Trajectory of a lifeboat in a surface wave
Raman-Nair, W.; Billard, R.; Veitch, B.; Simões Ré, A.

Publisher's version / Version de l'éditeur:

7th Canadian Marine Hydromechanics and Structures Conference [Proceedings], 2005

Trajectory of a lifeboat in a Surface Wave

W. Raman-Nair[1] , R. Billard[2], B. Veitch[2] and A. Simoes-Re[1]

[1] *Institute for Ocean Technology, National Research Council Canada, St. John's, NL, A1B 3T5, Canada*

[2] *Memorial University of Newfoundland, St. John's, NL, A1B 3T5, Canada*

Email: *wayne.raman-nair@nrc-cnrc.gc.ca*

Abstract

The three-dimensional trajectory of a small lifeboat in a surface wave is computed via the methods of Lagrangian dynamics. It is assumed that the motion normal to the wave surface is small and can be neglected, i.e. the boat moves along the propagating wave profile. Wave diffraction and reflection are also assumed to be negligible. A Stokes' second order wave is used and the wave forces are applied using Morison's equation for a body in accelerated flow. Wind loads are similarly modeled using drag coefficients. The equations are solved numerically for various initial conditions in a typical severe sea state. The model is expected to be useful for predicting the motions of small bodies such as bergy bits and lifeboats in waves.

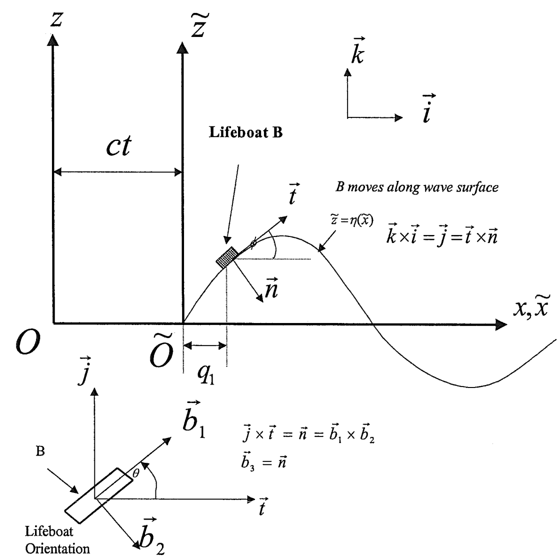

Figure 1: Problem Definition

1 Introduction

The present study is motivated by the research programmes at the National Research Council of Canada dealing with the motions of small bodies such as lifeboats and bergy bits in severe seas. The first attempt at addressing the problem seems to be Rumer et al.[1] who derived a slope sliding model for predicting ice transport in waves. However, as pointed out by Grotmaack [2], the model of Rumer et al.[1] does not account for the normal component of the body's acceleration as it moves along the curved wave profile. The problem was also considerd by Marchenko[3] who used a vector based approach but neglected the inertia aspects of the wave loads on the body. A thorough comparison of these models was presented by Grotmaack [2]. In the above models the body is considered to be a point mass and the motion is two-dimensional, i.e. confined to a vertical plane. Here we consider the rotational inertia of the boat as well as the three dimensional nature of the trajectory on the wave surface. The governing equations are derived using Lagrange's equations of motion. It is assumed that the body's dimensions are small relative to the wavelength so that wave reflection and diffraction are negligible. We also assume that the motion of the body normal to the wave surface is small and can be neglected. Numerical results are presented for various initial conditions in a typical severe sea state.

2 Equations of Motion

The problem is illustrated in Fig. 1 Point O is the origin of a fixed inertial coordinate system with x and z axes, and a small boat B moves along the surface of a wave which is propagating in the positive x direction with speed c. Point $\widetilde{O}$ is the origin of a coordinate system (axes $\widetilde{x}, \widetilde{y}, \widetilde{z}$) moving with the wave speed c in the positive x direction. The unit vectors of both systems are $\mathbf{i}, \mathbf{j}, \mathbf{k}$ in x, y, z directions respectively and $\widetilde{x} = x - ct$ where t is time. The equation of the wave profile in the moving coordinate system is

$$\widetilde{z} = \eta\left(\widetilde{x}\right) \tag{1}$$

where η is a specified function. The unit tangential and normal vectors to the wave surface at point $(\widetilde{x}, \eta)$ in the moving coordinate system are denoted by $\mathbf{t}$ and $\mathbf{n}$ respectively. Fig. 1 also illustrates the orientation of B relative to the wave tangent $\mathbf{t}$. The unit vectors $\mathbf{b}_1, \mathbf{b}_2$ and $\mathbf{b}_3$ are fixed in B and the $\mathbf{b}_1 - \mathbf{b}_2$ plane is parallel to the wave surface. The mean boat heading is specified by the angle θ (assumed constant) measured anticlockwise from $\mathbf{t}$. If boat B is at point $(\widetilde{x}, \widetilde{y}, \widetilde{z})$ on the wave surface (relative to the moving coordinate system), its position vector relative to the fixed coordinate system is

$$\mathbf{r}\left(t\right) = \left(q_1 + ct\right)\mathbf{i} + q_2\mathbf{j} + \eta\left(q_1\right)\mathbf{k} \tag{2}$$

where $q_1\left(t\right) = \widetilde{x}\left(t\right)$ and $q_2\left(t\right) = \widetilde{y}\left(t\right)$ are the generalised coordinates of the motion.. We denote differentiation with respect to q_1 and t by primes and overdots respectively. The absolute velocity of B is

$$\mathbf{v} = \left(\dot{q}_1 + c\right)\mathbf{i} + \dot{q}_2\mathbf{j} + \eta'\dot{q}_1\mathbf{k} \tag{3}$$

where $\eta' = \frac{\partial \eta}{\partial q_1}$ and $\dot{q}_1 = \frac{dq_1}{dt}$. The unit vectors $\mathbf{t}, \mathbf{n}$ are given by

$$\mathbf{t} = \frac{\widetilde{\mathbf{r}}'}{|\widetilde{\mathbf{r}}'|} = \frac{\mathbf{i} + \eta'\mathbf{k}}{\sqrt{1 + (\eta')^2}} \tag{4}$$

$$\mathbf{n} = \frac{\mathbf{t}'}{|\mathbf{t}'|} = \frac{(-\eta'\mathbf{i} + \mathbf{k})\,\mathrm{sign}\,(\eta'')}{\sqrt{1 + (\eta')^2}} \quad (\eta'' \neq 0) \tag{5}$$

where $\widetilde{\mathbf{r}}$ is the boat's position vector relative to the moving $\widetilde{x} - \widetilde{z}$ coordinate system. If ϕ is the angle made by the tangent vector $\mathbf{t}$ with the positive x direction, the angular velocity of the boat is

$$\boldsymbol{\omega} = -\dot{\phi}\mathbf{j} = -\left\{\frac{\eta''\dot{q}_1}{1 + (\eta')^2}\right\}\mathbf{j} \tag{6}$$

The Lagrangian L is the difference between the kinetic and potential energies and is written as

$$L = \frac{1}{2}m\left|\mathbf{v}\right|^2 + \frac{1}{2}\left\{\omega\right\}^T\left[I\right]\left\{\omega\right\} - mg\eta \tag{7}$$

where $\{\omega\}$ and $[I]$ are respectively the angular velocity vector and inertia matrix of the boat in the B frame (unit vectors $\mathbf{b}_i$), m is the mass of the boat and g is the acceleration due to gravity. In terms of the generalised coordinates (7) becomes

$$L = \frac{1}{2}m\left\{\left(\dot{q}_1 + c\right)^2 + \dot{q}_2^2 + \left(\eta'\dot{q}_1^2\right)^2\right\} + \alpha f\left(q_1\right)\dot{q}_1^2 - mg\eta \tag{8}$$

where

$$f\left(q_1\right) = \frac{1}{2}\left(\frac{\eta''}{1 + (\eta')^2}\right)^2 \tag{9}$$

$$\alpha = I_{11}\sin^2\left(\theta\right) + I_{22}\cos^2\left(\theta\right) \tag{10}$$

Here, I_{11} and I_{22} are the moments of inertia of B about the $\mathbf{b}_1, \mathbf{b}_2$ axes respectively. Let the net non-conservative external force on B be $\mathbf{F}^E$ and let its components in the $\mathbf{t}, \mathbf{n}$ and $\mathbf{j}$ directions be F_t^E,F_n^E and F_y^E respectively, i.e.

$$\mathbf{F}^E = F_t^E\mathbf{t} + F_n^E\mathbf{n} + F_y^E\mathbf{j} \tag{11}$$

The virtual work of the non-conservative force $\mathbf{F}^E$ is

$$\delta W_{nc} = \mathbf{F}^E \cdot \boldsymbol{\delta}\mathbf{r} \tag{12}$$

where the virtual displacement $\delta\mathbf{r}$ is given as

$$\delta\mathbf{r} = \boldsymbol{\delta} q_1(\mathbf{i} + \eta'\mathbf{k}) + \boldsymbol{\delta} q_2\mathbf{j} \tag{13}$$

Using (11), (4) and (5) we write (12) as

$$\delta W_{nc} = Q_1\delta q_1 + Q_2\delta q_2 \tag{14}$$

where the generalised non-conservative forces Q_1, Q_2 are

$$Q_1 = F_t^E\sqrt{1 + (\eta')^2} \;\; ; \;\; Q_2 = F_y^E \tag{15}$$

The equations of motion are

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = Q_k \;\; ; \;\; (k = 1, 2) \tag{16}$$

This is written from (8) and (15) as

$$\ddot{q}_1\left\{m\left[1 + (\eta')^2\right] + 2\alpha f\right\} + \dot{q}_1^2\left\{m\eta'\eta'' + \alpha f'\right\} + mg\eta' = F_t^E\sqrt{1 + (\eta')^2} \tag{17}$$

$$m\ddot{q}_2 = F_y^E \tag{18}$$

We write the external force $\mathbf{F}^E$ as the sum of forces due to waves ($\mathbf{F}^{\text{wave}}$), wind ($\mathbf{F}^{\text{wind}}$) and propulsion ($\mathbf{F}^P$) so that

$$F_t^E = \mathbf{F}^{\text{wave}} \cdot \mathbf{t} + \mathbf{F}^{\text{wind}} \cdot \mathbf{t} + \mathbf{F}^P \cdot \mathbf{t} \quad (19)$$
$$F_y^E = \mathbf{F}^{\text{wave}} \cdot \mathbf{j} + \mathbf{F}^{\text{wind}} \cdot \mathbf{j} + \mathbf{F}^P \cdot \mathbf{j} \quad (20)$$

The wave force $\mathbf{F}^{\text{wave}}$ is written as (Sumer and Fredsoe[4])

$$\mathbf{F}^{\text{wave}} = \mathbf{F}^{FK} + \mathbf{F}^A + \mathbf{F}^D \quad (21)$$

where $\mathbf{F}^{FK}$ is the Froude-Krylov force, $\mathbf{F}^A$ is the added mass force and $\mathbf{F}^D$ is the wave drag force. Let $\mathbf{a}$ be the acceleration of B, and let $\mathbf{v}_w, \mathbf{a}_w$ be the water particle velocity and acceleration respectively at B. The x and z components of $\mathbf{v}_w, \mathbf{a}_w$ are denoted by (u, w) and (a_x, a_z) respectively. The Froude-Krylov force is $\mathbf{F}^{FK} = \rho V_B \mathbf{a}_w$ which gives

$$\mathbf{F}^{FK} \cdot \mathbf{t} = \rho V_B \left(a_x + \eta' a_z\right) Z^{-\frac{1}{2}} \quad (22)$$
$$\mathbf{F}^{FK} \cdot \mathbf{j} = 0 \quad (23)$$

where ρ is the water density, V_B is the submerged volume of B and

$$Z = 1 + (\eta')^2 \quad (24)$$

We refer to the reference frame with unit vectors $\mathbf{t}, \mathbf{n}, \mathbf{j}$ attached to B by the superscript W. The added mass force in this frame is

$$\left\{{}^W F^A\right\} = -\left[{}^W M^A\right] \left\{{}^W a^R\right\} \quad (25)$$

where $\left[{}^W M^A\right]$ is the added mass matrix of B and $\left\{{}^W a^R\right\}$ is the acceleration of B (column vector) relative to the water, both in the W frame. The components of $\mathbf{F}^A$ in the $\mathbf{t}$ and $\mathbf{j}$ directions are found from (25) as

$$\mathbf{F}^A \cdot \mathbf{t} = -\beta_1 \ddot{q}_1 - \beta_2 \ddot{q}_2 + \beta_3 \quad (26)$$
$$\mathbf{F}^A \cdot \mathbf{j} = -\gamma_1 \ddot{q}_1 - \gamma_2 \ddot{q}_2 + \gamma_3 \quad (27)$$

where

$$\begin{aligned} \beta_1 &= Z^{\frac{1}{2}}\, {}^W m_{11} \\ \beta_2 &= {}^W m_{12} \\ \beta_3 &= Z^{-\frac{1}{2}} \left(a_x + \eta' a_z - \eta' \eta'' \dot{q}_1^2\right)\, {}^W m_{11} \end{aligned} \quad (28)$$

and

$$\begin{aligned} \gamma_1 &= Z^{\frac{1}{2}}\, {}^W m_{12} \\ \gamma_2 &= {}^W m_{22} \\ \gamma_3 &= Z^{-\frac{1}{2}} \left(a_x + \eta' a_z - \eta' \eta'' \dot{q}_1^2\right)\, {}^W m_{22} \end{aligned} \quad (29)$$

Here

$$ {}^W m_{11} = \left(m_{11} \cos^2\theta + m_{22} \sin^2\theta\right) \quad (30)$$
$$ {}^W m_{12} = \left(m_{11} - m_{22}\right) \sin\theta \cos\theta \quad (31)$$
$$ {}^W m_{22} = \left(m_{11} \sin^2\theta + m_{22} \cos^2\theta\right) \quad (32)$$

and m_{11}, m_{22} are the added masses of B in directions $\mathbf{b}_1, \mathbf{b}_2$ respectively. The wave drag force is written in the B frame as

$$\mathbf{F}^D = {}^B F_1^D \mathbf{b}_1 + {}^B F_2^D \mathbf{b}_2 \quad (33)$$

and the components are evaluated for $i = 1, 2$ as

$${}^B F_i^D = -\frac{1}{2} \rho C_i^D A_i^B \left(\mathbf{v} - \mathbf{v}_w\right) \cdot \mathbf{b}_i \left|\left(\mathbf{v} - \mathbf{v}_w\right) \cdot \mathbf{b}_i\right| \quad (34)$$

where $A_i^B, (i = 1, 2)$ is the projected wetted area of B normal to $\mathbf{b}_i$ and C_i^D is the associated drag coefficient. This is written in terms of the generalised coordinates using

$$\begin{aligned} (\mathbf{v} - \mathbf{v}_w) \cdot \mathbf{b}_1 &= \left(\dot{q}_1 Z + c - u - \eta' w\right) Z^{-\frac{1}{2}} \cos\theta \\ &\quad + \dot{q}_2 \sin\theta \quad (35) \\ (\mathbf{v} - \mathbf{v}_w) \cdot \mathbf{b}_2 &= \left(\dot{q}_1 Z + c - u - \eta' w\right) Z^{-\frac{1}{2}} \sin\theta \\ &\quad - \dot{q}_2 \cos\theta \quad (36) \end{aligned}$$

The components of $\mathbf{F}^D$ in the $\mathbf{t}$ and $\mathbf{j}$ directions are then given by

$$\mathbf{F}^D \cdot \mathbf{t} = {}^B F_1^D \cos\theta + {}^B F_2^D \sin\theta \quad (37)$$
$$\mathbf{F}^D \cdot \mathbf{j} = {}^B F_1^D \sin\theta - {}^B F_2^D \cos\theta \quad (38)$$

The components of the wind drag in the B frame are similarly written for $i = 1, 2$ as

$${}^B F_i^{\text{wind}} = -\frac{1}{2} \rho_{\text{air}} C_i^{\text{wind}} A_i^{\text{wind}} \mathbf{u}_R \cdot \mathbf{b}_i \left|\mathbf{u}_R \cdot \mathbf{b}_i\right| \quad (39)$$

where $\mathbf{u}_R = \mathbf{v} - \mathbf{v}_{\text{wind}}$ is the velocity of the body relative to the wind, A_i^{wind} is the projected area of B normal to $\mathbf{b}_i$ exposed to the wind and C_i^{wind} is the associated wind drag coefficient. This is written in terms of the generalised coordinates using

$$\begin{aligned} \mathbf{u}_R \cdot \mathbf{b}_1 &= \left(\dot{q}_1 Z + c - v_x^{\text{wind}}\right) Z^{-\frac{1}{2}} \cos\theta \\ &\quad + \left(\dot{q}_2 - v_y^{\text{wind}}\right) \sin\theta \quad (40) \\ \mathbf{u}_R \cdot \mathbf{b}_2 &= \left(\dot{q}_1 Z + c - v_x^{\text{wind}}\right) Z^{-\frac{1}{2}} \sin\theta \\ &\quad - \left(\dot{q}_2 - v_y^{\text{wind}}\right) \cos\theta \quad (41) \end{aligned}$$

The components of $\mathbf{F}^{\text{wind}}$ in the $\mathbf{t}$ and $\mathbf{j}$ directions are given by

$$\mathbf{F}^{\text{wind}} \cdot \mathbf{t} = {}^B F_1^{\text{wind}} \cos\theta + {}^B F^{\text{wind}} \sin\theta \quad (42)$$
$$\mathbf{F}^{\text{wind}} \cdot \mathbf{j} = {}^B F_1^{\text{wind}} \sin\theta - {}^B F_2^{\text{wind}} \cos\theta \quad (43)$$

Similar expressions hold for the propulsive thrust components $\mathbf{F}^P \cdot \mathbf{t}$ and $\mathbf{F}^P \cdot \mathbf{j}$. We can now write equations (19) and(20) as

$$
\begin{aligned}
F_t^E &= -\beta_1 \ddot{q}_1 - \beta_2 \ddot{q}_2 + \psi_t && (44)\\
F_y^E &= -\gamma_1 \ddot{q}_1 - \gamma_2 \ddot{q}_2 + \psi_y && (45)
\end{aligned}
$$

where

$$
\begin{aligned}
\psi_t &= \beta_3 + \mathbf{F}^{FK} \cdot \mathbf{t} + \mathbf{F}^D \cdot \mathbf{t} + \mathbf{F}^{\text{wind}} \cdot \mathbf{t} \\
&\quad + \mathbf{F}^P \cdot \mathbf{t} && (46)\\
\psi_y &= \gamma_3 + \mathbf{F}^{FK} \cdot \mathbf{j} + \mathbf{F}^D \cdot \mathbf{j} + \mathbf{F}^{\text{wind}} \cdot \mathbf{j} \\
&\quad + \mathbf{F}^P \cdot \mathbf{j} && (47)
\end{aligned}
$$

From (17),(18),(44) and (45) the equations of motion are written as

$$
\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} \qquad (48)
$$

where

$$
\begin{array}{ll} a_{11} = mZ + 2\alpha f + \beta_1 Z^{\frac{1}{2}} & a_{12} = \beta_2 Z^{\frac{1}{2}} \\ a_{21} = \gamma_1 & a_{22} = m + \gamma_2 \end{array} \qquad (49)
$$

and

$$
\begin{array}{l} d_1 = Z^{\frac{1}{2}} \psi_t - \dot{q}_1^{\,2} \left(m\eta'\eta'' + \alpha f' \right) - mg\eta' \\ d_2 = \psi_y \end{array} \qquad (50)
$$

Equation (48) is converted to an equivalent set of first order equations and solved numerically using the Runge-Kutta routine "ode45" of MATLAB, subject to specified initial conditions.

3 Results

We consider the motion of a typical fully loaded Totally Enclosed Motor Propelled Survival Craft (TEMPSC) of mass $m = 12,500$ kg in a wave of length $\lambda = 190$ m, height $H = 7.6$ m, period $T = 11$ sec moving in the positive x direction with speed $c = 17.2$ m/s in water of depth 500 m. The wind speed is 19 m/s in the x direction. These conditions are representative of a Beaufort 8 sea state. The boat geometry is approximated by a cylinder of length 10 m and radius 1.64 m. The added mass coefficient for motion along either axis is taken as 0.8. Drag coefficients are estimated as 1.2 for water and 1.0 for air. The body areas exposed to air and water are assumed to be in a 2:1 ratio. The boat heading is $\theta = 150^0$ under a propulsive force which is linearly ramped from 0 to 5000 N over a 15 sec interval. The boat heading is therefore at an angle of 30^0 against the wave direction. The wave profile and fluid velocity and acceleration fields are obtained from the standard formulae for a Stokes second order wave (Wilson[5]). The wave profile is illustrated in Fig. 2. The initial conditions are $q_1(0) = \frac{\lambda}{8}$, $\dot{q}_1(0) = -c$, $q_2(0) = 0$, $\dot{q}_2(0) = 0$. The starting condition thus corresponds to point B in Fig. 2 with zero forward speed. The boat trajectory is illustrated in figures 3 and 4.

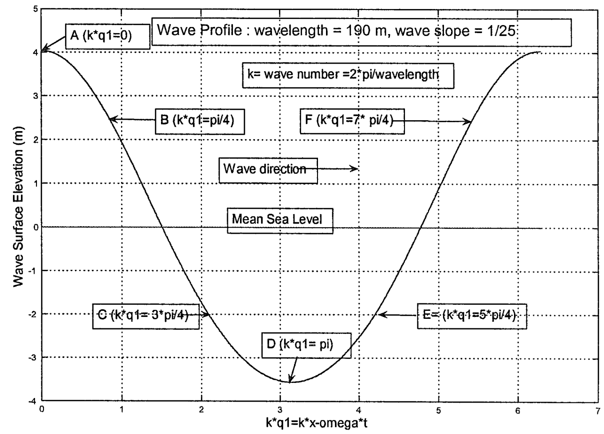

Figure 2: Wave Profile

3.1 Lifeboat Setback

We consider that the wave is propagating toward the launching platform or vessel and the possibility of collision with the platform presents a significant safety hazard. To quantify this hazard, we define the term *setback* to be the distance that the lifeboat is carried in the wave direction (and therefore toward the platform) before it's direction is reversed and it is propelled to safety. The setback distance depends on the initial positiion on the wave profile. For instance, Fig. 3 illustrates a setback distance of about 3 m. Figures 5 and 6 illustrate the boat trajectories starting from the points A to F shown in Fig. 2. When the boat starts from A or F, there is no setback. However, starting positions such as C, D or E will result in significant initial travel in the wave direction and increase the possibility of catastrophic collision with the launching platform.

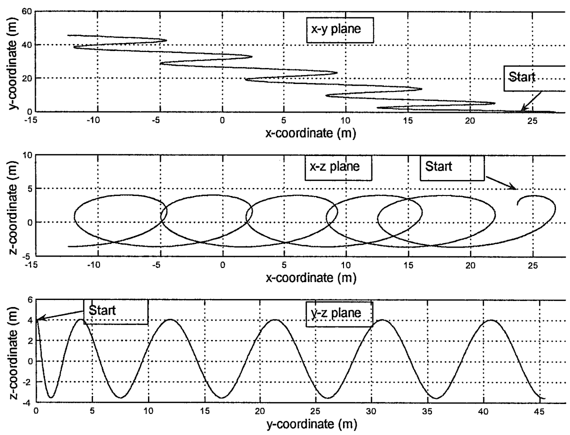

Figure 3: Boat Trajectory projected onto coordinate planes

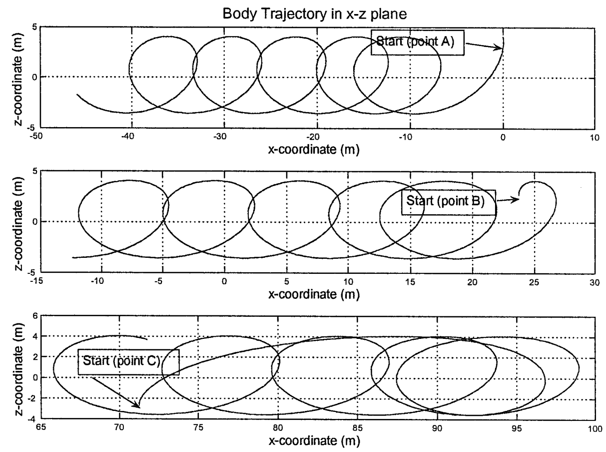

Figure 5: Effect of start positions A,B,C on wave

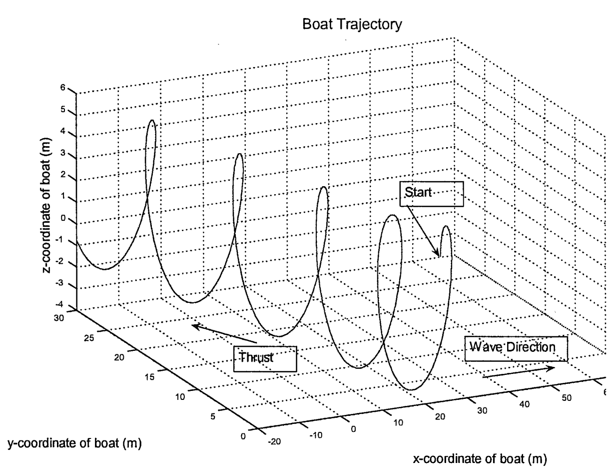

Figure 4: Boat Trajectory in 3D

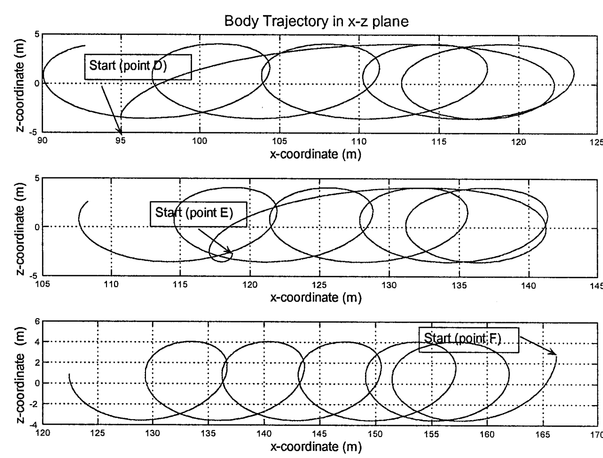

Figure 6: Effect of start positions D,E,F on wave

4 Conclusions

We have presented the equations of motion for the motion of a small floating body in a surface wave using Lagrange's equations. It is assumed that the body is small relative to the wavelength and that its motion normal to the wave profile is negligible. The wave forces on the body are modeled using a standard Morison's equation formulation for a moving body in an accelerated flow.The formulation may be used for predicting the motion of small floating objects such as lifeboats and bergy bits. One useful application is the assessment of lifeboat safety when deployed in severe sea states. Results have been presented for a Stokes' second order wave but it is clearly possible to examine the motion in other known wave profiles. Experimental validation of the model is proposed at the Institute for Ocean Technology, National Research Council, St. John's, NL.

References

[1] Rumer,R; Crissman, R.; Wake, A. "Ice Transport in Great Lakes", *Water Resources and Environmental Engineering Research Report* 79-3, NY State University at Buffalo, 1979.

[2] Grotmaack, R. "Small rigid floating bodies under the influence of water waves", *Research Letters in the Information and Mathematical Sciences*, Vol. 5, Massey University, Auckland, New Zealand, pp.143-157, 2003.

[3] Marchenko, A.V. "The Floating Behaviour of a Small Body Acted Upon by a Surface Wave", *Journal of Applied Mathematics and Mechanics,* Vol. 63, No.3, pp.471-478, 1999.

[4] Sumer, B.M. and Fredsoe, J.*Hydrodynamics Around Cylindrical Structures*, World Scientific Pub. Co., Singapore, pp. 130-131, 1997.

[5] Wilson, J.*Dynamics of Offshore Structures,* 2nd ed., Wiley and Sons Inc., New Jersey, 2003.