Supplemental Material

Tesla-Scale Terahertz Magnetic Impulses

Shawn Sederberg[1*], Fanqi Kong[1], Paul B. Corkum[1]

[1]*Joint Attosecond Science Laboratory, University of Ottawa and National Research Council Canada, 25 Templeton Street, Ottawa, ON K1N 7N9 Canada*

**msederbe@uottawa.ca*

Simulation Details

Particle-in-cell calculations were performed using a full-wave, 2.5-dimensional, moving window code including plasma dispersion. Azimuthal vector beams are propagated through the gas medium using full-wave finite-difference time-domain simulations in cylindrical coordinates [32]. Briefly, this is a spatio-temporal discretization of Maxwell's equations:

$$\boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \mu_0 \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}$$

As the name implies, it is a time-domain technique that enables a closed-form calculation of the six field components at a particular time step based on the field components and current density from the previous time step. The simulation space is discretized with a radial grid spacing, Δr = 100nm and a longitudinal grid spacing, Δz = 100nm. A corresponding time-step of Δt = 190as is used to ensure numerical stability.

At each time-step and grid position, the ionization rate of the gas species is evaluated using the following expression for DC tunnel ionization [33]:

$$w(t) = 4\omega_0 \left(\frac{E_i}{E_h}\right)^{5/2} \left(\frac{E_a}{E(t)}\right) \exp\left[-\frac{2}{3}\left(\frac{E_i}{E_h}\right)^{3/2} \left(\frac{E_a}{E(t)}\right)\right],$$

where $\omega_0 = me^4/\hbar^3$ is the atomic unit of frequency, E_h = 13.6eV is the ionization potential of atomic hydrogen, E_i is the ionization potential of the atom of interest, $E_a = m^2e^5/\hbar^4$ is the atomic unit of the electric field, and $E(t)$ is the time-varying electric field incident on the medium.

Classical electron trajectories are updated at each time-step via the Lorentz force. The field components are interpolated to the position of each electron. At the end of each time-step, the existing electrons are distributed into their adjacent grid points to form a current density mapping, ***J***, in the simulation space. This solenoidal current density is used in the evaluation of the next time-step of the algorithm, resulting in the excitation of a time-dependent longitudinal magnetic field. Because Faraday's Law is accounted for in Maxwell's equations, the spatio-temporal coupling of the current density, magnetic field, and back-EMF is accurately represented.

These simulations concern only the dynamics that turn on the magnetic field, when the optical fields dominate the electron motion. Interparticle interactions such as space charge and collisions are not accounted for, but these ultimately lead to the relaxation of the solenoidal current and magnetic field.

References

31. A. Taflove, *Computational Electromagnetics: The Finite-Difference Time-Domain Method* (Artech House, 1995).

32. L. D. Landau and E. M. Lifshitz, *Quantum Mechanics* (Pergamon, 1965).