003360CC
0003380CC
0003400CC
0003402CC
0003404CC
INTEGRATE OVER SINGULARITIES BY TAKING PRINCIPAL VALUES.
THE INTEGRAL HAS FOUR PARTS: THE FIRST TWO ARE THE NORMAL
INTEGRALS OVER (PHI(N),D) AND (C,PHI(N+1)) WHICH AVOID

```

0003406CC  
0003408CC  
0003410CC  
0003420CC  
0003500C175  
0003520C  
0003540C  
0003560C  
0003580C  
0003600C  
0003601C  
0003620C  
0003640C  
0003660C  
0003720CC  
0003740C  
0003760C  
0003780C  
0003820C  
0003840C  
0003860C  
0003880C  
0003890C  
0003900C  
0003905C  
0003920CC  
0003940C  
0003960C  
0003980C  
0004020C  
0004040C  
0004060C  
0004080C  
0004090C  
0004100C  
0004105C  
0004120CC  
0004140C  
0004160C  
0004180C  
0004220C  
0004240C  
0004260C  
0004280C  
0004290C  
0004300C  
0004320C  
0004420CC  
0004440C  
0004460C  
0004480C  
0004520C  
0004540C  
0004560C

THE SINGULARITY. THE THIRD TERM INVOLVES THETA AND IS EXPLAINED IN THE LITERATURE. THE FOURTH TERM IS THE ERROR IN THE INTEGRAL DUE TO THE CHOICE OF EPSILON.

```
CALL ROMBRG( XSIGX,PHI(N),D,1.0,1.0-7,15,SI,SIM1,INDIC,I)
IF (INDIC.EQ.1) GO TO 400
FORCE(M-1,N) = SI -1.
CALL ROMBRG( XSIGX,C,PHI(N+1),1.0,1.0-7,15,SI,SIM1,INDIC,I)
IF (INDIC.EQ.1) GO TO 400
FORCE(M-1,N)=FORCE(M-1,N)+SI-1.0
      2 - (5+4*DCOS(2*THETA)-DCOS(4*THETA))/16
      3 + EPSIL*T/(4*PI*(S**2+T**2)**3)*(6*S**6+(3-2*T**2)*S**4
      4 -6*S**2*T**2-T**4)
FORCE(MM-1,NN)=FORCE(M-1,N)

CALL ROMBRG( XSIGY,PHI(N),D,1.0,1.0-7,15,SI,SIM1,INDIC,I)
IF (INDIC.EQ.1) GO TO 400
FORCE(M-1,K) = SI -1.
CALL ROMBRG( XSIGY,C,PHI(N+1),1.0,1.0-7,15,SI,SIM1,INDIC,I)
IF (INDIC.EQ.1) GO TO 400
FORCE(M-1,K)=FORCE(M-1,K)+SI-1.0
      2 +(1-DCOS(4*THETA))/16
      3 + EPSIL*T/(4*PI*(S**2+T**2)**3)*(-2*S**6-(1-6*T**2)*S**4
      4 +6*S**2*T**2-T**4)
FORCE(MM-1,KK)=FORCE(M-1,K)

CALL ROMBRG( XTAU,PHI(N),D,1.0,1.0-7,15,SI,SIM1,INDIC,I)
IF (INDIC.EQ.1) GO TO 400
FORCE(M-1,L) = SI -1.
CALL ROMBRG( XTAU,C,PHI(N+1),1.0,1.0-7,15,SI,SIM1,INDIC,I)
IF (INDIC.EQ.1) GO TO 400
FORCE(M-1,L)=FORCE(M-1,L)+SI-1.0
      2 -(2*DSIN(2*THETA)-DSIN(4*THETA))/16
      3 + EPSIL*T**2/(4*PI*(S**2+T**2)**3)*(-9*S**5-(6+2*T**2)
      4 *S**3 + T**2*S*(2-T**2))
FORCE(MM-1,LL) = -FORCE(M-1,L)

CALL ROMBRG( YSIGX,PHI(N),D,1.0,1.0-7,15,SI,SIM1,INDIC,I)
IF (INDIC.EQ.1) GO TO 400
FORCE(J-1,N) = SI -1.
CALL ROMBRG( YSIGX,C,PHI(N+1),1.0,1.0-7,15,SI,SIM1,INDIC,I)
IF (INDIC.EQ.1) GO TO 400
FORCE(J-1,N)=FORCE(J-1,N)+SI-1.0
      2 +(1-DCOS(4*THETA))/16
      3 + EPSIL*T/(4*PI*(S**2+T**2)**3)*(-S**6+(6-T**2)*S**2
      4 *T**2 + 6*S**4*T**2-2*T**4)
FORCE(JJ-1,NN)=FORCE(J-1,N)

CALL ROMBRG( YSIGY,PHI(N),D,1.0,1.0-7,15,SI,SIM1,INDIC,I)
IF (INDIC.EQ.1) GO TO 400
FORCE(J-1,K) = SI -1.
CALL ROMBRG( YSIGY,C,PHI(N+1),1.0,1.0-7,15,SI,SIM1,INDIC,I)
IF (INDIC.EQ.1) GO TO 400
FORCE(J-1,K)=FORCE(J-1,K)+SI-1.0
```

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0004580C      2      -(5-4*DCOS(2*THETA)-DCOS(4*THETA))/16
0004600C      3      + EPSIL*T/(4*PI*(S**2+T**2)**3)*(-S**6+(2-T**2)*S**2
0004620C      4      *T**2 - 6*S**4*T**2+6*T**4)
0004625C      FORCE(JJ-1,KK)=FORCE(J-1,K)
0004640CC
0004660C      CALL ROMBERG( YTAU,PHI(N),D,1.0,1.0-7,15,SI,SIM1,INDIC,I)
0004680C      IF (INDIC.EQ.1) GO TO 400
0004700C      FORCE(J-1,L) = SI -1.
0004740C      CALL ROMBERG( YTAU,C,PHI(N+1),1.0,1.0-7,15,SI,SIM1,INDIC,I)
0004760C      IF (INDIC.EQ.1) GO TO 400
0004780C      FORCE(J-1,L)=FORCE(J-1,L)+SI-1.0
0004800C      2      -(2*DSIN(2*THETA)+DSIN(4*THETA))/16
0004820C      3      - EPSIL/(4*PI*(S**2+T**2)**3)*((1-2*T**2)*S**5
0004840C      4      +(1+3*T**2)*2*T**2*S**3 + 9*S*T**4)
0004860C      FORCE(JJ-1,LL) = -FORCE(J-1,L)
0004960CC
0004980CC
0005000C200    CONTINUE
0005040CC
0005180CC
0005200CC      INSERT INFLUENCE COEFFICIENTS AND STRESSES AT INFINITY
0005220CC      INTO EVAL MATRIX ACCORDING TO HOW THEY MUST APPEAR
0005240CC      IN THE SYSTEM OF LINEAR EQUATIONS.
0005245CC
0005246CC      THETA IS AGAIN THE ANGLE BETWEEN THE NORMAL TO THE ELLIPSE
0005247CC      AND THE X AXIS BUT IS IN THE INTERVAL (-PI/2,3*PI/2).
0005248CC      THIS IS THE CORRECT VALUE OF THETA.
0005249CC
0005250C      INCREM = 2*INDEX +1
0005260CC
0005280C      DD 250 M=1,INDEX
0005300C      J=M+INDEX
0005320C      THETA = DATAN(DTAN(MIDPT(M+1))/T)
0005325C      IF (MIDPT(M+1).LE.PI/2) GO TO 230
0005326C      IF (MIDPT(M+1).GT.3*PI/2) GO TO 230
0005340CC
0005360C      THETA = THETA + PI
0005380CC
0005400C230    EVAL(M,INCREM) = -STRESS*DCOS(THETA)
0005420C      EVAL(J,INCREM) = 0.0
0005425CC
0005440C      DD 250 N=1,INDEX
0005460C      K = N+INDEX
0005480C      L = N+2*INDEX
0005500C      EVAL(M,N) = FORCE(M,N)*DCOS(THETA)
0005520C      2      + FORCE(M,L)*DSIN(THETA)
0005540CC
0005560C      EVAL(J,N) = FORCE(M,K)*DSIN(THETA)
0005580C      2      + FORCE(M,L)*DCOS(THETA)
0005600CC
0005620C      EVAL(M,K) = FORCE(J,N)*DCOS(THETA)
0005640C      2      + FORCE(J,L)*DSIN(THETA)
0005660CC
0005680C      EVAL(J,K) = FORCE(J,K)*DSIN(THETA)

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0005700C      2      + FORCE(J,L)*DCOS(THETA)
0005720CC
0005740C250  CONTINUE
0005760CC
0005780CC
0005860CC
0005880CC      CALL SOLVD SUBROUTINE TO SOLVE SYSTEM OF LINEAR EQUATIONS.
0005900CC      ON RETURN FROM SOLVD, THE PART OF THE MATRIX THAT BEFORE
0005920CC      CONTAINED THE APPLIED STRESSES WILL NOW CONTAIN THE
0005940CC      SOLUTIONS FOR RO-SUB-X AND RO-SUB-Y, THE DENSITIES OF
0005945CC      THE BODY FORCES.
0005960CC
0005980C      CALL SOLVD(EVAL,200,INCREM-1,INCREM,.5D-6,DET,TEST)
0006000C      IF (TEST.NE.0) GO TO 450
0006020CC
0006022CC
0006024C      WRITE(6,1055)
0006025C      IF (DIST.EQ.0.0) GO TO 340
0006026C      WRITE(6,1060)
0006027C      GO TO 345
0006040C340  WRITE(6,1065)
0006060C345  WRITE(6,1010)
0006080CC
0006100C      DO 350 M=1,INDEX
0006120C          J=M+INDEX
0006140C350  WRITE(6,600) M,EVAL(M,INCREM),EVAL(J,INCREM)
0006160CC
0007320CC
0007340CC      PERFORM NUMERICAL INTEGRATIONS TO DETERMINE INFLUENCE
0007360CC      COEFFICIENTS. FOR EXAMPLE, PSIGY IS THE COEFFICIENT IN
0007380CC      THE Y DIRECTION DUE TO A POINT FORCE P.
0007400CC
0007420C      INDNEW = INT(INDEX/2.0)
0007460C      X = 0.0
0007465C      IF (DIST.EQ.0.0) GO TO 360
0007480C      Y = DIST - LENGTH
0007481C      NEW = 1
0007482C      C = -PI/2.0 + EPSIL
0007485C      GO TO 365
0007490C360  Y = LENGTH
0007492C      NEW = INDNEW
0007540C      D = PI/2.0 - EPSIL
0007560CC
0007580C365  DO 380 N=1,INDNEW
0007600CC
0007620C      IF (N.EQ.NEW) GO TO 375
0007640CC
0007660C      CALL ROMBRG( XSIGX,PHI(N),PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,1)
0007680C      IF (INDIC.EQ.1) GO TO 400
0007700C      FORCE(1,N) = SI- 1.0
0007720CC
0007740C      CALL ROMBRG( YSIGX,PHI(N),PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,1)
0007760C      IF (INDIC.EQ.1) GO TO 400
0007780C      FORCE(2,N) = SI- 1.0

```

```

0007800CC
0007820CC
0007840C      GO TO 380
0007860CC
0007880CC
0007900CC      INTEGRATE OVER SINGULARITIES BY TAKING PRINCIPAL VALJES.
0007920CC
0007925C375    IF (DIST.EQ.0.0) GO TO 376
0007926C      CALL ROMBRG( XSIGX,C,PHI(N+1),1.0,1.0-7,15,SI,SIM1,INDIC,I)
0007928C      GO TO 377
0007940C376    CALL ROMBRG( XSIGX,PHI(N),0.1,0.1,0-7,15,SI,SIM1,INDIC,I)
0007960C377    IF (INDIC.EQ.1) GO TO 400
0008040C      FORCE(1,N)= SI-1.0 + 3*EPSIL*T/(2*PI)
0008060CC
0008080CC
0008100CC
0008102C      IF (DIST.EQ.0.0) GO TO 378
0008104C      CALL ROMBRG( YSIGX,C,PHI(N+1),1.0,1.0-7,15,SI,SIM1,INDIC,I)
0008106C      GO TO 379
0008120C378    CALL ROMBRG( YSIGX,PHI(N),D,1.0,1.0-7,15,SI,SIM1,INDIC,I)
0008140C379    IF (INDIC.EQ.1) GO TO 400
0008220C      FORCE(2,N) = SI-1.0 - EPSIL*T/(4*PI)
0008240CC
0008280CC
0008300C380    CONTINUE
0008320CC
0008340CC
0008400CC
0008420C      XSUM90 = STRESS
0008440CC
0008460C      DO 370 M=1,INDNEW
0008480C          J=M+INDEX
0008500C          XSUM90 = XSUM90 + 2.0 * EVAL(M,INCREM) * FORCE(1,M)
0008520C          2 + 2.0 * EVAL(J,INCREM) * FORCE(2,M)
0008560C370    CONTINUE
0008580CC
0008600CC      DETERMINE STRESS CONCENTRATION FACTOR BY DIVIDING THE EVALUATED
0008620CC      STRESS IN THE X DIRECTION AT 90 DEGREES BY THE STRESS
0008640CC      AT INFINITY.
0008660CC
0008680CC
0008700C      WRITE(6,1010)
0008720CC
0008740C      XSUM90 = XSUM90/STRESS
0008760C      WRITE(6,1085) XSUM90
0008780C      WRITE(6,1010)
0008800C      WRITE(6,1090) DET
0008820CC
0008840CC
0008860CC
0008880C500    FORMAT(14,5(1X,F10.0))
0008885 740    FORMAT(8(1X,F6.3))
0008900C555    FORMAT(14)
0008920C600    FORMAT(14,2(1X,F12.6))

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```
0008940C866  FORMAT(2(1X,F12.6))
0008960C800  FORMAT(1X,F12.6)
0008980C1000  FORMAT(1X,'SOLUTION TO ELLIPSE IN SEMIINFINITE PLATE')
0009000C1005  FORMAT(3X,'USING NISITANI BODY FORCE METHOD')
0009020C1010  FORMAT(1X)
0009040C1020  FORMAT(1X,'THE NUMBER OF DIVISIONS IN THE ELLIPSE IS ',I4)
0009060C1025  FORMAT(1X,'HALF THE WIDTH OF THE ELLIPSE IS ',F12.6)
0009080C1030  FORMAT(1X,'HALF THE HEIGHT OF THE ELLIPSE IS ',F12.6)
0009100C1035  FORMAT(1X,'THE DISTANCE FROM THE CENTRE OF ELLIPSE TO')
0009120C1037  FORMAT(3X,'THE EDGE OF THE PLATE IS ',F12.6)
0009140C1040  FORMAT(1X,'THE APPLIED STRESS AT INFINITY IS ',F12.6)
0009160C1045  FORMAT(1X,'THE VALUE OF EPSILON IN THE NUMERICAL')
0009180C1050  FORMAT(3X,'INTEGRATION IS ',F12.6)
0009200C1055  FORMAT(1X,'THE X AND Y BODY FORCE DENSITIES, STARTING')
0009220C1060  FORMAT(3X,'WITH THE BOTTOM OF THE ELLIPSE, ARE')
0009240C1065  FORMAT(3X,'WITH THE SIDE OF THE ELLIPSE, ARE')
0009260C1085  FORMAT(1X,'THE STRESS CONCENTRATION FACTOR IS ',2(F12.6,1X))
0009280C1090  FORMAT(1X,'THE DETERMINANT OF COEFFICIENT MATRIX IS ',E12.6)
0009300C1100  FORMAT(1X,'ERROR: NUMERICAL INTEGRATION DID NOT CONVERGE.')
0009320C1110  FORMAT(1X,'ERROR: COEFFICIENT MATRIX IS SINGULAR.')
0009340CC
0009360CC
0009380C      STOP
0009400CC
0009420CC
0009440C400  WRITE(6,1100)
0009460C      STOP
0009480CC
0009500CC
0009520C450  WRITE(6,1110)
0009540C      STOP
0009560CC
0009580C      END
```

### 8.2 TYPICAL INPUT

INDEX (MM) = 12

WIDTH (a) = 1.0

LENGTH (b) = 4.0

DIST (e) = 0.0

STRESS ( $\sigma_x^0$ ) = 1.0

FRAC = 0.01

FORMAT (I4, 5(IX, F10.0))

### 8.3 TYPICAL OUTPUT

SOLUTION TO ELLIPSE IN SEMIINFINITE PLATE  
USING NISITANI BODY FORCE METHOD

THE NUMBER OF DIVISIONS IN THE ELLIPSE IS 12

HALF THE WIDTH OF THE ELLIPSE IS 1.000000

HALF THE HEIGHT OF THE ELLIPSE IS 4.000000

THE DISTANCE FROM THE CENTRE OF ELLIPSE TO  
THE EDGE OF THE PLATE IS 0.000000

THE APPLIED STRESS AT INFINITY IS 1.000000

THE VALUE OF EPSILON IN THE NUMERICAL  
INTEGRATION IS 0.002618

THE X AND Y BODY FORCE DENSITIES, STARTING  
WITH THE SIDE OF THE ELLIPSE, ARE

1	10.479455	6.235629
2	10.021310	1.518940
3	9.752178	0.286842
4	9.599312	-0.247939
5	9.515675	-0.492201
6	9.478575	-0.589385
7	9.478575	-0.589385
8	9.515675	-0.492201
9	9.599312	-0.247939
10	9.752178	0.286842
11	10.021310	1.518940
12	10.479455	6.235629

THE STRESS CONCENTRATION FACTOR IS 9.499096

THE DETERMINANT OF COEFFICIENT MATRIX IS 0.139862E-16

9. PROGRAM FOR SEMI-ELLIPSE AND CRACK IN A SEMI-INFINITE PLATE (ELCRA I)

9.1 LISTING

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0000100CC PROGRAM TO COMPUTE SIF FOR CRACK IN A SEMIINFINITE PLATE
0000120CC HAVING A SEMIELLIPTICAL NOTCH.
0000140CC
0000202CC METHOD OF SOLUTION IS THE BODY FORCE METHOD (H. NISITANI).
0000203CC
0000204CC NUMERICAL INTEGRATION IS USED INSTEAD OF ANALYTICAL
0000205CC INTEGRATION TO DETERMINE INFLUENCE COEFFICIENTS.
0000210CC
0000220CC INDEX1 IS THE NUMBER OF INTERVALS INTO WHICH THE SEMIELLIPSE
0000240CC HAS BEEN PARTITIONED.
0000241CC INDEX2 IS THE NUMBER OF INTERVALS INTO WHICH THE CRACK
0000242CC HAS BEEN PARTITIONED.
0000246CC WIDTH IS HALF THE WIDTH OF THE SEMIELLIPSE ALONG
0000260CC THE X AXIS.
0000280CC LENGTH IS THE HEIGHT OF THE SEMIELLIPSE.
0000290CC CRAC IS HALF THE LENGTH OF THE CRACK.
0000300CC HEIGHT IS THE DISTANCE FROM THE CENTRE OF THE CRACK TO THE EDGE
0000305CC OF THE PLATE.
0000312CC STRESS IS THE APPLIED STRESS AT INFINITY.
0000320CC FRAC IS A NUMBER MUCH LESS THAN 1.0 USED IN THE
0000340CC NUMERICAL INTEGRATION ABOUT A SINGULARITY.
0000360CC
0000380C INTEGER INDEX1,INDEX2,INCREM,HALF1,HALF2
0000400C REAL*8 PI,WIDTH,LENGTH,X,Y,STRESS,EPSIL,S1,SIM1,C,D,PREC,BET,TEST
0000405C REAL*8 S,T,THETA,DIST,CRAC,A1,A2,A3,A4,A5,A6,A7,A8,A9
0000410C REAL*8 FRAC,HEIGHT,SIF1,SIF2
0000420C REAL*8 PHIT(101),MIDPT(101),FORCE(200,300),EVAL(300,301)
0000425C REAL*8 ETA(101),CENTRE(101),FUNC(100,101),FORCE2(100,300)
0000440C COMMON PI,X,Y /ELLIP/ WIDTH,LENGTH,DIST /CRACK/ CRAC,HEIGHT
0000460C EXTERNAL XSIGX,XSIGY,YSIGX,YSIGY,XTAU,YTAU,XCRAC,YCRAC,TALCRA
0000480CC
0000500CC
0000520C READ(5,600) INDEX1,INDEX2,WIDTH,LENGTH,CRAC,HEIGHT
0000525C READ(5,650) STRESS,FRAC
0000540CC
0000560C WRITE(6,1000)
0000580C WRITE(6,1005)
0000600C WRITE(6,1010)
0000620C WRITE(6,1015) INDEX1
0000640C WRITE(6,1010)
0000645C WRITE(6,1020) INDEX2
0000650C WRITE(6,1010)
0000660C WRITE(6,1025) WIDTH
0000680C WRITE(6,1010)
0000685C WRITE(6,1030) LENGTH
0000686C WRITE(6,1010)
0000720C WRITE(6,1035) CRAC
0000725C WRITE(6,1010)
0000730C WRITE(6,1037) HEIGHT
0000740C WRITE(6,1010)
0000760C WRITE(6,1040) STRESS
0000780C WRITE(6,1010)
0000800CC
0000820CC DEFINE PI, EPSILON, EPSILON IS USED IN INTEGRATION

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0000840CC      ABOUT SINGULARITIES ON THE SEMIELLIPSE.
0000860CC
0000880CC      PI = 3.14159265359
0000940CC      EPSIL = FRAC * PI/INDEX1
0000960CC      WRITE(6,1048)
0000980CC      WRITE(6,1050) EPSIL
0000990CC      WRITE(6,1010)
0001020CC
0001025CC      DIST = 0.0
0001030CC      T = LENGTH/WIDTH
0001040CC
0001060CC      COMPUTE VALUE OF PHI(N) FOR N=1 TO INDEX1 + 1.
0001080CC      PHI(N) AND PHI(N+1) ARE PARAMETERS FOR THE VALUE OF
0001100CC      THE ENDPOINTS OF THE NTH INTERVAL ON THE ELLIPSE:
0001120CC      X = WIDTH * COS(PHI); Y = LENGTH * SIN(PHI)
0001122CC
0001124CC      COMPUTE ETA(N) FOR N=1 TO INDEX2 + 1.
0001126CC      ETA(N) AND ETA(N+1) ARE THE Y-VALUES OF THE ENDPOINTS OF
0001128CC      THE NTH INTERVAL ON THE CRACK.
0001130CC
0001140CC
0001160CC      MIDPT(N) IS THE VALUE OF PHI AT THE MIDPOINT
0001180CC      OF THE (N-1)TH INTERVAL ON THE ELLIPSE.
0001182CC
0001184CC      CENTRE(N) IS THE Y-VALUE OF THE MIDPOINT OF THE (N-1)TH
0001186CC      INTERVAL ON THE CRACK.
0001200CC
0001220CC      X AND Y ARE THE X AND Y VALUES OF THE POINT ON THE ELLIPSE
0001240CC      HAVING MIDPT(M) AS ITS PARAMETER VALUE.
0001260CC
0001280CC      THETA IS THE ANGLE THE NORMAL ON THE ELLIPSE MAKES WITH THE X
0001300CC      AXIS. IT IS ALWAYS TAKEN TO BE IN THE INTERVAL (-PI/2,PI/2).
0001320CC      THIS IS NOT THE TRUE VALUE OF THETA IN ALL QUADRANTS, BUT
0001340CC      NO CORRECTION IS NEEDED AT THIS POINT.
0001360CC
0001380CC
0001385CC      INCREM=INDEX1+1
0001400CC      PHI(1) = 0.0
0001480CC
0001500CC      DO 100 N=2,INCREM
0001520CC          PHI(N) = PI*(N-1)/INDEX1
0001540CC          MIDPT(N) = PHI(N) - PI/(2*INDEX1)
0001560C100      CONTINUE
0001680CC
0001681CC
0001700CC      INCREM=INDEX2+1
0001720CC      ETA(1) = HEIGHT - CRAC
0001721CC
0001840CC
0001860CC      DO 150 N=2,INCREM
0001880CC          ETA(N) = ETA(1) + 2.0*CRAC*(N-1)/INDEX2
0001900CC          CENTRE(N) = ETA(N) - CRAC/INDEX2
0001920C150      CONTINUE
0001940CC

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0001960CC
0001980CC
0002000CC
0002005 C
0002010 C
0002020CC
0002040CC
0002060C
0002080C
0002100CC
0002120C
0002140C
0002160C
0002180C
0002200C
0002220C
0002240C
0002260CC
0002280C
0002300CC
0002320C
0002340C
0002360CC
0002600C
0002630C
0002640C
0002650C
0002660C
0002670C
0002700CC
0002720C
0002740CC
0002800C
0003000C
0003005C
0003006C
0003010CC
0003011C
0003013C
0003014C
0003020C
0003040CC
0003060C
0003080C
0003100C
0003105C
0003120CC
0003140C
0003160C
0003180C
0003185C
0003200CC
0003201C
0003203C

```

PERFORM NUMERICAL INTEGRATIONS TO DETERMINE INFLUENCE  
COEFFICIENTS ON THE BOUNDARY OF THE SEMIELLIPSE DUE TO  
POINT FORCES IN THE X AND Y DIRECTION ON THE ELLIPSE.  
FOR EXAMPLE, XSIGY IS THE FORCE IN THE Y DIRECTION  
DUE TO A POINT FORCE X IN THE X DIRECTION.

INCREM=INDEX1+1  
INDNEW=INT(INDEX1/2.0)

DO 200 M=2,INCREM  
J=M+INDEX1  
MM = INDEX1 -M +3  
JJ = MM + INDEX1  
X = WIDTH \* DCOS(MIDPT(M))  
Y = LENGTH \* DSIN(MIDPT(M))  
S = DTAN(MIDPT(M))  
THETA = DATAN(S/T)

C = MIDPT(M) + EPSIL  
D = MIDPT(M) - EPSIL

DO 200 N=1,INDNEW  
K=N+INDEX1  
L=N+2\*INDEX1  
NN = INDEX1 -N +1  
KK = NN + INDEX1  
LL = NN + 2\*INDEX1

IF (M-1.EQ.N) GO TO 175

CALL ROMBRG( XSIGX,PHI(N),PHI(N+1),1.0,1.0-7,10,SI,SIMI,INDIC,I)  
IF (INDIC.EQ.1) GO TO 400  
FORCE(M-1,N) = SI- 1.0  
FORCE(MM-1,NN) = SI- 1.0

CALL ROMBRG( XSIGY,PHI(N),PHI(N+1),1.0,1.0-7,10,SI,SIMI,INDIC,I)  
IF (INDIC.EQ.1) GO TO 400  
FORCE(M-1,K) = SI- 1.0  
FORCE(MM-1,KK) = SI- 1.0

CALL ROMBRG( XTAU,PHI(N),PHI(N+1),1.0,1.0-7,10,SI,SIMI,INDIC,I)  
IF (INDIC.EQ.1) GO TO 400  
FORCE(M-1,L) = SI- 1.0  
FORCE(MM-1,LL) = 1.0 - SI

CALL ROMBRG( YSIGX,PHI(N),PHI(N+1),1.0,1.0-7,10,SI,SIMI,INDIC,I)  
IF (INDIC.EQ.1) GO TO 400  
FORCE(J-1,N) = SI- 1.0  
FORCE(JJ-1,NN) = SI- 1.0

CALL ROMBRG( YSIGY,PHI(N),PHI(N+1),1.0,1.0-7,10,SI,SIMI,INDIC,I)  
IF (INDIC.EQ.1) GO TO 400

```

0003204C      FORCE(J-1,K) = SI- 1.0
0003210C      FORCE(JJ-1,KK) = SI- 1.0
0003240CC
0003260C      CALL ROMBRG( YTAU,PHI(N),PHI(N+1),1.0,1.0-7,10,SI,SIM1,INDIC,I)
0003280C      IF (INDIC.EQ.1) GO TO 400
0003300C      FORCE(J-1,L) = SI- 1.0
0003365C      FORCE(JJ-1,LL) = 1.0 - SI
0003320CC
0003620CC
0003640C      GO TO 200
0003660CC
0003680CC
0003700CC
0003720CC      INTEGRATE OVER SINGULARITIES BY TAKING PRINCIPAL VALUES.
0003740CC      THE INTEGRAL HAS FOUR PARTS: THE FIRST TWO ARE THE NORMAL
0003760CC      INTEGRALS OVER (PHI(N),D) AND (C,PHI(N+1)) WHICH AVOID
0003780CC      THE SINGULARITY. THE THIRD TERM INVOLVES THETA AND IS
0003800CC      EXPLAINED IN THE LITERATURE. THE FOURTH TERM IS THE
0003820CC      ERROR IN THE INTEGRAL DUE TO THE CHOICE OF EPSILON.
0003840C:75    CALL ROMBRG( XSIGX,PHI(N),D,1.0,1.0-7,10,SI,SIM1,INDIC,I)
0003860C      IF (INDIC.EQ.1) GO TO 400
0003880C      FORCE(M-1,N) = SI -1.
0003900C      CALL ROMBRG( XSIGX,C,PHI(N+1),1.0,1.0-7,10,SI,SIM1,INDIC,I)
0003920C      IF (INDIC.EQ.1) GO TO 400
0003940C      FORCE(M-1,N)=FORCE(M-1,N)+SI-1.0
0003960C      2      - (5+4*DCOS(2*THETA)-DCOS(4*THETA))/16
0003980C      3      + EPSIL*T/(4*PI*(S**2+T**2)**3)*((6*S**6+(3-2*T**2)*S**4
0004000C      4      -6*S**2*T**2-T**4)
0004020C      FORCE(MM-1,NN)=FORCE(M-1,N)
0004040CC
0004060C      CALL ROMBRG( XSIGY,PHI(N),D,1.0,1.0-7,10,SI,SIM1,INDIC,I)
0004080C      IF (INDIC.EQ.1) GO TO 400
0004100C      FORCE(M-1,K) = SI -1.
0004120C      CALL ROMBRG( XSIGY,C,PHI(N+1),1.0,1.0-7,10,SI,SIM1,INDIC,I)
0004140C      IF (INDIC.EQ.1) GO TO 400
0004160C      FORCE(M-1,K)=FORCE(M-1,K)+SI-1.0
0004180C      2      +(1-DCOS(4*THETA))/16
0004200C      3      + EPSIL*T/(4*PI*(S**2+T**2)**3)*(-2*S**6-(1-6*T**2)*S**4
0004220C      4      +6*S**2*T**2-T**4)
0004240C      FORCE(MM-1,KK)=FORCE(M-1,K)
0004260CC
0004280C      CALL ROMBRG( XTAU,PHI(N),D,1.0,1.0-7,10,SI,SIM1,INDIC,I)
0004300C      IF (INDIC.EQ.1) GO TO 400
0004320C      FORCE(M-1,L) = SI -1.
0004340C      CALL ROMBRG( XTAU,C,PHI(N+1),1.0,1.0-7,10,SI,SIM1,INDIC,I)
0004360C      IF (INDIC.EQ.1) GO TO 400
0004380C      FORCE(M-1,L)=FORCE(M-1,L)+SI-1.0
0004400C      2      -(2*DSIN(2*THETA)-CSIN(4*THETA))/16
0004420C      3      + EPSIL*T**2/(4*PI*(S**2+T**2)**3)*(-9*S**5-(6+2*T**2)
0004440C      4      *S**3 + T**2*S*(2-T**2))
0004460C      FORCE(MM-1,LL) = -FORCE(M-1,L)
0004480CC
0004500C      CALL ROMBRG( YSIGX,PHI(N),D,1.0,1.0-7,10,SI,SIM1,INDIC,I)
0004520C      IF (INDIC.EQ.1) GO TO 400

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0004540C      FORCE(J-1,N) = SI -1.
0004560C      CALL ROMBRG( YSIGX,C,PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
0004580C      IF (INDIC.EQ.1) GO TO 400
0004600C      FORCE(J-1,N)=FORCE(J-1,N)+SI-1.0
0004620C      2      +(1-DCOS(4*THETA))/16
0004640C      3      + EPSIL*T/(4*PI*(S**2+T**2)**3)*(-S**6+(6-T**2)*S**2
0004660C      4      *T**2 + 6*S**4*T**2-2*T**4)
0004680C      FORCE(JJ-1,NN)=FORCE(J-1,N)
0004700C
0004720C      CALL ROMBRG( YSIGY,PHI(N),D,1.0,1.D-7,10,SI,SIM1,INDIC,I)
0004740C      IF (INDIC.EQ.1) GO TO 400
0004760C      FORCE(J-1,K) = SI -1.
0004780C      CALL ROMBRG( YSIGY,C,PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
0004800C      IF (INDIC.EQ.1) GO TO 400
0004820C      FORCE(J-1,K)=FORCE(J-1,K)+SI-1.0
0004840C      2      -(5-4*DCOS(2*THETA)-DCOS(4*THETA))/16
0004860C      3      + EPSIL*T/(4*PI*(S**2+T**2)**3)*(-S**6+(2-T**2)*S**2
0004880C      4      *T**2 - 6*S**4*T**2+6*T**4)
0004900C      FORCE(JJ-1,KK)=FORCE(J-1,K)
0004920C
0004940C      CALL ROMBRG( YTAU,PHI(N),D,1.0,1.D-7,10,SI,SIM1,INDIC,I)
0004960C      IF (INDIC.EQ.1) GO TO 400
0004980C      FORCE(J-1,L) = SI -1.
0005000C      CALL ROMBRG( YTAU,C,PHI(N+1),1.0,1.D-7,10,SI,SIM1,INDIC,I)
0005020C      IF (INDIC.EQ.1) GO TO 400
0005040C      FORCE(J-1,L)=FORCE(J-1,L)+SI-1.0
0005060C      2      -(2*DSIN(2*THETA)+LSIN(4*THETA))/16
0005080C      3      - EPSIL/(4*PI*(S**2+T**2)**3)*((1-2*T**2)*S**5
0005100C      4      +(1+3*T**2)*2*T**2*S**3 + 9*S*T**4)
0005120C      FORCE(JJ-1,LL) = -FORCE(J-1,L)
0005160CC
0005165C200  CONTINUE
0005180CC
0005185CC
0005190CC
0005192CC      EVALUATE THE STRESSES ON THE ELLIPTIC INTERVALS DUE TO
0005200CC      POINT FORCES ON THE CRACK INTERVALS.
0005260CC
0005280CC
0005300C      INDNEW = INT(INDEX1/2.0) + 1
0005320CC
0005340C      DD 210 M=2,INDNEW
0005360C      MM = INDEX1 -M +3
0005380C      X = WIDTH * DCOS(MIDPT(M))
0005400C      Y = LENGTH * DSIN(MIDPT(M))
0005420C      DO 210 N=1,INDEX2
0005440C      K = N+INDEX2
0005460C      L = N+2*INDEX2
0005480CC
0005500C      CALL ROMBRG(XCRAC,ETA(N),ETA(N+1),1.0,1.D-7,20,SI,SIM1,INDIC,I)
0005520C      IF (INDIC.EQ.1) GO TO 400
0005540C      FORCE2(M-1,N) = SI - 1.0
0005560C      FORCE2(MM-1,N) = SI - 1.0
0005580CC

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```

0005600C      CALL ROMBRG(YCRAC,ETA(N),ETA(N+1),1.0,1.0-7,20,SI,SIMI,INDIC,I)
0005620C      IF (INDIC.EQ.1) GO TO 400
0005640C      FORCE2(M-1,K) = SI- 1.0
0005660C      FORCE2(MM-1,K) = SI- 1.0
0005680CC
0005700C      CALL ROMBRG(TAUCRA,ETA(N),ETA(N+1),1.0,1.0-7,20,SI,SIMI,INDIC,I)
0005720C      IF (INDIC.EQ.1) GO TO 400
0005740C      FORCE2(M-1,L) = SI- 1.0
0005760C      FORCE2(MM-1,L) = 1.0 - SI
0005780CC
0005800C210    CONTINUE
0005820CC
0005840CC
0005860CC
0005880CC      INSERT INFLUENCE COEFFICIENTS AND STRESSES AT INFINITY
0005900CC      INTO EVAL MATRIX ACCORDING TO HOW THEY MUST APPEAR
0005920CC      IN THE SYSTEM OF LINEAR EQUATIONS.
0005940CC
0005960CC      THETA IS AGAIN THE ANGLE BETWEEN THE NORMAL TO THE ELLIPSE
0005980CC      AND THE X AXIS BUT IS IN THE INTERVAL (-PI/2,PI/2).
0006000CC      THIS IS THE CORRECT VALUE OF THETA.
0006020CC
0006040C      INCREM = 2*INDEX1 + INDEX2 +1
0006060CC
0006080C      DO 250 M=1,INDEX1
0006100C          J=M+INDEX1
0006120C          THETA = DATAN(DTAN(MIDPT(M+1)))/I)
0006140C          IF (MIDPT(M+1).LE.PI/2) GO TO 230
0006160C          IF (MIDPT(M+1).GT.3*PI/2) GO TO 230
0006180CC
0006200C          THETA = THETA + P
0006220CC
0006240C230    EVAL(M,INCREM) = -STRESS*DCOS(THETA)
0006260C          EVAL(J,INCREM) = 0.0
0006280CC
0006300C          DO 240 N=1,INDEX1
0006320C              K = N+INDEX1
0006340C              L = N+2*INDEX1
0006360C              EVAL(M,N) = FORCE(M,N)*DCOS(THETA)
0006380C          2          + FORCE(M,L)*DSIN(THETA)
0006400CC
0006420C              EVAL(J,N) = FORCE(M,K)*DSIN(THETA)
0006440C          2          + FORCE(M,L)*DCOS(THETA)
0006460CC
0006480C              EVAL(M,K) = FORCE(J,N)*DCOS(THETA)
0006500C          2          + FORCE(J,L)*DSIN(THETA)
0006520CC
0006540C              EVAL(J,K) = FORCE(J,K)*DSIN(THETA)
0006560C          2          + FORCE(J,L)*DCOS(THETA)
0006580CC
0006600C240    CONTINUE
0006620C      DO 250 N=1,INDEX2
0006640C          K = N + INDEX2
0006660C          L = N + 2*INDEX2

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0006665C      I = N + 2*INDEX1
0006685C      EVAL(M,I) = FORCE2(M,N)*DCOS(THETA)
0006700C      2      + FORCE2(M,L)*DSIN(THETA)
0006705CC
0006720C      EVAL(J,I) = FORCE2(M,K)*DSIN(THETA)
0006740C      2      + FORCE2(M,L)*DCOS(THETA)
0006760CC
0006780C250  CONTINUE
0006800CC
0006820CC
0006840CC
0006860C      INCREM = INDEX2 + 1
0006880CC
0006900CC      CALCULATE FUNC(M,N). THIS IS THE ANALYTICALLY INTEGRATED
0006920CC      FUNCTION FOR THE EVALUATION OF THE CRACK INFLUENCE
0006940CC      COEFFICIENTS.
0006945CC      FUNC(M,N+1) - FUNC(M,N) WILL BE THE STRESS IN THE X
0006950CC      DIRECTION AT THE (M-1)TH CRACK INTERVAL DUE TO A POINT
0006952CC      FORCE X AT THE NTH CRACK INTERVAL.
0006954CC
0006960CC
0006980C      DO 300 M=2,INCREM
0007000CC
0007020CC      A1,A2,A3,A4,A5,A6,A7,A8,A9 ARE INTERMEDIARY FUNCTIONS.
0007040CC
0007050CC
0007060C      A1 = CENTRE(M) + HEIGHT
0007070C      A2 = CENTRE(M) - HEIGHT
0007080C      A3 = DSGRT(CRAC**2 - A2**2)
0007090C      A4 = A1**2 - CRAC**2
0007100CC
0007110CC
0007120C      DO 300 N=1,INCREM
0007130C      A5 = ETA(N) - HEIGHT
0007140C      A6 = ETA(N) + CENTRE(M)
0007150C      A7 = CENTRE(M) - ETA(N)
0007160C      A8 = DSGRT(CRAC**2 - A6**2)
0007170C      A9 = DARSIN((CRAC**2 + A1*A6)/(CRAC*A6))
0007180CC
0007190CC
0007200C      FUNC(M,N) = 1/ PI *(A8/A7 + A2/A3
0007210C      1      * DLOG(DABS((CRAC**2 - A2*A5 + A3*A8)/(CRAC*A7))))
0007220C      2      - 1/ PI *(-A8/A6 + A1/DSGRT(A4) * A9)
0007230C      3      + 5* CENTRE(M)/ PI * (-A8/A6**2 *(1 - A1*A6/A4)
0007240C      4      + CRAC**2/A4**1.5 * A9 ) - 2*CENTRE(M)**2/ PI
0007250C      5      *(A8/A6**3 *(-2 + A1*A6/A4+(2*CRAC**2 + A1**2)*A6**2/A4**2)
0007260C      6      + 3*CRAC**2 *A1/A4**2.5 * A9)
0007400CC
0007420C300  CONTINUE
0007440CC
0007460CC
0007480CC
0007485CC      THE STRESSES ON THE CRACK DUE TO THE CRACK ARE NOW INSERTED
0007490CC      IN EVAL MATRIX. ALSO EVALUATED ARE THE STRESSES ON THE

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0007492CC      CRACK IN THE X DIRECTION DUE TO POINT FORCES ON THE ELLIPSE
0007500C      IN THE X AND Y DIRECTIONS. THESE STRESSES ARE
0007520CC      XSIGX AND YSIGX WITH X=0 AND Y=CENTRE(M)
0007540CC
0007560C      INCREM = 2*INDEX1 + INDEX2 + 1
0007580C      INDNEW = INDEX1/2
0007600C      X = 0.0
0007620CC
0007640CC
0007660C      DO 500 M=1,INDEX2
0007680C      Y = CENTRE(M+1)
0007700C      J = M + 2*INDEX1
0007720C      EVAL(J,INCREM) = -STRESS
0007740CC
0007760C      DO 475 N=1,INDNEW
0007780C      K = N + INDEX1
0007800C      NN = INDEX1 -N +1
0007820C      KK = NN + INDEX1
0007840C      CALL ROMBRG(XSIGX,PHI(N),PHI(N+1),1.0,1.D-7,15,SI,SIM1,INDIC,1)
0007860C      IF (INDIC.EQ.1) GO TO 400
0007880C      EVAL(J,N) = SI - 1.0
0007900C      EVAL(J,NN) = SI - 1.0
0007920CC
0007940C      CALL ROMBRG(YSIGX,PHI(N),PHI(N+1),1.0,1.D-7,15,SI,SIM1,INDIC,1)
0007960C      IF (INDIC.EQ.1) GO TO 400
0007980C      EVAL(J,K) = SI - 1.0
0008000C      EVAL(J,KK) = SI - 1.0
0008020C475    CONTINUE
0008040CC
0008060C      DO 500 N=1,INDEX2
0008080C      L = N + 2*INDEX1
0008100C      EVAL(J,L) = FUNC(M+1,N+1) - FUNC(M+1,N)
0008120C500    CONTINUE
0008140CC
0008160CC
0008180CC
0008200CC
0008220CC
0008240CC      CALL SOLVD SUBROUTINE TO SOLVE SYSTEM OF LINEAR EQUATIONS.
0008260CC      ON RETURN FROM SOLVD, THE PART OF THE MATRIX THAT BEFORE
0008280CC      CONTAINED THE APPLIED STRESSES WILL NOW CONTAIN THE
0008300CC      SOLUTIONS FOR RO-SUB-X, RO-SUB-Y (THE DENSITIES OF THE
0008320CC      BODY FORCES ALONG THE ELLIPSE), AND THE CRACK CORRECTION
0008340CC      FACTORS.
0008360CC
0008380CC
0008400C      CALL SOLVD(EVAL,300,INCREM-I,INCREM,.5D-6,DET,TEST)
0008420C      IF (TEST.NE.0) GO TO 450
0008440CC
0008460CC
0008480C      WRITE(6,1055)
0008500C      WRITE(6,1065)

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0008560C      DO 530 M=1,INDEX1
0008580C          J=M+INDEX1
0008600C530      WRITE(6,700) M,EVAL(M,INCREM),EVAL(J,INCREM)
0008620CC
0008640C          WRITE(6,1010)
0008660C          WRITE(6,1070)
0008680C          WRITE(6,1075)
0008700CC
0008720C      DO 550 M=1,INDEX2
0008740C          K = M + 2*INDEX1
0008760C          EVAL(K,INCREM) = EVAL(K,INCREM)/STRESS
0008780C          WRITE(6,750) M,EVAL(K,INCREM)
0008800C550      CONTINUE
0008820CC
0008822 C      CALCULATE THE STRESS INTENSITY FACTORS SIF1,SIF2 AT BOTH
0008824 C          TIPS OF THE CRACK.
0008826 C
0008840CC
0008860C      II=2*INDEX1
0008880 C
0008900C      SIF1 = EVAL(II+1,INCREM) * DSQRT(PI*CRAC) * STRESS
0008920C      SIF2 = EVAL(INCREM-1,INCREM) * DSQRT(PI*CRAC) * STRESS
0008940CC
0008960CC
0008980C          WRITE(6,1010)
0009000C          WRITE(6,1080)
0009020C          WRITE(6,1085) SIF1
0009040C          WRITE(6,1080)
0009060C          WRITE(6,1088) SIF2
0009080C          WRITE(6,1010)
0009100C          WRITE(6,1090) DET
0009120CC
0009140CC
0009160CC
0009180C600      FORMAT(14,1X,14,4(1X,F10.0))
0009200C650      FORMAT(F10.0,1X,F10.0)
0009220C700      FORMAT(14,2(1X,F12.6))
0009240C740      FORMAT(8(1X,F6.3))
0009260C750      FORMAT(14,1X,F12.6)
0009280C800      FORMAT(1X,E24.18)
0009300C1000     FORMAT(1X,'SOLUTION TO CRACK IN SEMIINFINITE PLATE HAVING A ')
0009320C1005     FORMAT(3X,'SEMIELLIPTICAL NOTCH USING NISITANI BODY FORCE METHOD')
0009340C1010     FORMAT(1X)
0009360C1015     FORMAT(1X,'THE NUMBER OF DIVISIONS IN THE SEMIELLIPSE IS ',I4)
0009380C1020     FORMAT(1X,'THE NUMBER OF DIVISIONS IN THE CRACK IS ',I4)
0009400C1025     FORMAT(1X,'HALF THE WIDTH OF THE SEMIELLIPSE IS ',F12.6)
0009420C1030     FORMAT(1X,'THE HEIGHT OF THE SEMIELLIPSE IS ',F12.6)
0009440C1035     FORMAT(1X,'THE LENGTH OF THE CRACK IS ',F12.6)
0009460C1037     FORMAT(1X,'THE DISTANCE FROM THE EDGE TO CRACK CENTRE IS ',F12.6)
0009480C1040     FORMAT(1X,'THE APPLIED STRESS AT INFINITY IS ',F12.6)
0009500C1045     FORMAT(1X,'THE VALUE OF EPSILON IN THE NUMERICAL')
0009520C1050     FORMAT(3X,'INTEGRATION IS ',F12.6)
0009540C1055     FORMAT(1X,'THE X AND Y BODY FORCE DENSITIES, STARTING')
0009560C1060     FORMAT(3X,'WITH THE SIDE OF THE ELLIPSE, ARE')

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0009580C1070  FORMAT(1X,'THE CORRECTION FACTORS FOR EACH CRACK INTERVAL.')
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```
0009600C1070  FORMAT(3X,'STARTING WITH THE ONE NEAREST THE ELLIPSE, ARE')
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```
0009620C1080  FORMAT(1X,'THE STRESS INTENSITY FACTOR AT THE TIP OF')
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```
0009640C1080  FORMAT(3X,'THE CRACK NEAREST THE EDGE IS ',F12.6)
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```
0009660C1088  FORMAT(3X,'THE CRACK FURTHEST FROM THE EDGE IS ',F12.6)
```

```
0009680C1090  FORMAT(1X,'THE DETERMINANT OF COEFFICIENT MATRIX IS ',E12.6)
```

```
0009700C1100  FORMAT(1X,'ERROR: NUMERICAL INTEGRATION DID NOT CONVERGE.')
```

```
0009720C1110  FORMAT(1X,'ERROR: COEFFICIENT MATRIX IS SINGULAR.')
```

```
0009740CC
```

```
0009760CC
```

```
0009780C      STOP
```

```
0009800CC
```

```
0009820CC
```

```
0009840C400  WRITE(6,1100)
```

```
0009860C      STOP
```

```
0009880CC
```

```
0009900CC
```

```
0009920C450  WRITE(6,1110)
```

```
0009940C      STOP
```

```
0009960CC
```

```
0009980C      END
```

## 9.2 TYPICAL INPUT

INDEX 1 (M1) = 12

INDEX 2 (M2) = 12

WIDTH (a) = 1000.0\*

LENGTH (b) = 1000.0\*

CRAC (c) = 100.0\*

HEIGHT (e) = 1500.0\*

STRESS ( $\sigma_x$ ) = 1.0

FRAC = 0.05

FORMAT (I4, 1X, I4, 4(1X, F10.0)

FORMAT (F10.0, 1X, F10.0)

\* Program stops due to roundoff errors if smaller values of a,b,c and e are chosen.

## 9.3 TYPICAL OUTPUT

SOLUTION TO CRACK IN SEMIINFINITE PLATE HAVING A  
SEMIELLIPTICAL NOTCH USING NISITANI BODY FORCE METHOD

THE NUMBER OF DIVISIONS IN THE SEMIELLIPSE IS 12  
THE NUMBER OF DIVISIONS IN THE CRACK IS 12  
HALF THE WIDTH OF THE SEMIELLIPSE IS 1000.000000  
THE HEIGHT OF THE SEMIELLIPSE IS 1000.000000  
THE LENGTH OF THE CRACK IS 100.000000  
THE DISTANCE FROM THE EDGE TO CRACK CENTRE IS 1500.000000  
THE APPLIED STRESS AT INFINITY IS 1.000000  
THE VALUE OF EPSILON IN THE NUMERICAL  
INTEGRATION IS 0.013090  
THE X AND Y BODY FORCE DENSITIES, STARTING  
WITH THE SIDE OF THE ELLIPSE, ARE

1	3.036725	1.454622
2	3.172604	0.424099
3	3.265192	0.089852
4	3.335564	-0.079543
5	3.368478	-0.178077
6	3.175544	-0.148367
7	3.175544	-0.148367
8	3.368478	-0.178077
9	3.335564	-0.079543
10	3.265192	0.089852
11	3.172604	0.424099
12	3.036725	1.454622

THE CORRECTION FACTORS FOR EACH CRACK INTERVAL,  
STARTING WITH THE ONE NEAREST THE ELLIPSE, ARE

1	1.637645
2	1.625330
3	1.613797
4	1.602758
5	1.592126
6	1.581854
7	1.571910
8	1.562267
9	1.552898
10	1.543769
11	1.534818
12	1.525850

THE STRESS INTENSITY FACTOR AT THE TIP OF  
THE CRACK NEAREST THE EDGE IS 29.026504  
THE STRESS INTENSITY FACTOR AT THE TIP OF

THE CRACK FURTHEST FROM THE EDGE IS 27.044994  
THE DETERMINANT OF COEFFICIENT MATRIX IS 0.597485E-05

## 10. REFERENCES

1. Nisitani, H., 'Solutions of Notch Problems by Body Force Method', Mechanics of Fracture Vol. 5, ed. G.C. Sih, Sijthoff and Noordhoff, The Netherlands, 1978.
2. Fraga, W.E. and Hewitt, R.L., 'Implementation of Nisitani's Body Force Method for the Solution of Notch Problems', Aeronautical Note AN-17, National Research Council of Canada, October 1983.