nature portfolio

Peer Review File

Effect of aberrations on 3D optical topologies

Reviewers' comments:

Reviewer #1 (Remarks to the Author):

This paper focuses on experimental and numerical studies of optical phase aberrations on linked and knotted optical fields. The researchers create knotted and linked in linear optics and investigate their robustness against perturbations.

The work appears to have been carefully performed, with visually well-constructed images of the setup and the shapes of vortex singularities. There have been numerous studies on shaping singular vortex lines into links and knots for linear optical fields that satisfy Maxwell's equations. Since these processes are well known and the linear system can be accurately and straightforwardly simulated using numerical software packages, it is not entirely obvious what additional knowledge the experiment would bring, aside from confirming the technological feasibility of creating these structures and perturbations using spatial light modulators. The more aspect lies in their experimental detection, but it seems that the images are created from simulation data(?)

The structures studied essentially involve linear optical vortices where the phase singularity takes the form of a loop or a knot. It is trivially known that such linear vortices, as well as their more complex nonlinear counterparts, undergo vortex reconnections, so claims of "mathematical stability" in this context are unsubstantiated. There are also highly exaggerated statements of linking the work to topological quantum computation and various quantum systems that could potentially only undermine the credibility of the study. Topological quantum computation proposals refer to strongly interacting systems where braiding operations are protected in parameter space. The study is rather detached from any theoretical work on knots but can, nevertheless, be of interest among researchers specifically working on structured light manipulation technology.

I would support publication, provided that any exaggerated claims and connections are removed from the presentation. The relationship between numerical simulations and experimental detection should be clarified and explicitly stated in the manuscript, with some comments more added about detection and challenges. One important point to consider in conclusions: where does the work realistically (avoiding hyperbole) lead both on short and longer timescales?

Reviewer #2 (Remarks to the Author):

Report for Effect of Aberrations on 3D optical topologies.

This article demonstrates that paraxial optical knots and links can be robust under controlled low order aberrations with a strength similar to 1. The authors show experimental results and compare with simulations. I think this article is interesting but I think it is missing a more realistic scenario for one of the knots, like propagating it through several relay systems and comparing it with a regular beam.

Specific comments

-In the introduction you mention the topological robustness of these beams. However, if you are talking about stability under phase perturbations you should mention propagation invariant fields which are probably the most stable beams under aberrations and have self-healing properties.

-As I understand your beam is shaped by the slm and the aberrations are added with the same element. This is not clear in the experimental description (page 2, lines 24-44). Please add a sentence that clarifies that point. You mention that in the Methods section but it would be better to make it clear in the main text.

- It would be good to see a more realistic scenario with the aberration in a different place of the optical path or by propagating the beam through several relay systems. You could compare the unperturbed knot or link and a Gaussian with the aberrated versions with a few errors in the optical system.

-In figures 2, 3, 4 and figures S1, S2, S3. What is the size of the box that contains your structures? It would be good to clarify in the figure caption if the results are experimental. You also do not mention the insets contained in a circle which are a different view of the knots and links.

-What happens when you have aberrations with orders greater than 5? Since you don’t mention I would assume that the knots break. But do you have an explanation for that? Why do you think you will not encounter these orders in a real application?

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The work appears to have been carefully performed, with visually well-constructed images of the setup and the shapes of vortex singularities. There have been numerous studies on shaping singular vortex lines into links and knots for linear optical fields that satisfy Maxwell's equations. Since these processes are well known and the linear system can be accurately and straightforwardly simulated using numerical software packages, it is not entirely obvious what additional knowledge the experiment would bring, aside from confirming the technological feasibility of creating these structures and perturbations using spatial light modulators. The more aspect lies in their experimental detection, but it seems that the images are created from simulation data(?)

Reply: We agree with the Reviewer about the fact that the stability of these structures can be investigated with standard free space propagation, for which one can reach arbitrary accuracy. The reason why we decided to perform the experiment, is to make sure that, when discussing robustness, this is practically observed in an experimental realization, where uncontrolled aberrations, misalignments, and additional effects (for instance, the pixelation of the SLM and the influence of finite aperture) can play a role. All these effects can also be taken into account in simulations. However, one cannot rule out the possibility of experimentally meeting a situation in which the combination of all these effects was not taken into account and gives unexpected results. Therefore, one can show robustness by simulations, but when the experiment is performed, it might not be possible to retrieve the full topology. Performing an experiment was, thus, for us a direct way to face these doubts.

As the Reviewer notes, the experimental detection can be another factor affecting the successful reconstruction of the topology. Data with insufficient resolution or scanning along a too narrow region can yield the wrong result. Our results show how, in reasonable scenarios, one does not need to go through the impractical process of adjusting the scanning volume depending on the aberration type. For each knot type, the number of observation planes and the distances between each of these were the same for all the applied distortions.

All the images shown in the main part of the manuscript are obtained from experimental data. The singular skeletons plotted are obtained after a process of sorting and joining the experimental singular points. After the sorting (which is the fundamental operation for determining the topology), the data are smoothed by a Gaussian filter to remove fast oscillations due to camera pixelation and vibrations in the setup. The only simulated results are in Figure S2 of the supplementary materials and the Fig.5 of the updated manuscript, Simulating the effects of diffraction from the SLM panels **a** and **b**.

The structures studied essentially involve linear optical vortices where the phase singularity takes the form of a loop or a knot. It is trivially known that such linear vortices, as well as their more complex nonlinear counterparts, undergo vortex reconnections, so claims of "mathematical stability" in this context are unsubstantiated.

Reply: The vortex reconnection is the main phenomenon behind the appearance of knotted structures in optical beams. However, we are unsure if we understood the reasoning of the Reviewer that made them doubt about the mathematical stability of these structures. Here, we try to clarify our understanding of the topic.

From a mathematical point of view, knots are objects to which one can attribute a topological invariant (which can be either the number of crossings or more advanced objects such as the Alexander and Jones polynomials). Topological objects are "mathematically stable" in the sense that, to alter their topology (i.e., changing the associated invariant), one must apply a transformation going through a singular point in the parameter space (for example, a deformation that makes the knot self-intersecting), i.e., a point where the invariant itself is ill-defined. Optical knots are curves formed by the union of the trajectories, in free space, of many vortex singularities that split or annihilate at different propagation planes. The same above-mentioned topological invariants can be calculated for these structures. The possibility of associating an invariant to a physical object is what makes us attribute to it a feature of mathematical stability, which, however, does not necessarily translate into *physical* stability. This is the main point that we address in our manuscript. Physical perturbation can be, in principle, strong enough to make any mathematical stability irrelevant for all practical purposes. Before our work, the question concerning '*how strong should an optical aberration be in order to alter the topology of an optical knot?*' was still unaddressed.

Another feature that makes us consider the optical knots as mathematically stable objects is the fact that they are trajectories traced by elementary vortex singularities, i.e., singularities with the lowest possible topological charge. These singularities are also mathematically stable since they cannot split into lower-order ones. Hence, the knot formed by these singularities cannot unfold into multiple curves due to the splitting of these singularities (as it would happen if, instead, one tried to generate knots with high-order optical vortices or V-points). The topology may be, however, affected by the appearance of additional vortex-antivortex pairs in the propagation that link or join with the original curve, thus changing its topology. Nevertheless, our observations show that this effect can be observed only under particularly strong aberrations, thus only if altering strongly the wavefront of the beam.

There are also highly exaggerated statements of linking the work to topological quantum computation and various quantum systems that could potentially only undermine the credibility of the study. Topological quantum computation proposals refer to strongly interacting systems where braiding operations are protected in parameter space.

Reply: We understand and thank the Reviewer for pointing this out. We recognize now that the link between topological quantum computation and optical knots is too weak and mainly consistent in some common mathematical tools. We have removed any reference to this topic in the new version of the manuscript.

The study is rather detached from any theoretical work on knots but can, nevertheless, be of interest among researchers specifically working on structured light manipulation technology.

Reply: The purpose of this study was indeed to understand the limits and possibilities of practical applications of knotted curves generated with paraxial structured light.

I would support publication, provided that any exaggerated claims and connections are removed from the presentation. The relationship between numerical simulations and experimental detection should be clarified and explicitly stated in the manuscript, with some comments more added about detection and challenges. One important point to consider in conclusions: where does the work realistically (avoiding hyperbole) lead both on short and longer timescales?

Reply: We hope to have properly addressed these points in our revised manuscript. In the new version of the manuscript, we also added simulation results about the effect on the structure of aberrations applied in multiple, separate planes, an effect that is important when considering communication in turbulent channels or through multiple aberration planes. Based on these findings, we added in the conclusions the following sentence:

> "*This work addressed one important question for what concerns using optical knots for communications. It was shown that these structures can be altered without affecting their topology, and thus the encoded information, when propagating in media that introduce relatively strong aberrations. However, other practical challenges need attention, starting on the practical implementation of higher order structures, and a fast and reliable reconstruction. The first challenge can be faced either by the nested knot approach, which allows to scale the prime number encoding by accessing multiple wavelengths. The second challenge can be addressed by introducing single or few-planes knot detection based either on the post-processed digital propagation of the detected amplitude and phase or on phase retrieval from multiple plane intensity detection.*"

Reviewer #2 (Remarks to the Author):

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This article demonstrates that paraxial optical knots and links can be robust under controlled low order aberrations with a strength similar to 1. The authors show experimental results and compare with simulations. I think this article is interesting but I think it is missing a more realistic scenario for one of the knots, like propagating it through several relay systems and comparing it with a regular beam.

Reply: We thank the Reviewer for their positive comments and suggestions. We now included a simulation analysis where the knot propagates through several, equally spaced phase masks obtained by random superpositions of elementary aberrations. Knot survival can still be observed in some configurations despite the strong deformations applied. We report some more details below.

Specific comments

-In the introduction you mention the topological robustness of these beams. However, if you are talking about stability under phase perturbations you should mention propagation invariant fields which are probably the most stable beams under aberrations and have self-healing properties.

Reply: We thank the reviewer for pointing out this interesting application. We would, however, like to point out how self-healing beams, like the Bessel beams, in our understanding, are robust under the effect of strong obstacles, e.g. opaque apertures. Still, it is not clear to us what would be the influence of aberrations that can distort the wavefront in such a way as to eliminate their "self-healing" properties. Indeed, in J. Opt. Soc. Am. A **35**, 1021-1027 (2018); it is shown how, for what concerns transparent phase screens, Bessel beams do not exhibit the *self-healing* properties observed in the case of opaque apertures (more precisely, it is noted how "strong aberrations may show self-healing while weak aberrations will not"). In Opt. Express **26**, 26946-26960 (2018); it has been shown that vector Bessel beams can be exploited in Quantum Key Distribution schemes and yield a better QBER (compared with LG modes) when a circular obstacle is placed in the communication channel. However, the OAM information encoding in Bessel beams can be easily affected by environmental distortions that break cylindrical symmetry and in any other kind of OAM carrying beam, in agreement with J. Opt. Soc. Am. A **35**, 1021-1027 (2018).

-As I understand your beam is shaped by the SLM and the aberrations are added with the same element. This is not clear in the experimental description (page 2, lines 24-44). Please add a sentence that clarifies that point. You mention that in the Methods section but it would be better to make it clear in the main text.

Reply: We thank the Reviewer for the suggestion. We clarified this point in the caption of Figure 1 and in the main text:

> *"Aberrations effects can be introduced in a controlled way adding on the same SLM the aberration phase to the knot hologram."*

- It would be good to see a more realistic scenario with the aberration in a different place of the optical path or by propagating the beam through several relay systems. You could compare the unperturbed knot or link and a Gaussian with the aberrated versions with a few errors in the optical system.

Reply: We thank the reviewer for this suggestion. One situation in which the aberrations are introduced in multiple propagation planes arises when considering a turbulent channel. This is of particular interest when considering the propagation of optical knots in air for communication purposes. Although we already considered this situation in an isoplanatic approximation (where the effect of turbulence is encoded as a phase distortion introduced in a single plane), we believe that going beyond this limit is an important step. We thus performed some simulations where multiple-phase masks were applied in the propagation channel. We considered a few different scenarios, in two of them, the phase masks were generated according to the theory presented by Noll describing the statistics of Zernike decomposition in the case of Kolmogorov turbulence. The spacing between different planes was chosen to be equal to the Fried parameter, which gives the average size of turbulent eddies (cells of uniform refractive index). We also considered two other cases in which the masks were obtained as superpositions of Zernike polynomials with random coefficients having either zero mean or being uniformly distributed between zero and 1. The results are described in the text as follows and have been added to the manuscript:

> *"For instance, turbulent channels are better modeled by considering that phase distortions are not introduced in a single plane. One can ask if a knot field propagating through several random phase masks can still keep its topology unaltered. We carried a few simulations of this kind for some examples in which we considered the propagations through multiple, equally spaced phase masks. The masks were obtained as superpositions of Zernike polynomials with coefficients extracted from a Gaussian distribution with zero mean and standard deviation calculated based on Ref. \cite{noll1976zernike}. In our simulations, we chose typical values of the Fried parameter $r_0=180, 10$ mm and the spacing between the planes $\Delta z=r_0$. The field was numerically propagated over a range of 2×75 cm (corresponding to twice the focal length $f_1=f_2$ of the lenses shown in Fig. 6. In these configurations, we observed the survival of cinquefoil knots. This is somewhat not surprising since the individual phase masks introduce relatively weak phase distortions. We thus expect that moderate levels of turbulence, which can be compensated by an adaptive optics system, will not present a serious obstacle to the transmission of either classical or quantum information by means of three-dimensional optical topological structures. We also considered two other cases simulating a generic system with multiple wavefront-distortion-inducing slabs (not related to turbulence models): first the variance of the normal distribution (with zero mean) of the coefficients was increased to 1, second we considered random strengths values uniformly distributed between 0 and 1 for the first 9 Zernike polynomials. We observed a full knot with 10 aberration masks for both of these cases."*

A new figure (Fig. 5) summarises the results of these simulations and has been added to the manuscript.

-In figures 2, 3, 4 and figures S1, S2, S3. What is the size of the box that contains your structures? It would be good to clarify in the figure caption if the results are experimental. You also do not mention the insets contained in a circle which are a different view of the knots and links.

Reply: Each knot is plotted in a box of approximately 0.5mm x 0.5mm x 500mm. These values can change for different knots. For example, the Cinquefoil is much more compact than Trefoil or Hopflink (we inserted the precise numbers in the figure caption). For the reconstruction of each structure we took 80 measurements, each of them consisting of 4 interference images plus the intensity of the knotted beam.

-What happens when you have aberrations with orders greater than 5? Since you don't mention it, I would assume that the knots break. But do you have an explanation for that? Why do you think you will not encounter these orders in a real application?

Reply: Looking at higher order aberration phase masks, we can see that, by increasing the order of the aberration, just as we discussed for $\square < \square_{\square}$ (The critical aperture), the variations in the phase become more significant around the singularities. Therefore, they experience bigger "kicks" in their trajectory and the topology starts to break up. This means increasing the order, increasing the strength or decreasing the aperture show similar effects on the knots and link mainly coming from the effect of the local phase distortions on the wavevector of the singularities.

We stressed this point out in the following sentence,

> "*Beyond the 4th order, for the chosen maximum apertures we observed breakups for some specific aberrations. This is not surprising, since, to a first approximation, these distortions resemble coma aberrations with higher strength.*"

Moreover, we added in the supplementary a figure summarizing the results for some cases (up to order 9) and illustrating how the effect of some higher order aberrations is expected to be similar of lower order ones with similar patterns but reduced aperture.

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Reviewer #1 (Remarks to the Author):

The authors have improved the manuscript. Topological stability has a different practical meaning in linear and nonlinear system. The knots are line singularities whose shape is not protected in nonlinear light fields because there is nothing protecting against singularities crossing, so the knots will reconnect to simpler line singularities in the presence of nonlinearities. The authors may consider commenting this when addressing any nonlinear media, e.g, communication fibres.

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Reviewer #2 (Remarks to the Author):

The authors have answered my comments and revised the manuscript.
In my opinion the manuscript is ready for publication.

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Reply: We thank the reviewer for their positive feedback. We added the following sentence in the conclusions to address the case of nonlinearities: *We also note that results in the nonlinear regime can be expected to be completely different. Effects such as self-focusing may cause singularity crossing and thus the transition to trivial line singularities.*

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Reply: We thank the reviewer for their positive evaluation.